9.2: Critical-Value Approach to Hypothesis Testing

How do we decide whether the null hypothesis is tenable? Or whether there is evidence in favor of the alternative hypothesis? There are two approaches. In both approaches, we set the α -level and state the null and alternative hypotheses before the sample data is analyzed. Then:

Approach #1: *p*-value approach to hypothesis testing (we'll omit):

First, we calculate the test statistic. If we are interested in testing a hypothesis about the population mean, then the test statistic is: $\dfrac{\overline{x}-\mu_0}{\sigma_{\overline{x}}}$

Then we use the test statistic to calculate a *p*-value, often by using technology. The *p*-value, called the *observed significance level*, is the probability of obtaining a sample statistic at least as extreme as that observed in the sample, given that the null hypothesis is true.

A result is said to be *statistically significant* if the *p*-value is less than the predetermined α level.

If the *p*-value is less then or equal to α , we reject the null hypothesis.
If the *p*-value is greater than α , we fail to reject the null hypothesis.

Note: We cannot prove the null hypothesis is true. We never accept the null hypothesis. The closest we can come to accepting the null hypothesis is to conclude that there is not enough evidence to reject it.

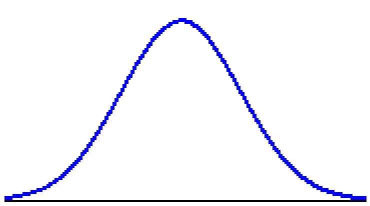
Approach #2: Critical-value approach to hypothesis testing (we'll use):

Determine the rejection region, nonrejection region, and critical value(s).

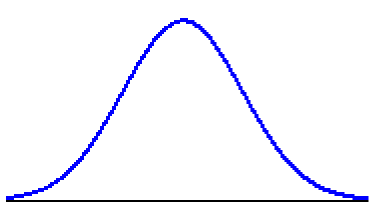
Rejection region: Set of values for the test statistic that lead to rejection of the null hypothesis.

Nonrejection region: Set of values for the test statistic that do not lead to rejection of the null hypothesis.

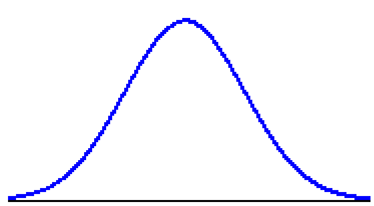
Critical value(s): Value(s) of the test statistic that separate the rejection region from the nonrejection region. (The critical values are considered to be included in the rejection region.) We'll use a table to find them.



Two-Tailed



Left-Tailed



Right-Tailed

Calculate the test statistic and compare it to the critical value.

- If the test statistic falls in the rejection region, reject the null hypothesis.
- If the test statistic falls in the nonrejection region, do not reject the null hypothesis.

Obtaining critical values for a one-mean *z*-test:

We test a hypothesis about one population mean, in which the null hypothesis H_0 is $\mu \neq \mu_0$, $\mu < \mu_0$, or $\mu > \mu_0$.

The one-mean *z*-test is used when the population standard deviation is known and the variable under consideration is normally distributed. The test statistic is

$$z = \frac{\overline{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}.$$

Example 1: Determine the critical value(s) for a two-tailed *z*-test with $\alpha = 0.01$. Sketch the rejection region.

Example 2: Determine the critical value(s) for a right-tailed *z*-test with $\alpha = 0.10$. Sketch the rejection region.

Example 3: Determine the critical value(s) for a left-tailed *z*-test with $\alpha = 0.05$. Sketch the rejection region.

<u>Example 4:</u> Suppose that a random sample of 40 fifth graders in a particular region is given a national standardized test. On this particular test, a score of 100 represents mastery of the subject area. Based on historical records, we can assume that the standard deviation of fifth graders' scores is 18. The mean score for this sample of fifth graders was 95. Can it be concluded that this group of students differed from the mastery score on their knowledge of the subject area covered by the test? Use a significance level of 0.05.

Example 5: The concentration of an airborne particulate was measured at 42 randomly chosen industrial workplaces in a certain region. The mean of those measurements was 335 parts per million (ppm). Suppose it can be assumed that the standard deviation of the measurements at all such workplaces is 80 ppm. The Environmental Protection Agency (EPA) considers a level of 300 ppm or below to be safe. Can it be concluded that, as a group, industrial workplaces in this region have a level of this pollutant that is higher than what is considered safe? Use a significance level of 0.01.