9.3: Hypothesis Tests for One Population Mean, Sigma Unknown

In practice, when we are using a sample to make inferences about the population mean, it is rare for us to know the population standard deviation.

Instead, we must use the sample standard deviation, *s*, as a point estimate of the population standard deviation, σ.

When using *s* as an estimate for σ, we cannot use a *z*-test, because $\frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$ is not normally distributed.

The *t*-test for one population mean:

When using *s* as an estimate for σ, we use the Student *t*-distribution.

In order to use this procedure, we need to know (or be able to reasonably assume) that the variable of interest follows a normal distribution, or we must have a large sample size ($n \geq 30$).

In addition, the sample should be randomly obtained, observations within the sample must be independent of one another. This means that if we have a sample size that is more than 5% of the population, we should multiply the standard error by a finite population correction factor, $\sqrt{\frac{N-n}{n-1}}$. (In this class, I do not anticipate that we will encounter this situation.)

Hypothesis Testing for a Population Mean:

Step 1: Determine the significance level α .

Step 2: Determine the null and alternative hypotheses.

Two-Tailed Test (most common)	Left-Tailed Test (rare)	Right-Tailed Test (rare)
$H_0 : \mu = \mu_0$ $H_1 : \mu \neq \mu_0$	$H_0 : \mu = \mu_0$ $H_1 : \mu < \mu_0$	$H_0 : \mu = \mu_0$ $H_1 : \mu > \mu_0$
Rejection Region	Rejection Region	Rejection Region

Note: One tailed tests assume that the scenario not listed ($\mu > \mu_0$ for a left-tailed test or $\mu < \mu_0$ for a right-tailed test) is not possible or is of zero interest.

Step 3: Use your α level and hypotheses, sketch the rejection region.

Step 4: Compute the test statistic $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$.

Step 5: Use a table (Table IV, on page A-13) to determine the critical value for *t* associated with your rejection region.

Step 6: Determine whether the value of *t* calculated from your sample (in Step 3) is in the rejection region.

- If *t* is in the rejection region, reject the null hypothesis.
- If *t* is not in the rejection region, do not reject the null hypothesis.

Step 7: State your conclusion.

Example 1: The normal human body temperature is widely accepted to be $98.6° F$ and can be assumed to follow a normal distribution. A medical researcher wants to know whether a certain geographical community of Native Alaskans has a mean body temperature of $98.6° F$. A sample of 20 members of the Native Alaskan geographical community resulted in a mean body temperature of $98.3° F$ with a standard deviation of $0.7° F$. Perform an appropriate hypothesis test at the 95% confidence level.

Example 2: The average amount of lead in the blood of young children is 2 micrograms per deciliter (mcg/dL). A city has recently changed its water supply, and there have been widespread reports of increased lead levels in the water. A concerned doctor wants to dig into the city's medical records to find out whether the children in the city have blood lead levels above the average level of 2 mcg/dL. In a sample of 35 children, she found a mean lead level of 2.60 mcg/dL with a standard deviation of 1.9 mcg/dL. Perform an appropriate hypothesis test at the 95% confidence level.

Example 3: Suppose a manufacturer claims on the label that a package contains 8 ounces of potato chips. A customer (or a FDA analyst) buys 50 bags of chips, weighs them on a high-accuracy scale, and obtains a sample mean of 7.89 ounces with a sample standard deviation of 0.2 ounces. Does this sample provide evidence that the manufacturer's labeling may be inaccurate? Use the $\alpha = 0.10$ level of significance. Assume that the package weights are normally distributed.

Example 4: Suppose a manufacturer claims on the label that a package contains 8 ounces of potato chips. Again, a customer (or a FDA analyst) wonders whether the package size is accurate. This time, the analysist only buys 10 bags of chips, and obtains a sample mean of 7.89 ounces with a sample standard deviation of 0.2 ounces. Does this sample provide evidence that the manufacturer's labeling may be inaccurate? Use the $\alpha = 0.10$ level of significance. Assume that the package weights are normally distributed.