

Factoring Review

GCF's and Grouping

1. $8x^2y - 4xy^3$ **GCF** 2. $-4x^3 - 8x^2 + 24x$ **GCF**
 $= 4xy(2x - y^2)$ $= -4x(x^2 + 2x - 6)$

3. $3x(a+2b) + y(a+2b)$ 4. $12x^2(3x-y)^2 - 6x(3x-y)$
 $= (a+2b)(3x+y)$ $= 6x(3x-y)[2x(3x-y) - 1]$
 $a+2b$ is GCF **$6x(3x-y)$ is GCF**
 $= 6x(3x-y)(6x^2 - 2xy - 1)$

5. $24x^3y^4 - 12x^2y^3 - 8xy^5$ 6. $ax - 2xb + ay - 2by$
 $= 4xy^3(6x^2y - 3x - 2y^2)$ **use grouping**
GCF → $= x(a-2b) + y(a-2b)$
 $= (a-2b)(x+y)$

7. $x^3 - 4x^2 + 2xy - 8y$ **use grouping**
 $= x^2(x-4) + 2y(x-4)$
 $= (x-4)(x^2 + 2y)$

8. $2bx^3 - 4x^3 - 8bx^2y + 16x^2y$ **GCF first**
 $= 2x^2(bx - 2x - 4by + 8y)$ **grouping**
 $= 2x^2[x(b-2) - 4y(b-2)]$
 $= 2x^2(b-2)(x-4y)$

Factoring Trinomials

$$9. x^2 - 2x - 3 \\ = (x-3)(x+1)$$

$$11. x^2 - 7x - 60 \\ = (x-12)(x+5)$$

$$13. -x^2 - 17x - 72 \\ \text{make sure } \rightarrow = -1(x^2 + 17x + 72) \\ \text{leading coefficient is positive} \\ = -1(x+8)(x+9)$$

$$15. 5x^2 - 14x - 3 \\ = (5x+1)(x-3)$$

$$10. x^2 - 5x + 6 \\ = (x-2)(x-3)$$

$$12. 2x^2 - 4x - 96 \\ = 2(x^2 - 2x - 48) \text{ GCF first} \\ = 2(x-8)(x+6)$$

$$14. 3x^2 + 10x - 8 \\ = (3x-2)(x+4)$$

$$16. 2x^2 + 9x - 5 \\ = (2x-1)(x+5)$$

$$17. 6x^2 - 19x - 20 \\ = (6x+5)(x-4)$$

$$18. -4x^3 + 10x^2 + 6x \text{ GCF first} \\ = -2x(2x^2 - 5x - 3) \\ = -2x(2x+1)(x-3)$$

$$19. 12x^2 + xy - 35y^2 \\ = (3x - 5y)(4x + 7y)$$

$$20. 20a^3 - 42a^2b - 20ab^2 \\ = 2a(10a^2 - 21ab - 10b^2) \\ = 2a(2a - 5b)(5a + 2b) \text{ GCF first}$$

Factoring Binomials

$$a^2 - b^2 = (a+b)(a-b) \quad a^2 + b^2 \Rightarrow \text{prime (unless a GCF)}$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$21. \quad x^2 - 36 \quad a^2 - b^2 \\ = (x+6)(x-6)$$

$$22. \quad 4x^2 - 81 \quad a^2 - b^2 \\ = (2x+9)(2x-9)$$

$$23. \quad 16x^2 - 25y^2 \quad a^2 - b^2 \\ = (4x+5y)(4x-5y)$$

$$24. \quad 16x^4 - 1 \quad a^2 - b^2 \\ = (4x^2+1)(4x^2-1) \quad a^2 - b^2 \\ = (4x^2+1)(2x+1)(2x-1)$$

$$25. \quad 18x^2 - 50 \quad \text{GCF first} \\ = 2(9x^2 - 25) \quad \text{then } a^2 - b^2 \\ = 2(3x+5)(3x-5)$$

$$26. \quad x^3 - 64 \quad a^3 - b^3 \\ = (x-4)(x^2+4x+16)$$

$$27. \quad x^3 + 27 \quad a^3 + b^3 \\ = (x+3)(x^2 - 3x + 9)$$

$$28. \quad 8x^3 - 125y^3 \quad a^3 - b^3 \\ = (2x-5y)(4x^2 + 10xy + 25y^2)$$

$$29. \quad 54x^3y - 16y^4 \quad \text{GCF } 2y \\ = 2y(27x^3 - 8y^3) \quad a^3 - b^3$$

$$30. \quad (x+1)^3 + y^3 \quad a^3 + b^3 \\ = (x+1+y)((x+1)^2 - (x+1)y + y^2) \\ = 2y(3x-2y)(9x^2 + 6xy + 4y^2) \\ = (x+1+y)(x^2 + 2x + 1 - xy - y + y^2)$$

"Harder" Grouping Problems

$$\begin{aligned} 31. \quad & x^2y + 2x^2 - 4y - 8 \\ & = x^2(y+2) - 4(y+2) \quad \text{grouping} \\ & = (y+2)(x^2-4) = (y+2)(x+2)(x-2) \\ & \quad \quad \quad a^2-b^2 \end{aligned}$$

$$\begin{aligned} 32. \quad & 4x^3 + 8x^2 - 9x - 18 \\ & = 4x^2(x+2) - 9(x+2) \quad \text{grouping} \\ & = (x+2)(4x^2-9) = (x+2)(2x+3)(2x-3) \\ & \quad \quad \quad a^2-b^2 \end{aligned}$$

$$\begin{aligned} 33. \quad & \underline{x^2 + 2x + 1} - \underline{4y^2} \quad \text{group 3 terms and 1} \\ & = (x+1)^2 - 4y^2 = [(x+1)+2y][(x+1)-2y] \\ & \quad \quad \quad = (x+1+2y)(x+1-2y) \end{aligned}$$

$$\begin{aligned} 34. \quad & \underline{4x^2 - 20xy + 25y^2} - \underline{81} \quad \text{group 3 terms and 1} \\ & = (2x-5y)^2 - 81 \\ & = [(2x-5y)+9][(2x-5y)-9] \\ & = (2x-5y+9)(2x-5y-9) \end{aligned}$$

$$\begin{aligned}
 35. \quad & 5x^3y - 40y - 6x^3 + 48 \\
 & = 5y(x^3 - 8) - 6(x^3 - 8) \quad \text{grouping} \\
 & = (x^3 - 8)(5y - 6) \\
 & = (x - 2)(x^2 + 2x + 4)(5y - 6)
 \end{aligned}$$

$a^3 - b^3$ →

Mixed Review of Factoring

$$36. \quad 25a^2 - b^2 = (5a + b)(5a - b)$$

$a^2 - b^2$

$$37. \quad x^2 - 10x + 9 = (x - 9)(x - 1)$$

$$38. \quad 36x^2 - 60x + 25 = (6x - 5)^2$$

perfect square trinomial

$$\begin{aligned}
 39. \quad & 16x^3y - 54y^4 \\
 & \xrightarrow{\text{GCF}} 2y(8x^3 - 27y^3) \\
 & \xrightarrow{a^3 - b^3} 2y(2x - 3y)(4x^2 + 6xy + 9y^2)
 \end{aligned}$$

$$40. \quad 14x^2 - 41x + 15 = (2x - 5)(7x - 3)$$

$$41. \quad 3x^2 - 5x + 1 \quad \text{prime}$$

$$\begin{aligned}
 42. \quad & x^3 - 2x^2 - 16x + 32 \\
 & = x^2(x-2) - 16(x-2) \quad \text{grouping} \\
 & = (x-2)(x^2-16) \leftarrow a^2 - b^2 \\
 & = (x-2)(x+4)(x-4)
 \end{aligned}$$

$$\begin{aligned}
 43. \quad & -5x^2 + 25x + 30 \\
 \text{GCF} & \text{ 1st} \\
 & = -5(x^2 - 5x - 6) \\
 & = -5(x-6)(x+1)
 \end{aligned}$$

$$\begin{aligned}
 44. \quad & 10x^2 - 17x - 20 \\
 & = (2x-5)(5x+4)
 \end{aligned}$$

$$\begin{aligned}
 45. \quad & 4x^4 - 25x^2 + 36 \\
 & = (4x^2 - 9)(x^2 - 4) \leftarrow \text{both are } a^2 - b^2 \\
 & = (2x+3)(2x-3)(x+2)(x-2)
 \end{aligned}$$

$$\begin{aligned}
 46. \quad & 20x^2 - 18xy - 12x^2y + 30x \\
 \text{GCF} & \rightarrow 2x(10x - 9y - 6xy + 15) \\
 & = 2x(10x + 15 - 6xy - 9y) \\
 & = 2x[5(2x+3) - 3y(2x+3)] \\
 & = 2x(2x+3)(5-3y)
 \end{aligned}$$

Rearrange terms then use grouping