Math 1316 Final Exam Review

Convert the angle to a decimal in degrees. Round the answer to two decimal places.

1) 291°26'12"

Answer: 291.44°

Convert the angle to D° M' S" form. Round the answer to the nearest second.

2) 183.82°

Answer: 183°49'12"

Solve the problem.

3) For a circle of radius 4 feet, find the arc length s subtended by a central angle of 30°. Round to the nearest hundredth.

Answer: 2.09 ft

Convert the angle to degrees or radians, as appropriate. Express the answer as multiple of π.

4) a) 54° (in terms of π) b) $\frac{12\pi}{7}$

Answer: $\frac{3\pi}{10}$ Answer: 308.57°

If A denotes the area of the sector of a circle of radius r formed by the central angle θ, find the missing quantity. If necessary, round the answer to two decimal places.

5) r = 7 feet, A = 56 square feet, θ = ?

Answer: 2.29 radians

Solve the problem.

6) A pick–up truck is fitted with new tires which have a diameter of 40 inches. How fast will the pick–up truck be moving when the wheels are rotating at 395 revolutions per minute? Express the answer in miles per hour rounded to the nearest whole number.

Answer: 47 mph

Use Fundamental Identities to find the exact value of the expression. Do not use a calculator.

7) a) $\sec^2 25° - \tan^2 25°$ b) $\tan 5° - \frac{\cos 85°}{\cos 5°}$

Answer: 1 Answer : (

Use the definition or identities to find the exact value of the indicated trigonometric function of the acute angle θ.

8) $\tan\theta = \frac{7}{\sqrt{15}}$ Find $\sin\theta$ and $\cos\theta$.

Answer: $\sin\theta = \frac{7}{8}$, $\cos\theta = \frac{\sqrt{15}}{8}$

Solve the problem.

9) Given the approximation $\sin 31° \approx 0.52$, use trigonometric identities to find the approximate value of $\cot 31°$. If necessary, round the answer to two decimal places.

Answer: 1.66

f(x) = sin x, g(x) = cos x, h(x) = 2x, and $p(x) = \frac{x}{2}$. Find the value of the following.

10) $(p \circ g)\left(\frac{\pi}{4}\right)$

Answer: $\frac{\sqrt{2}}{4}$

Solve the problem.

11) A boat leaves the entrance of a harbor and travels 83 miles on a bearing of N 19° E. How many miles north and how many miles east from the harbor has the boat traveled? Round to the nearest tenth of a mile.

Answer: 78.5 miles north and 27 miles east

With the information given, find the <u>exact</u> value of the indicated trigonometric function.

12) a) (4, –2) Find csc θ. b) cot 720° c) $\csc\frac{-2\pi}{3}$ d) $\cos\frac{8\pi}{3}$

Answer: $-\sqrt{5}$ Ans: undef Ans: $-\frac{2\sqrt{3}}{3}$ Ans: $-\frac{1}{2}$

Find the <u>exact</u> value of the indicated trigonometric function of θ.

13) a) $\cos\theta = \frac{2}{5}$, $\tan\theta < 0$ Find sin θ. b) $\tan\left(\frac{9\pi}{4}\right) - \cos\left(\frac{9\pi}{4}\right)$

Answer: $-\frac{\sqrt{21}}{5}$ Ans: $\frac{2-\sqrt{2}}{2}$

14) If f(θ) = tan θ and f(a) = 5, find the exact value of f(–a).

Answer: –5

Without graphing the function, determine its amplitude and period.

15) $y = \frac{7}{6}\sin\left(-\frac{6\pi}{5}x\right)$

Answer: period : $\frac{5}{3}$ amplitude: $\frac{7}{6}$

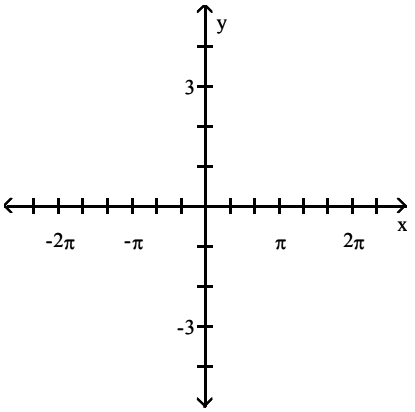
Find (i) the amplitude, (ii) the period, and (iii) the phase shift.

16) $y = -\frac{1}{2}\sin(4x + 3\pi)$

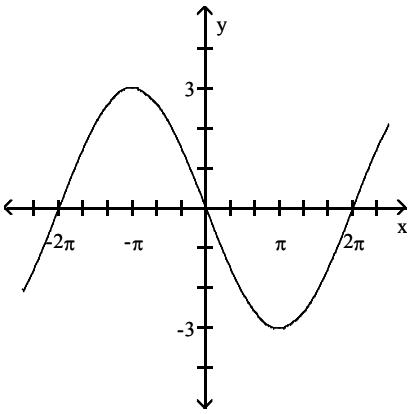
Answer: (i) $\frac{1}{2}$ (ii) $\frac{\pi}{2}$ (iii) $-\frac{3\pi}{4}$

Use transformations to graph the function.

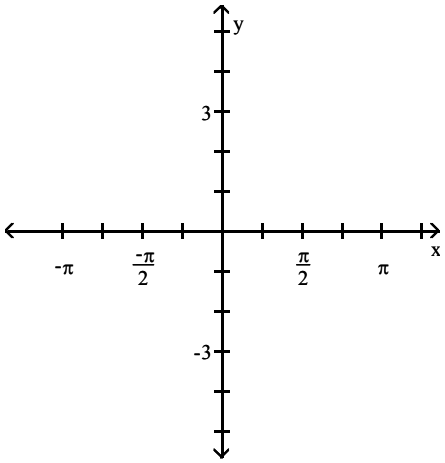
17) a) $y = -3 \sin\left(\frac{1}{2}x\right)$



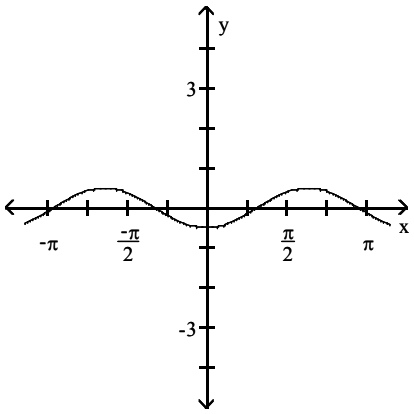
Answer:



b) $y = -\frac{1}{2} \cos\left(\frac{\pi}{2}x\right)$

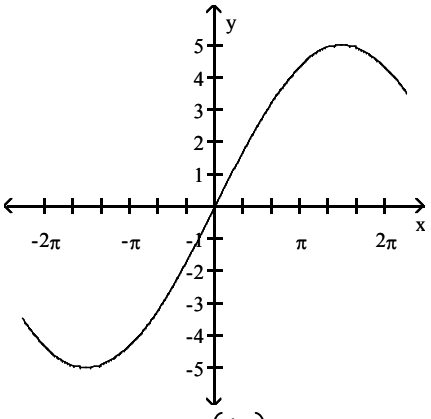


Answer



Find an equation for the graph.

18)



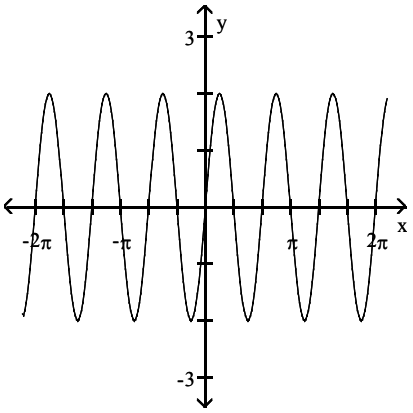
Answer: $y = 5 \sin\left(\frac{1}{3}x\right)$

Match the given function to its graph.

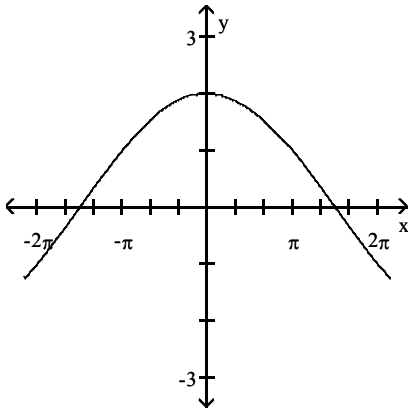
19) 1) $y = 2\sin(3x)$ 2) $y = 2\sin\left(\frac{1}{3}x\right)$

3) $y = 2\cos(3x)$ 4) $y = 2\cos\left(\frac{1}{3}x\right)$

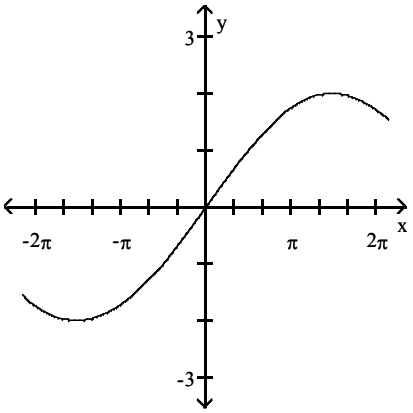
A



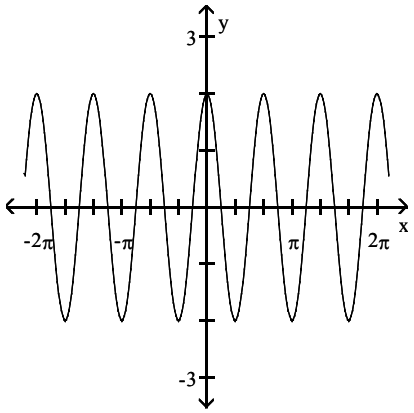
B



C



D



Answer: 1A, 2C, 3D, 4B

Write the equation of a sine function that has the given characteristics.

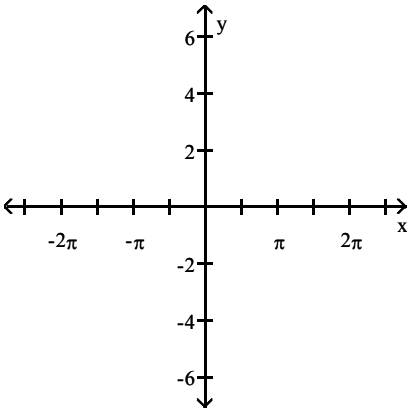
20) Amplitude: 5
Period: 3π

Phase Shift: $\frac{\pi}{3}$

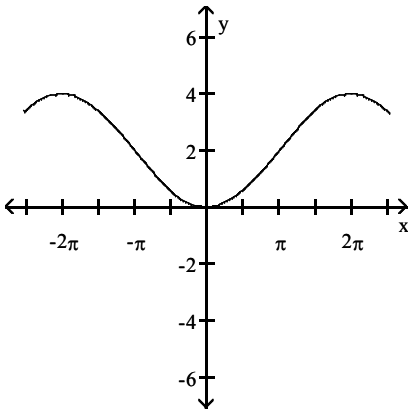
Answer: $y = 5\sin\left(\frac{2}{3}x - \frac{2}{9}\pi\right)$

Graph the sinusoidal function using key points.

21) $y = -2\cos\left(\frac{1}{2}x\right) + 2$



Answer:



Find the exact value of the expression.

22) a) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ b) $\sin^{-1}\left(\sin\frac{5\pi}{4}\right)$ c) $\csc\left(\cos^{-1}\frac{\sqrt{3}}{2}\right)$ d) $\cot\left[\cos^{-1}\left(-\frac{7}{25}\right)\right]$

Answer: $\frac{5\pi}{6}$ Answer: $-\frac{\pi}{4}$ Answer: 2 Answer: $-\frac{7}{24}$

Find the inverse function f^{-1} of the function f.

23) $f(x) = 7\tan(10x - 5)$

Answer: $f^{-1}(x) = \frac{1}{10}\left[\tan^{-1}\left(\frac{x}{7}\right) + 5\right]$

Find the exact solution of the equation.

24) $-3\sin^{-1}(2x) = \pi$

Answer: $x = -\frac{\sqrt{3}}{4}$

Establish the identity.

25) $\sec u + \tan u = \frac{\cos u}{1 - \sin u}$

Answer: $\sec u + \tan u = \frac{1}{\cos u} + \frac{\sin u}{\cos u} = \frac{1 + \sin u}{\cos u} = \frac{1 + \sin u}{\cos u} \cdot \frac{1 - \sin u}{1 - \sin u} = \frac{1 - \sin^2 u}{\cos u(1 - \sin u)} = \frac{\cos^2 u}{\cos u(1 - \sin u)} = \frac{\cos u}{1 - \sin u}$

26) $\dfrac{\sin\alpha+\sin\beta}{\csc\alpha+\csc\beta}=\sin\alpha\sin\beta$

Answer: $\dfrac{\sin\alpha+\sin\beta}{\csc\alpha+\csc\beta}=\dfrac{\sin\alpha+\sin\beta}{\frac{1}{\sin\alpha}+\frac{1}{\sin\beta}}=\dfrac{\sin\alpha+\sin\beta}{\frac{\sin\beta+\sin\alpha}{\sin\alpha\sin\beta}}=(\sin\alpha+\sin\beta)\cdot\dfrac{\sin\alpha\sin\beta}{\sin\beta+\sin\alpha}=\sin\alpha\sin\beta$

Find the exact value of the expression.

27) a) $\tan\dfrac{\pi}{12}$ b) $\cos 35^\circ\cos 25^\circ-\sin 35^\circ\sin 25^\circ$ c) $\tan 165^\circ$

Answer: $2-\sqrt{3}$ Answer: $\dfrac{1}{2}$ Answer: $-2+\sqrt{3}$

Find the exact value under the given conditions.

28) $\tan\alpha=\dfrac{15}{8},\ \pi<\alpha<\dfrac{3\pi}{2};\quad \cos\beta=-\dfrac{24}{25},\ \dfrac{\pi}{2}<\beta<\pi$ Find $\sin(\alpha+\beta)$.

Answer: $\dfrac{304}{425}$

Establish the identity.

29) $\sec\left(\dfrac{\pi}{2}+u\right)=-\csc u$

Answer: $\sec\left(\dfrac{\pi}{2}+u\right)=\dfrac{1}{\cos(\pi/2)\cos u-\sin(\pi/2)\sin u}=\dfrac{1}{0\cdot\cos u-1\cdot\sin u}=-\csc u.$

Find the exact value of the expression.

30) $\cos\left(\sin^{-1}\dfrac{1}{3}-\tan^{-1}\dfrac{1}{2}\right)$

Answer: $\dfrac{4\sqrt{10}+\sqrt{5}}{15}$

Use the information given about the angle θ, $0\le\theta\le 2\pi$, to find the exact value of the indicated trigonometric function.

31) a) $\cos\theta=\dfrac{20}{29},\ \dfrac{3\pi}{2}<\theta<2\pi$ Find $\sin(2\theta)$. b) $\tan\theta=\dfrac{12}{5},\ \csc\theta<0$ Find $\cos\dfrac{\theta}{2}$.

Answer: $-\dfrac{840}{841}$ Answer: $-\dfrac{2\sqrt{13}}{13}$

Solve the equation on the interval $0\le\theta<2\pi$.

32) a) $\tan(2\theta)=-1$ b) $2\cos\theta+2\sqrt{3}=\sqrt{3}$

Answer: $\dfrac{3\pi}{8},\dfrac{7\pi}{8},\dfrac{11\pi}{8},\dfrac{15\pi}{8}$ Answer: $\dfrac{5\pi}{6},\dfrac{7\pi}{6}$

33) a) $2\cos^2\theta-3\cos\theta+1=0$ b) $\sin^2\theta-\cos^2\theta+\cos\theta=0$

Answer: $0,\ \dfrac{\pi}{3},\dfrac{5\pi}{3}$ Answer: $0,\dfrac{2\pi}{3},\dfrac{4\pi}{3}$

34) $\cos(2\theta)=\sin\theta$

Answer: $\dfrac{\pi}{6},\dfrac{5\pi}{6},\dfrac{3\pi}{2}$

Use a calculator to solve the equation. Round the solution(s) to two decimal places if necessary.

35) $\cos x + \sin x = 2x$

Answer: 0.70

Use a calculator to solve the equation on the interval $0 \le \theta < 2\pi$. Round the answer to two decimal places.

36) $\sin \theta = -0.24$

Answer: 3.38, 6.04

Solve the <u>right</u> triangle using the information given. Round answers to two decimal places, if necessary.

37) $b = 6$, $A = 30°$; Find a, c, and B.

Answer: $a = 3.46$
$c = 6.93$
$B = 60°$

Solve the problem.

38) John (whose line of sight is 6 ft above horizontal) is trying to estimate the height of a tall oak tree. He first measures the angle of elevation from where he is standing as 35°. He walks 30 feet closer to the tree and finds that the angle of elevation has increased by 12°. Estimate the height of the tree rounded to the nearest whole number.

Answer: 67 ft

Solve the triangle.

39) $B = 20°$, $C = 60°$, $a = 5$

Answer: $A = 100°$, $b = 1.74$, $c = 4.4$

Two sides and an angle are given. Determine whether the given information results in one triangle, two triangles, or no triangle at all. Solve any triangle(s) that results.

40) $B = 15°$, $b = 9.3$, $a = 17.97$

Answer: two triangles
$A_1 = 30°$, $C_1 = 135°$, $c_1 = 25.4$ or
$A_2 = 150°$, $C_2 = 15°$, $c_2 = 9.3$

Solve the problem.

41) It is 4.7 km from Lighthouse A to Port B. The bearing of the port from the lighthouse is N7°E. A ship has sailed due west from the port and its bearing from the lighthouse is N31°E. How far has the ship sailed from the port? Round your answer to the nearest 0.1 km.

Answer: 3.7 km

Solve the triangle.

42) $b = 2$, $c = 4$, $A = 95°$

Answer: $a = 4.63$, $B = 25.5°$, $C = 59.5°$

43) $a = 9$, $b = 6$, $c = 5$

Answer: $A = 109.5°$, $B = 38.9°$, $C = 31.6°$

Solve the problem.

44) A ladder leans against a building that has a wall slanting away from the ladder at an angle of 96° with the ground. If the bottom of the ladder is 23 feet from the base of the wall and it reaches a point 52 feet up the wall, how tall is the ladder to the nearest foot?

Answer: 59 ft

Find the area of the triangle. If necessary, round the answer to two decimal places.

45) $A = 23°$, $b = 9$, $c = 2$

Answer: 3.52

Solve the problem.

46) A room in the shape of a triangle has sides of length 6 yd, 10 yd, and 13 yd. If carpeting costs $17.50 a square yard and padding costs $3.25 a square yard, how much to the nearest dollar will it cost to carpet the room, assuming that there is no waste?

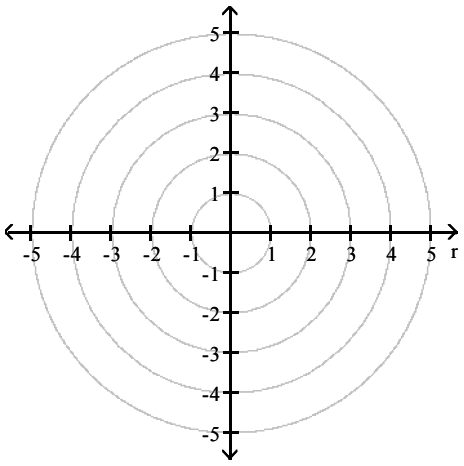
Answer: $598

47) Plot the point $\left(4, \frac{\pi}{6}\right)$ and find other polar coordinates (r, θ) of the point for which:

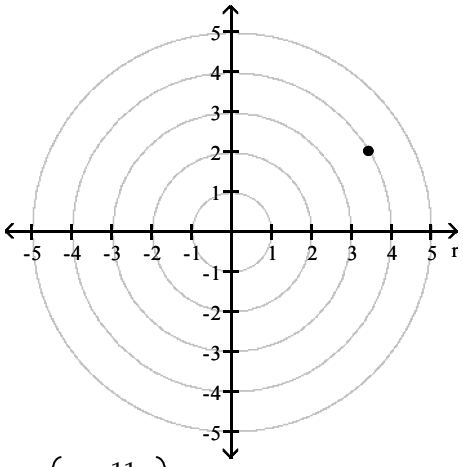
(a) $r > 0$, $-2\pi \le \theta < 0$

(b) $r < 0$, $0 \le \theta < 2\pi$

(c) $r > 0$ $2\pi \le \theta < 4\pi$



Answer:



(a) $\left(4, -\frac{11\pi}{6}\right)$

(b) $\left(-4, \frac{7\pi}{6}\right)$

(c) $\left(4, \frac{13\pi}{6}\right)$

The polar coordinates of a point are given. Find the rectangular coordinates of the point.

48) $\left(5, -\frac{4\pi}{3}\right)$

Answer: $\left(-\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$

The rectangular coordinates of a point are given. Find polar coordinates for the point.

49) (2, –2)

Answer: $\left(2\sqrt{2}, -\frac{\pi}{4}\right)$

The letters r and θ represent polar coordinates. Write the equation using rectangular coordinates (x, y).

50) $r = \frac{5}{1 + \cos\theta}$

Answer: $y^2 = 25 - 10x$

The letters x and y represent rectangular coordinates. Write the equation using polar coordinates (r, θ).

51) $2xy = 5$

Answer: $r^2 \sin 2\theta = 5$

Find $\frac{z}{w}$. Leave your answer in polar form.

52) $z = \sqrt{3}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

$w = \sqrt{6}\left(\cos\frac{9\pi}{4} + i\sin\frac{9\pi}{4}\right)$

Answer: $\frac{\sqrt{2}}{2}\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

53) $z = 1 + i$

$w = \sqrt{3} - i$

Find zw. Leave answer in polar form.

Answer: $2\sqrt{2}(\cos 15° + i\sin 15°)$

Write the expression in the standard form a + bi.

54) $(1 - i)^{10}$

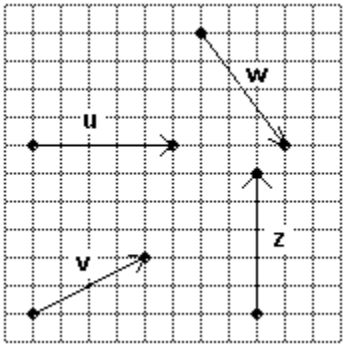
Answer: –32i

Find all the complex roots. Leave your answers in polar form with the argument in degrees.

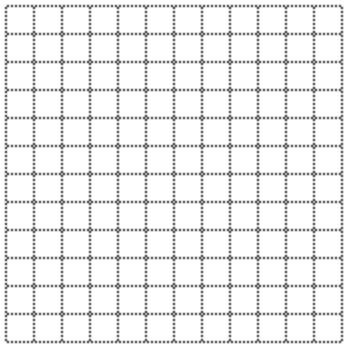
55) The complex fourth roots of –16

Answer: $2(\cos 45° + i\sin 45°)$, $2(\cos 135° + i\sin 135°)$, $2(\cos 225° + i\sin 225°)$, $16(\cos 315° + i\sin 315°)$

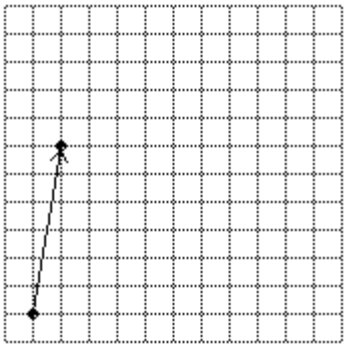
Use the vectors in the figure below to graph the following vector.



56) **v – w**



Answer:



The vector v has initial point P and terminal point Q. Write v in the form ai + bj; that is, find its position vector.

57) P = (3, 4); Q = (–2, –2)

Answer: $\mathbf{v} = -5\mathbf{i} - 6\mathbf{j}$

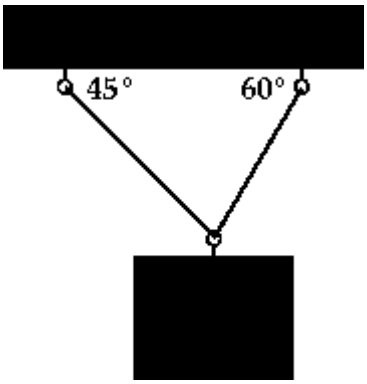
Find the unit vector having the same direction as v.

58) $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$

Answer: $\mathbf{u} = \frac{3}{5}\mathbf{i} - \frac{4}{5}\mathbf{j}$

Solve the problem.

59) A box of supplies that weighs 1500 kilograms is suspended by two cables as shown in the figure. To two decimal places, what is the tension in the two cables?



Answer: Tension in right cable: 1098.08 kg; tension in left cable: 776.46 kg

Find the angle between v and w. Round your answer to one decimal place, if necessary.

60) $\mathbf{v} = -6\mathbf{i} - 7\mathbf{j}$, $\mathbf{w} = -5\mathbf{i} - 6\mathbf{j}$

Answer: 0.8°

Solve the problem.

61) Which of the following vectors is parallel to $\mathbf{v} = \mathbf{i} - \mathbf{j}$?

A) $\mathbf{w} = \mathbf{i} - 2\mathbf{j}$ B) $\mathbf{w} = 2\mathbf{i} - 2\mathbf{j}$ C) $\mathbf{w} = \mathbf{i} + \mathbf{j}$ D) $\mathbf{w} = 2\mathbf{i} + 2\mathbf{j}$

Answer: B

62) Which of the following vectors is orthogonal to $20\mathbf{i} - 8\mathbf{j}$?

A) $\mathbf{w} = 15\mathbf{i} - 6\mathbf{j}$ B) $\mathbf{w} = 4\mathbf{i} + 3\mathbf{j}$ C) $\mathbf{w} = 20\mathbf{i} + 4\mathbf{j}$ D) $\mathbf{w} = -10\mathbf{i} - 25\mathbf{j}$

Answer: D

Decompose v into two vectors $\mathbf{v_1}$ and $\mathbf{v_2}$, where $\mathbf{v_1}$ is parallel to w and $\mathbf{v_2}$ is orthogonal to w.

63) $\mathbf{v} = \mathbf{i} + 4\mathbf{j}$, $\mathbf{w} = -3\mathbf{i} + \mathbf{j}$

Answer: $\mathbf{v}_1 = -\frac{3}{10}\mathbf{i} + \frac{1}{10}\mathbf{j}$, $\mathbf{v}_2 = \frac{13}{10}\mathbf{i} + \frac{39}{10}\mathbf{j}$