P6: Rational Expressions

Rational Number: Can be written as the ratio (fraction) of two integers

Rational Expression: Can be written as the ratio (fraction) of two polynomials.

Examples of rational expressions: $\frac{x^2+4}{x-8}$, $\frac{x^2-7x-13}{5-2x^2}$, $\frac{3}{x^2+5}$

Simplifying a rational expression means reducing the fraction by dividing ("cancelling") factors that appear in both the numerator and denominator.

Ex: $\frac{8}{20} = \frac{\cancel{4}^{1}\cdot 2}{\cancel{4}_{1}\cdot 5} = \boxed{\frac{2}{5}}$

Ex: Simplify $\frac{\cancel{24}^{3}\, x^4 y^3}{\cancel{32}_{4}\, x^5 y} = \boxed{\frac{3y^2}{4x}}$

Ex: Simplify.

$$\frac{x^2+8x+15}{x^2+5x+6} = \frac{\cancel{(x+3)}^{1}(x+5)}{(x+2)\cancel{(x+3)}_{1}} = \boxed{\frac{x+5}{x+2}}$$

throw out: $-2, -3$

Restrictions (Domain Restrictions) (values excluded from domain)

We must exclude (throw out) values that make the denominator 0.

In the previous example, what values should be excluded? $-2, -3$

Ex: Simplify.

$$\frac{x-3}{x^2-7x+12} = \frac{\cancel{x-3}}{\cancel{(x-3)}(x-4)} = \boxed{\frac{1}{x-4}}$$

Multiplying and Dividing Rational Expressions

Ex: $$\frac{2}{3}\cdot\frac{4}{5} = \left(\frac{2}{3}\right)\left(\frac{4}{5}\right) = \frac{2}{3}\left(\frac{4}{5}\right) = \boxed{\frac{8}{15}}$$

Ex: $$\frac{\overset{1}{\cancel{16}}}{\underset{5}{\cancel{15}}}\cdot\frac{\overset{7}{\cancel{21}}}{\underset{2}{\cancel{32}}} = \boxed{\frac{7}{10}}$$

Ex: $$\frac{3x^2y}{\underset{1}{\cancel{4}}z}\cdot\frac{\overset{4}{\cancel{16}}z^5}{xy^3} = \frac{12x^2yz^5}{xy^3z} = \boxed{\frac{12xz^4}{y^2}}$$

Ex: Simplify.

$$\frac{x^2+2x}{x^2-4}\cdot\frac{x^2-4x+4}{x^2-x}$$

[Factor all the numerators and denominators]

$$= \frac{\cancel{x}\cancel{(x+2)}}{\cancel{(x+2)}\cancel{(x-2)}}\cdot\frac{\cancel{(x-2)}(x-2)}{\cancel{x}(x-1)}$$

$$= \boxed{\frac{x-2}{x-1}}$$

Ex: Simplify.

$$(x+2)\cdot\frac{5}{4x^2-16} = \frac{x+2}{1}\cdot\frac{5}{4x^2-16}$$

$$= \frac{x+2}{1}\cdot\frac{5}{4(x^2-4)}$$

$$= \frac{\cancel{x+2}}{1}\cdot\frac{5}{4\cancel{(x+2)}(x-2)}$$

$$= \boxed{\frac{5}{4(x-2)}} \text{ OR } \boxed{\frac{5}{4x-8}}$$

Division:

Ex: $\frac{3}{4} \div \frac{5}{7} = \frac{3}{4}\cdot\frac{7}{5} = \boxed{\frac{21}{20}}$

Dividing by a number is the same as multiplying by its reciprocal

Ex: Simplify.

$$\frac{x^2+2x}{x+5} \div \frac{4-x^2}{3x-6}$$

$$= \frac{x^2+2x}{x+5}\cdot\frac{3x-6}{-x^2+4}$$

$$= \frac{x(x+2)}{x+5}\cdot\frac{3(x-2)}{-1(x^2-4)}$$

$$= \frac{x\cancel{(x+2)}}{x+5}\cdot\frac{3\cancel{(x-2)}}{-1\cancel{(x+2)}\cancel{(x-2)}}$$

$$= \frac{3x}{-1(x+5)} = \boxed{-\frac{3x}{x+5}} = \boxed{\frac{3x}{-x-5}}$$

Best way!

Note:

$$\frac{2}{8} = \frac{1}{4}$$

$$\frac{-2}{8} = -\frac{1}{4}$$

$$\frac{2}{-8} = -\frac{1}{4}$$

Adding and Subtracting Rational Expressions (with the same denominators)

Recall:

multiplication: $\frac{3}{5} \cdot \frac{4}{5} = \boxed{\frac{12}{25}}$

Addition:
(same denominators) $\frac{3}{5} + \frac{4}{5} = \frac{3+4}{5} = \boxed{\frac{7}{5}}$

LCD: 35
(Least Common Denominator)

Addition
(different denominators): $\frac{3}{5} + \frac{4}{7}$

$$= \frac{3}{5}\left(\frac{7}{7}\right) + \frac{4}{7}\left(\frac{5}{5}\right)$$

$$= \frac{21}{35} + \frac{20}{35} = \frac{21+20}{35} = \boxed{\frac{41}{35}}$$

Ex: $\frac{1}{6x} + \frac{7}{6x} = \frac{1+7}{6x} = \frac{\cancel{8}^{4}}{{}_{3}\cancel{6}x} = \boxed{\frac{4}{3x}}$

Ex $\frac{3}{x+5} - \frac{10}{x+5} = \frac{3-10}{x+5} = \boxed{\frac{-7}{x+5}} = \boxed{-\frac{7}{x+5}}$

Ex: $\frac{4x}{x+1} + \frac{x+3}{x+1} = \frac{4x+x+3}{x+1} = \boxed{\frac{5x+3}{x+1}}$

Ex. $\frac{3y-5}{y+4} - \frac{5y-7}{y+4}$

$$= \frac{3y-5}{y+4} + \frac{-5y+7}{y+4} = \frac{3y-5-5y+7}{y+4}$$

$$= \boxed{\frac{-2y+2}{y+4}} = \boxed{\frac{-2(y-1)}{y+4}}$$

could write it either way.

Ex. $\dfrac{2x+1}{x^2-1} + \dfrac{x+2}{x^2-1}$

$= \dfrac{2x+1+x+2}{x^2-1} = \dfrac{3x+3}{x^2-1} = \dfrac{3\cancel{(x+1)}}{(x-1)\cancel{(x+1)}} = \boxed{\dfrac{3}{x-1}}$

Adding and Subtracting when they don't already have common denominators.

$\dfrac{7x}{30} - \dfrac{11x}{12}$ LCD: 60

$= \dfrac{7x}{30}\left(\dfrac{2}{2}\right) + \dfrac{-11x}{12}\left(\dfrac{5}{5}\right)$

$= \dfrac{14x}{60} + \dfrac{-55x}{60} = \dfrac{14x-55x}{60} = \boxed{\dfrac{-41x}{60}}$

Ex: $\dfrac{5y}{6x^2} - \dfrac{y}{4x}$ LCD: $12x^2$

$= \dfrac{5y}{6x^2}\left(\dfrac{2}{2}\right) + \dfrac{-y}{4x}\left(\dfrac{3x}{3x}\right)$

$= \dfrac{10y}{12x^2} + \dfrac{-3xy}{12x^2} = \boxed{\dfrac{10y-3xy}{12x^2}} = \boxed{\dfrac{y(10-3x)}{12x^2}}$

both OK

Ex: $\dfrac{2}{xy^4} + \dfrac{3}{x^3y^2}$ LCD: x^3y^4

$= \dfrac{2}{xy^4}\left(\dfrac{x^2}{x^2}\right) + \dfrac{3}{x^3y^2}\left(\dfrac{y^2}{y^2}\right)$

$= \dfrac{2x^2}{x^3y^4} + \dfrac{3y^2}{x^3y^4} = \boxed{\dfrac{2x^2+3y^2}{x^3y^4}}$

Ex: $\dfrac{3}{x-3} - \dfrac{6}{x+5}$ LCD: $(x-3)(x+5)$

$= \dfrac{3}{x-3}\left(\dfrac{x+5}{x+5}\right) + \dfrac{-6}{x+5}\left(\dfrac{x-3}{x-3}\right)$

$= \dfrac{3x+15}{(x-3)(x+5)} + \dfrac{-6x+18}{(x+5)(x-3)} = \boxed{\dfrac{-3x+33}{(x+5)(x-3)}}$

$= \boxed{\dfrac{-3(x-11)}{(x+5)(x-3)}}$

Simplifying Complex Fractions

Complex Fraction: Fractions within a fraction

Example: Done on board…no online notes.