INTERMEDIATE ALGEBRA

Math 0310

Turnell

Updated July 2015

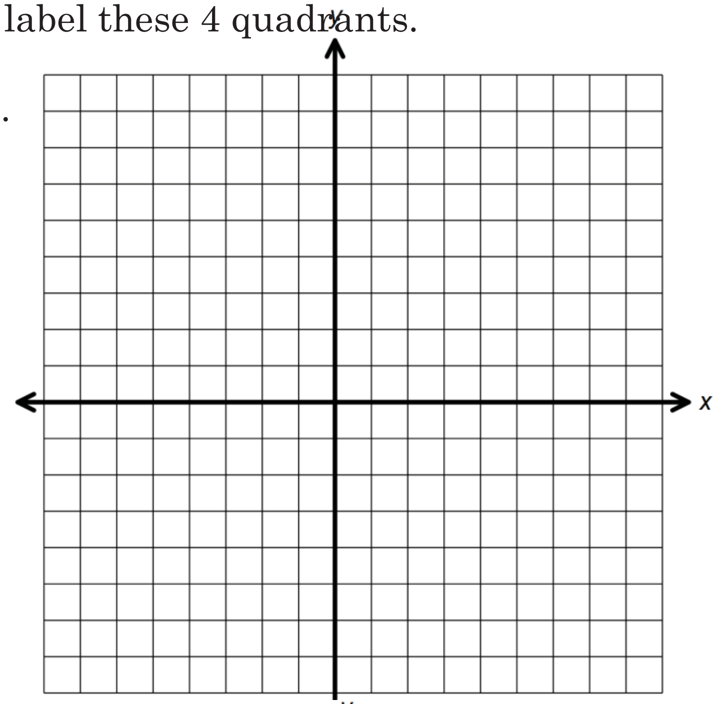
LONE STAR COLLEGE

Notes-- Cartesian Coordinate System

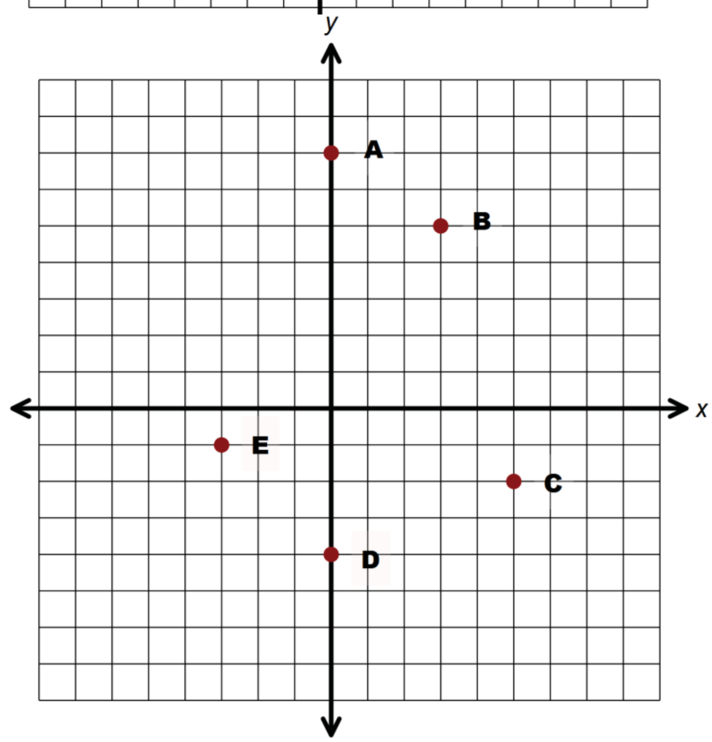
We use a rectangular coordinate system to help us "map" out relations. The coordinate grid has a horizontal axis and a vertical axis. Where these two axes intersect is called the origin. The grid is also divided into 4 quadrants. Traditionally, we use Roman numerals to label these 4 quadrants.

Let's begin by plotting some ordered pairs.

A=(1, 4)   B=(0,2)
C=(-3,5)   D=(-4,0)
E=(-2,-4)   F=(0,-3)
G=(3,-1)



Find the ordered pair associated with the given points.



We also use the coordinate system to graph solutions of equations in two variables.

One of the most common equations we graph is linear equations. LINES There are several methods to graph lines: plot points by creating a table of values, plot the intercepts, use the slope and y-intercept.

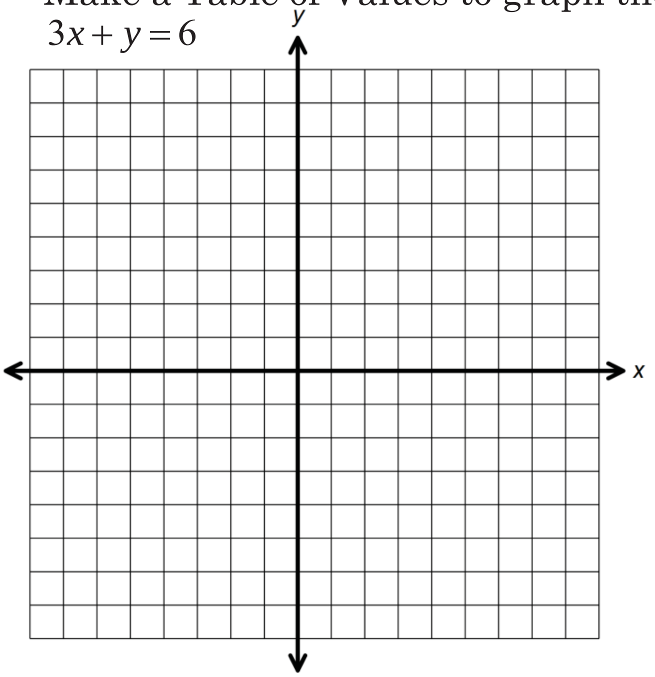
How do you know if the graph of an equation is a line?

$y+2x^2=5$ $\qquad$ $2y+4x=11$ $\qquad$ $y=\sqrt{2x+1}$

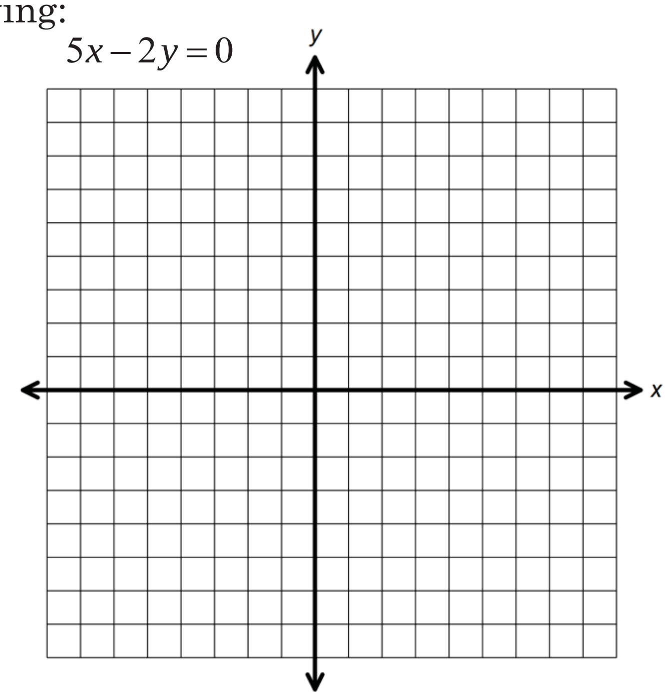
$x-9=0$ $\qquad$ $y=5$ $\qquad$ $y^2+2x^2=5$

Make a Table of Values to graph the following:

$3x+y=6$

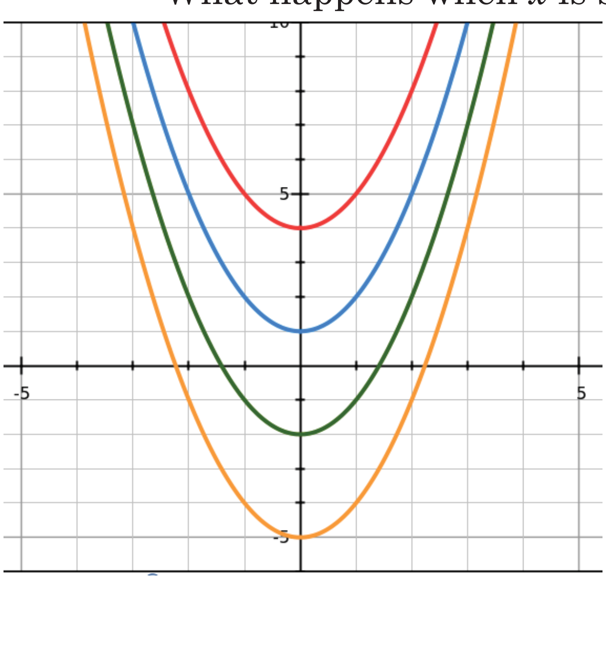


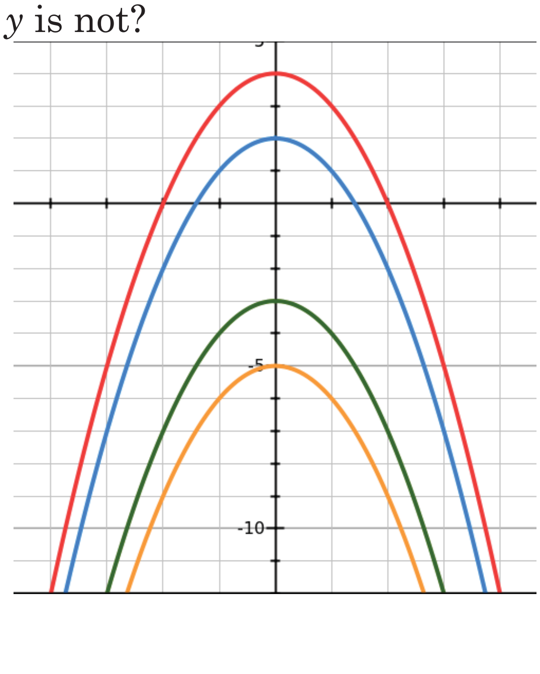
$5x-2y=0$



What happens when we do NOT have a linear equation?

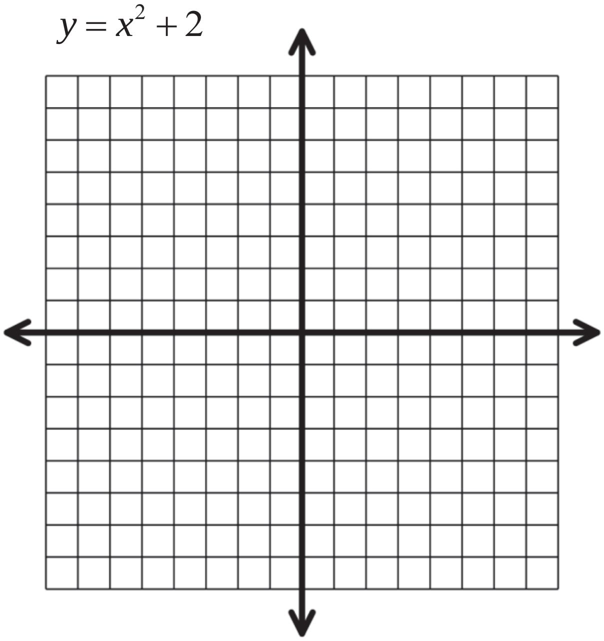
What happens when x is squared and y is not?



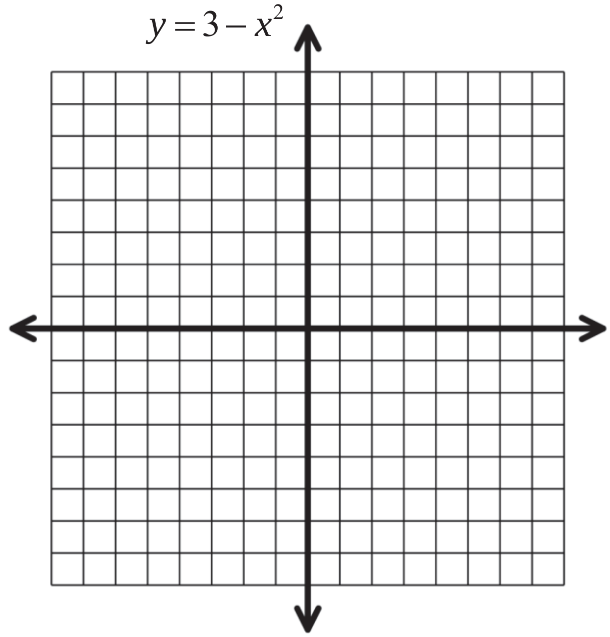


Make a Table of Values to graph the following:

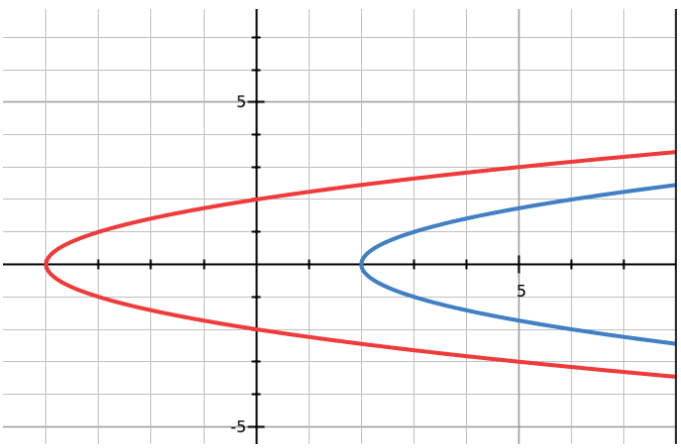
$y = x^2 + 2$

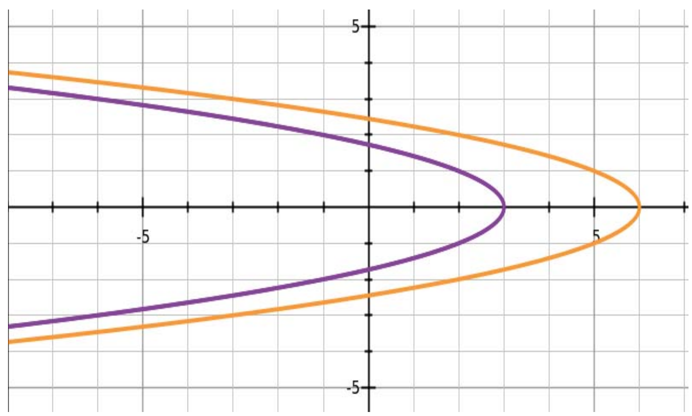


$y = 3 - x^2$



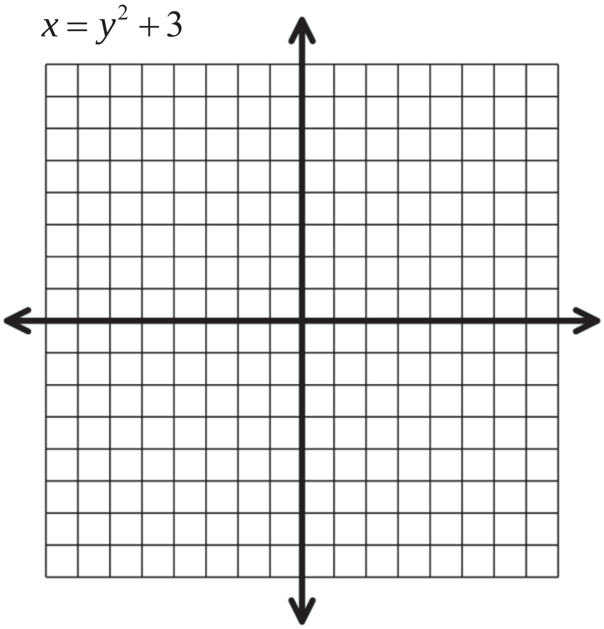
What happens when y is squared and x is not?



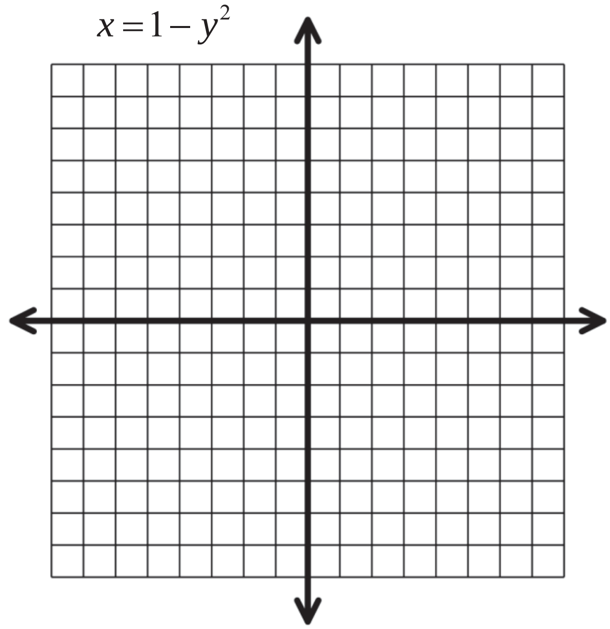


Make a Table of Values to graph the following:

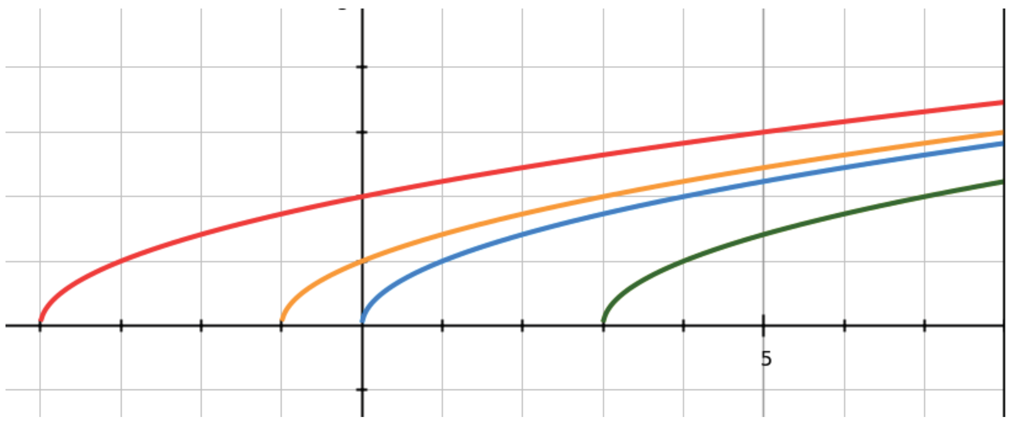
$x = y^2 + 3$



$x = 1 - y^2$

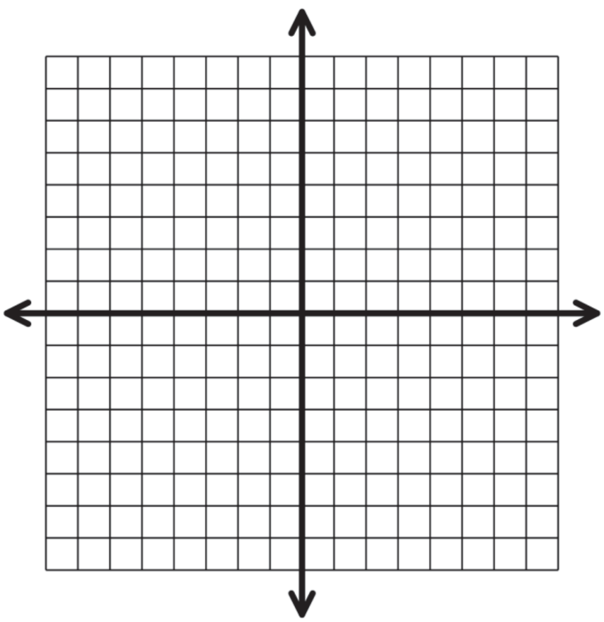


What happens when x is under a square root sign?

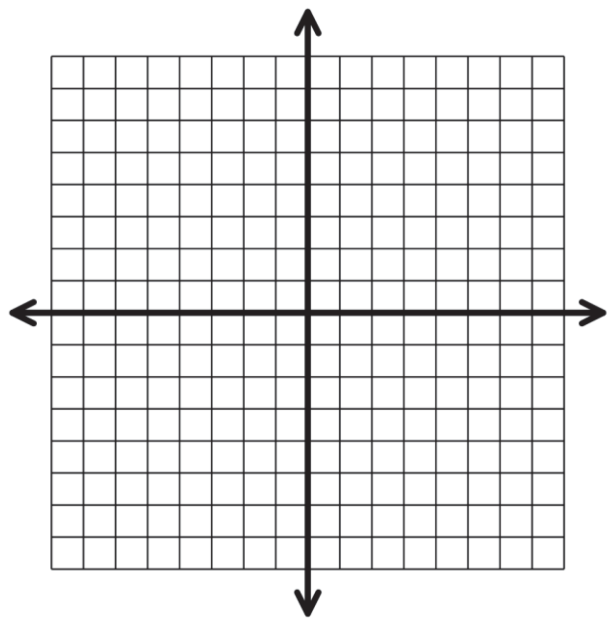


Make a Table of Values to graph the following:

$y=\sqrt{x+3}$

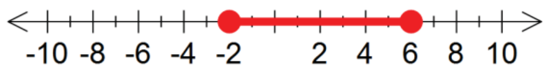


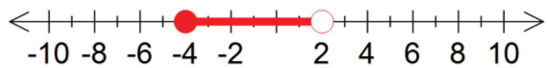
$y=\sqrt{x-2}$

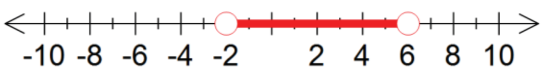


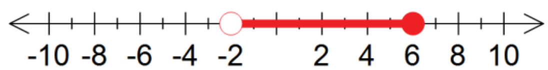
Interval Notation

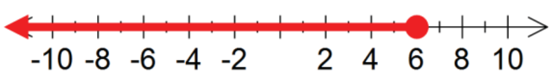
A filled in circle means "equal to". We use a bracket **[** or **]**
An open circle means "not equal to" (but as close as possible). We use a parenthesis: **(** or **)**
You can NEVER equal infinity. We ALWAYS use a parenthesis with the infinity symbol, ∞.

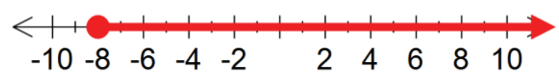


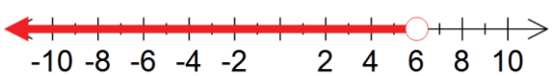


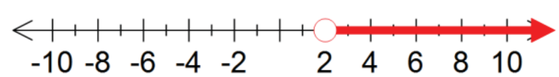


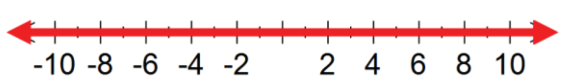








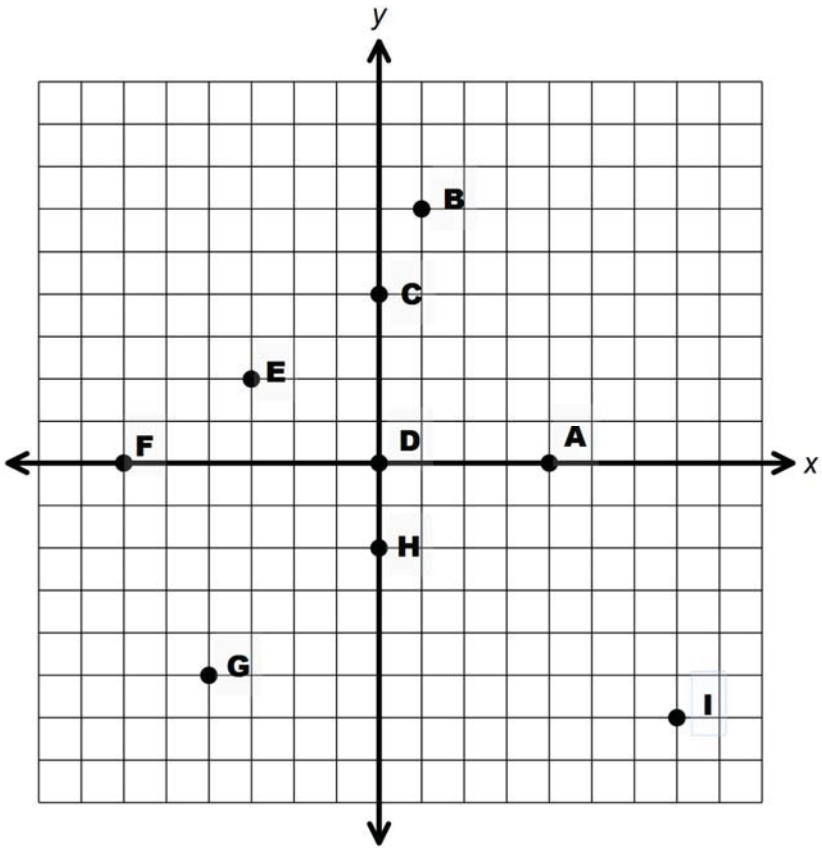




Cartesian Coordinate System

(You should complete the work on THIS paper)

1. Find the ordered pairs associated with the given points:



A:________ H:________

B:________ I:_________

C:________

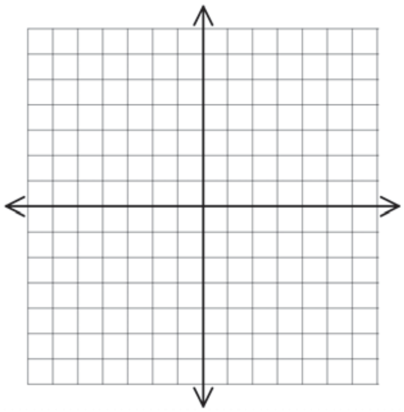
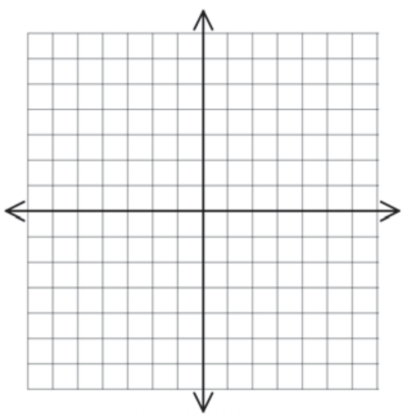
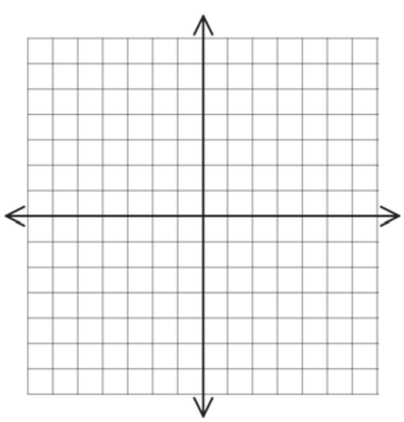
D:________

E:________

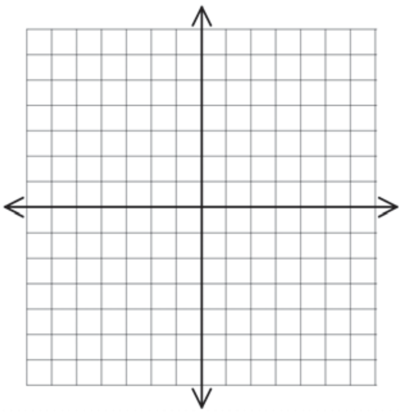
F:________

G:________

Make a table of values and graph the following equations on this paper provided.

2. $y = 2x - 3$ 	Complete the table. x y	Show work
3. $x + 2y = 4$ 	Complete the table. x y	Show work
4. $\frac{1}{2}x + 2 = y$ 	Complete the table. x y	Show work

5. $2x+3y=0$

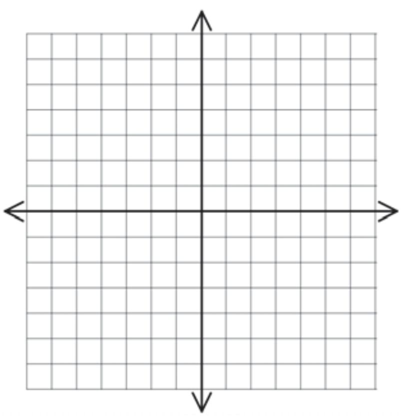


Complete the table.

x	y

Show work

6. $y=x^2-5$

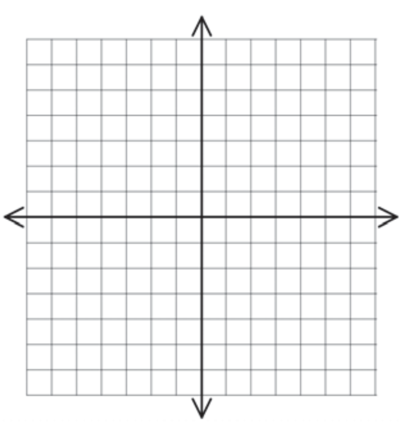


Complete the table.

x	y

Show work

7. $y=9-x^2$

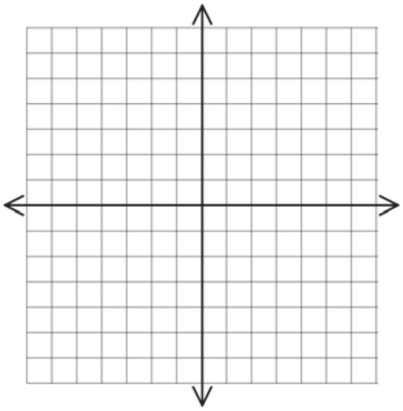


Complete the table.

x	y

Show work

8. $x = 1 - y^2$

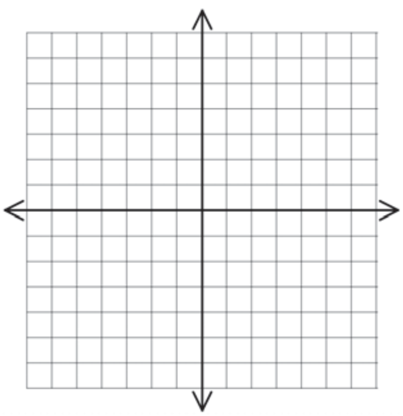


Complete the table.

x	y

Show work

9. $y = \sqrt{x-4}$

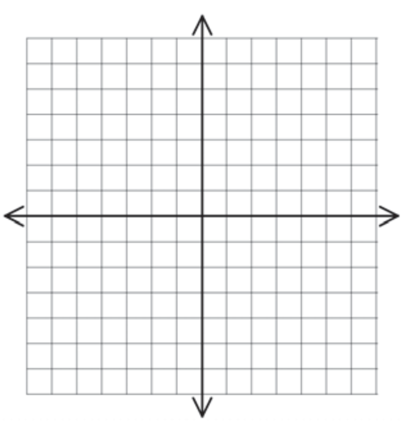


Complete the table.

x	y

Show work

10. $y = \sqrt{x+2}$



Complete the table.

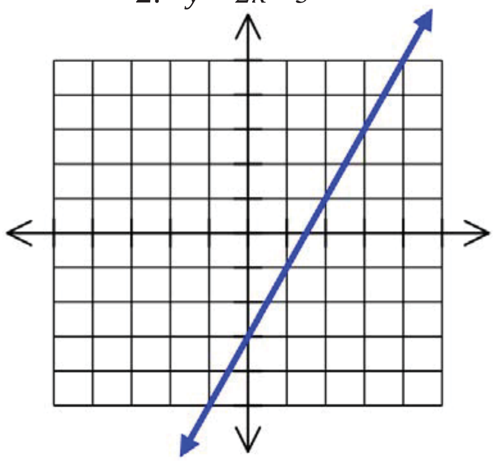
x	y

Show work

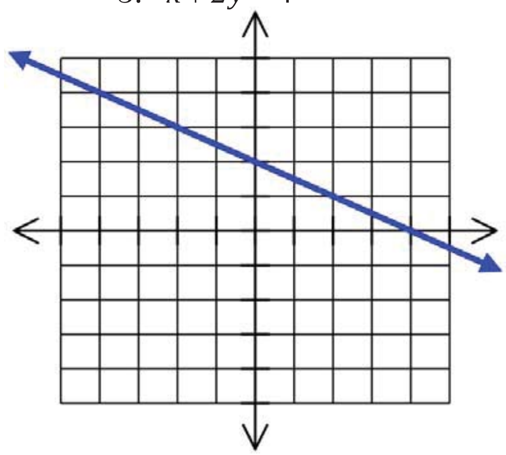
Cartesian Coordinate System-Answers

1. $A(4,0)$ $B(1,6)$ $C(0,4)$ $D(0,0)$ $E(-3,2)$ $F(-6,0)$ $G(-4,-5)$ $H(0,-2)$ $I(7,-6)$

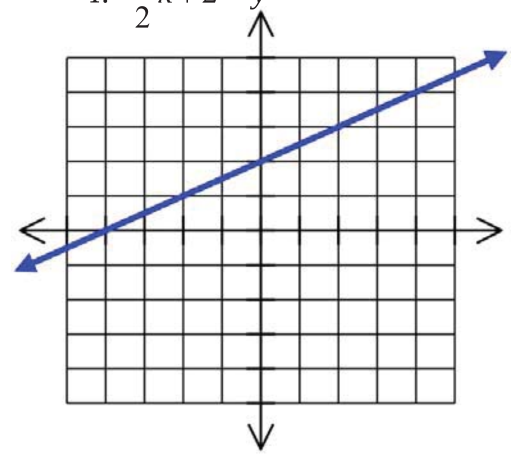
2. $y = 2x - 3$



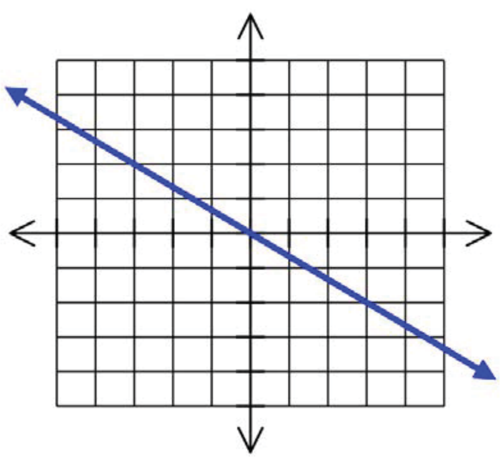
3. $x + 2y = 4$



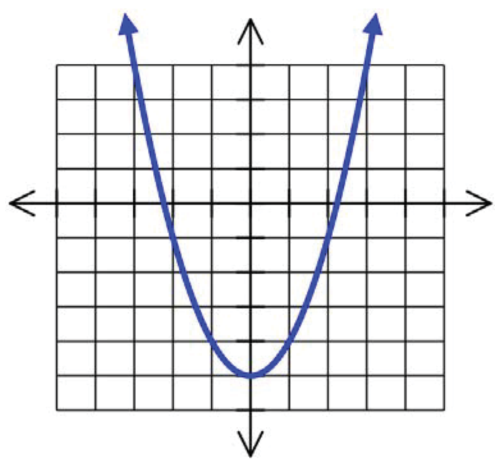
4. $\frac{1}{2}x + 2 = y$



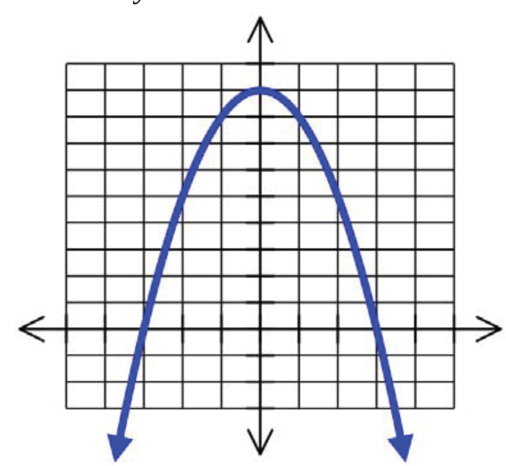
5. $2x + 3y = 0$



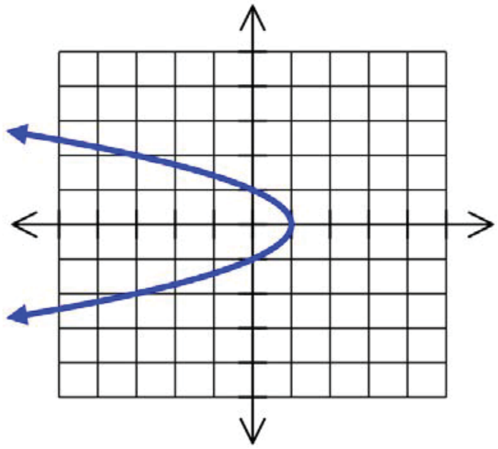
6. $y = x^2 - 5$



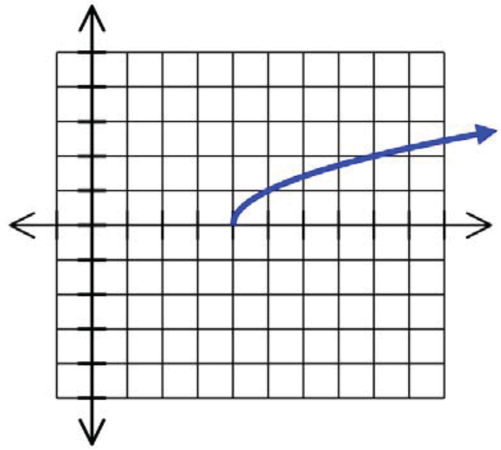
7. $y = 9 - x^2$



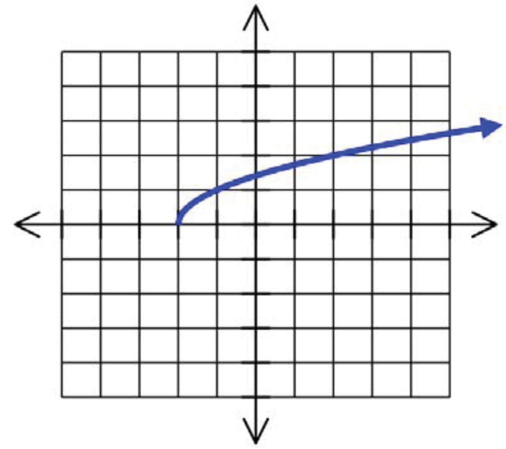
8. $x = 1 - y^2$



9. $y = \sqrt{x-4}$



10. $y = \sqrt{x+2}$



Notes-- Relations and Functions

A *relation* is a set of ordered pairs. The *domain* is the set of all first coordinates and the *range* is the set of all second coordinates.

{(Bob, tie), (Carly, bow),(Juan, flower),(Bob, scarf)}

The *domain* would be {Bob, Carly, Juan}

The *range* would be {tie, bow, flower, scarf}

A *function* is a relation such that no two ordered pairs have the same first coordinate.

An example of a *function* is:

{(Sally, pink), (Carly, blue), (Adam, green), (Bob, blue)}

The *domain* would be {Sally, Carly, Adam, Bob}

The *range* would be {pink, blue, green}

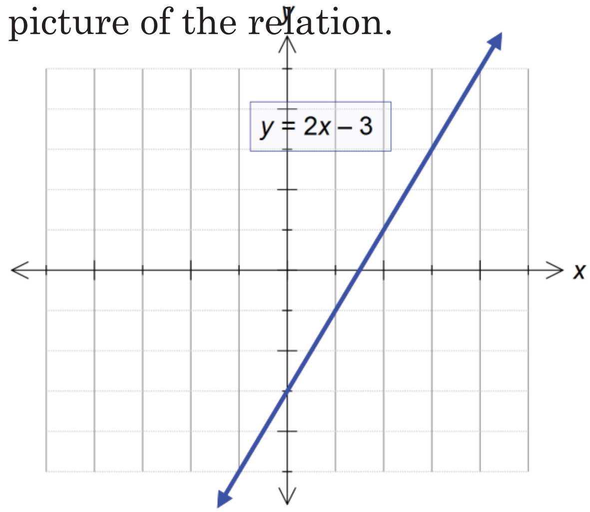
Another example of a function is:

$A = \{(x, y) : y = 3x - 5, x = -2, 0, 3\}$ $\qquad$ $B = \left\{(x, y) : y = \sqrt{x+10}, x = -6, -1, 6\right\}$

Find the domain and range.

What if we have an infinite number of ordered pairs?

We cannot make a list, but we can draw a picture of the relation.



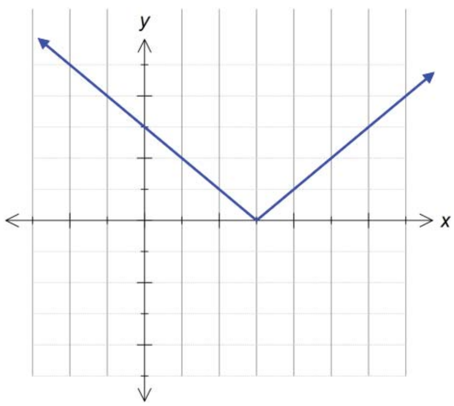
What is the domain?

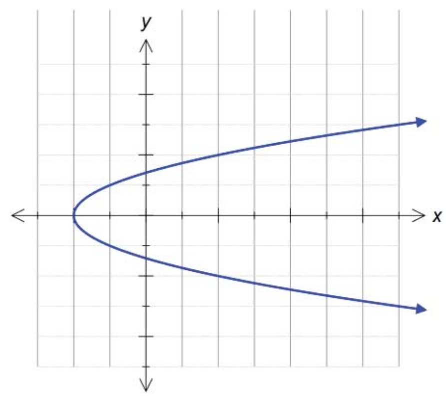
What is the range?

How can we determine if a graph is a function?

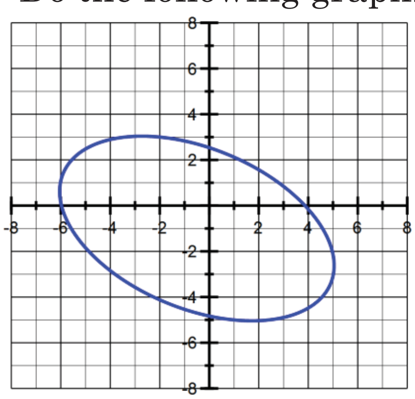
Remember definition: no two ordered pairs have the same first coordinate.

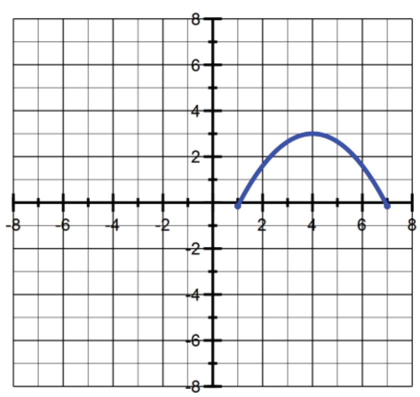
This leads to the ***vertical line test***.

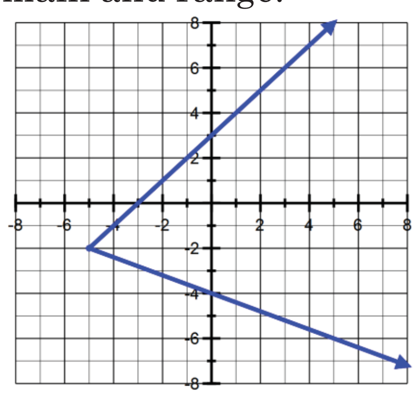


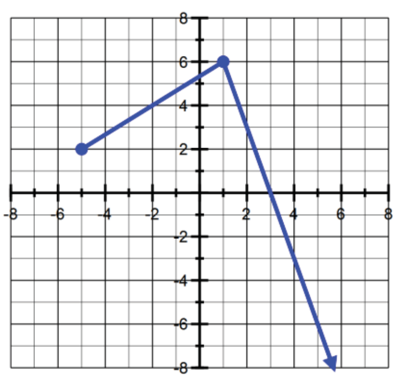


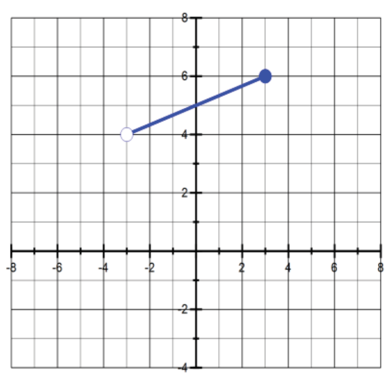
Do the following graphs represent a function? Find the domain and range.

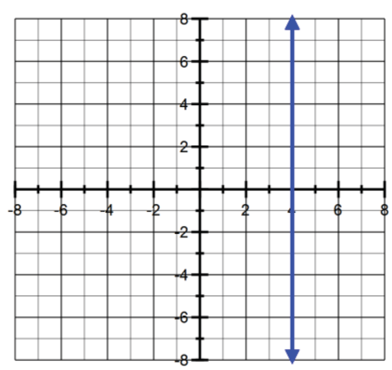












Relations and Functions

Do all work on notebook paper. All work should be neat and organized.

Write the following sets as sets of ordered pairs and identify the domain and range.

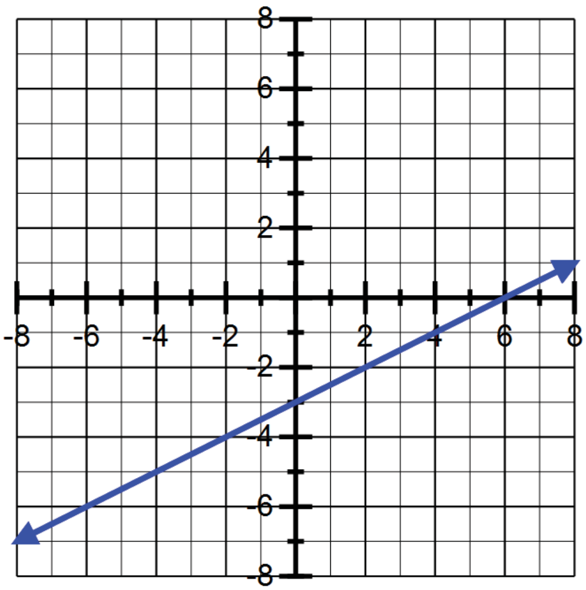
1. $V = \left\{(x, y) : y = -2x + 7, x = -5, 0, 2, 1, \frac{3}{2}\right\}$

2. $Z = \{(x, y) : y = |x| + 3, x = -2, 0, 2\}$

3. $X = \left\{(x, y) : y = \sqrt{2x - 1}, x = 1, 3, 5, \frac{1}{2}\right\}$

Determine the domain and range of each relation whose graph is given. Determine which of the following are graphs of functions.

4.



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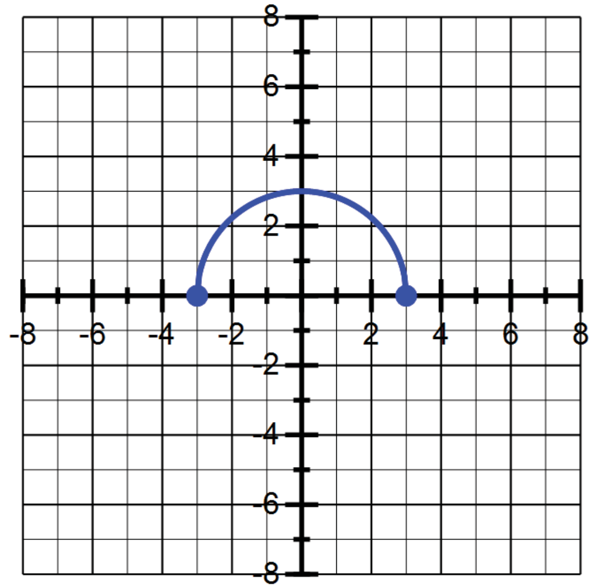

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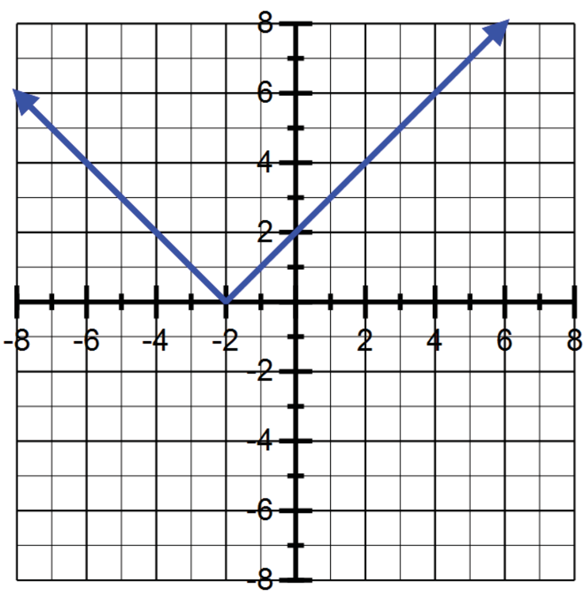
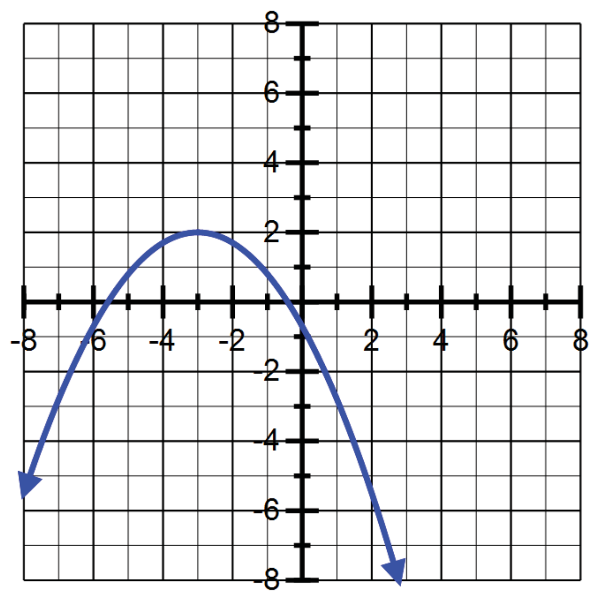
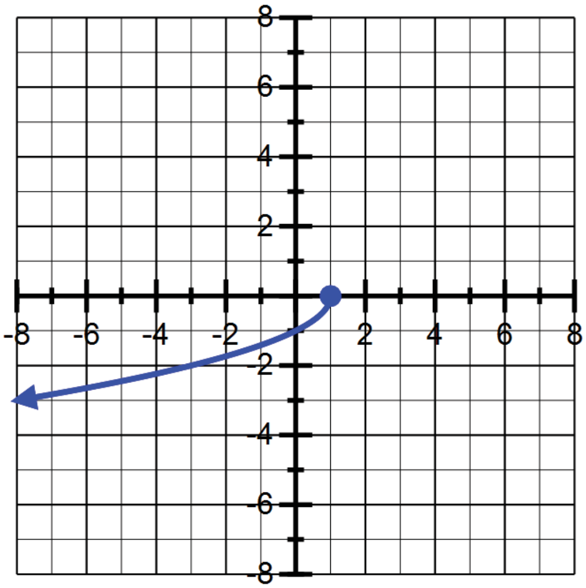


7.

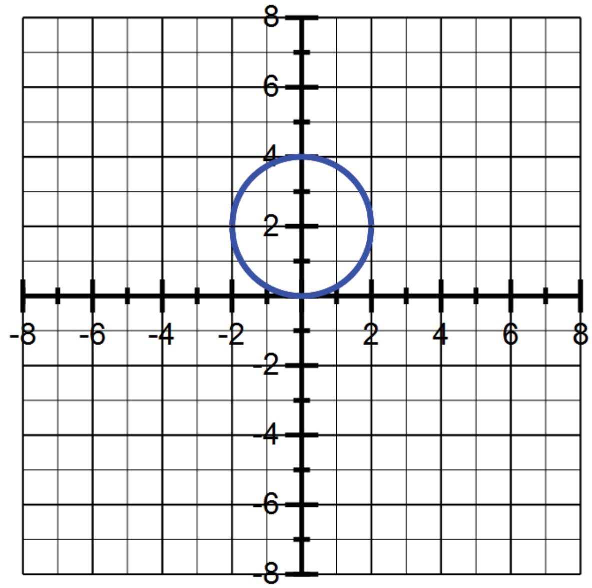


8.



8
6
4
2
-8 -6 -4 -2 2 4 6 8
-2
-4
-6
-8

9.



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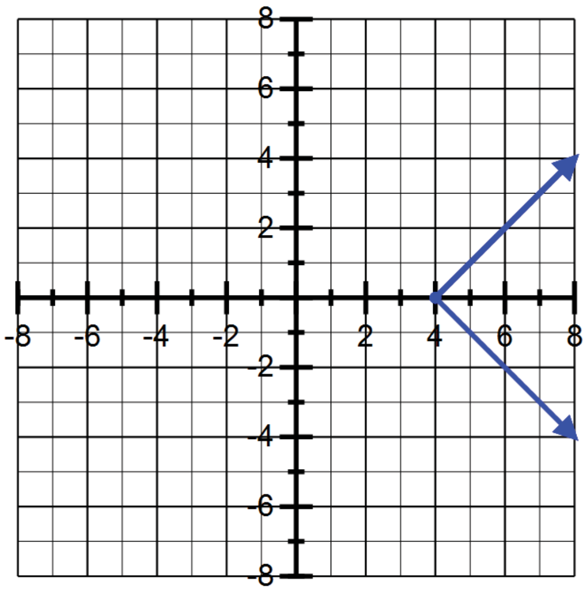
4
2
-8 -6 -4 -2 2 4 6 8
-2
-4
-6
-8

10.



8
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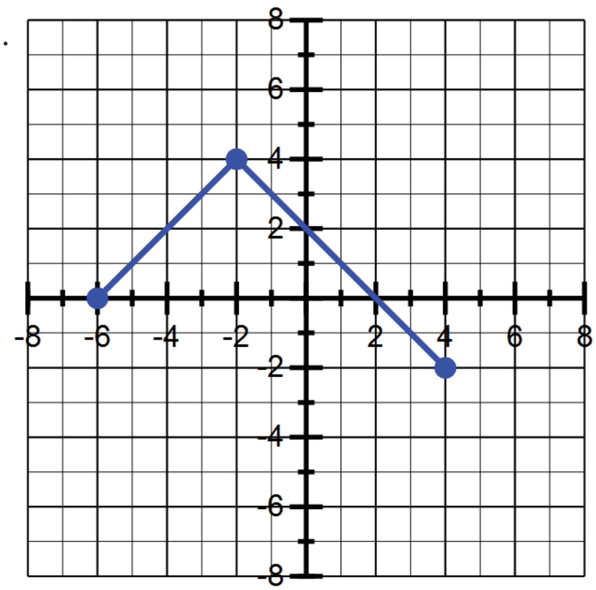
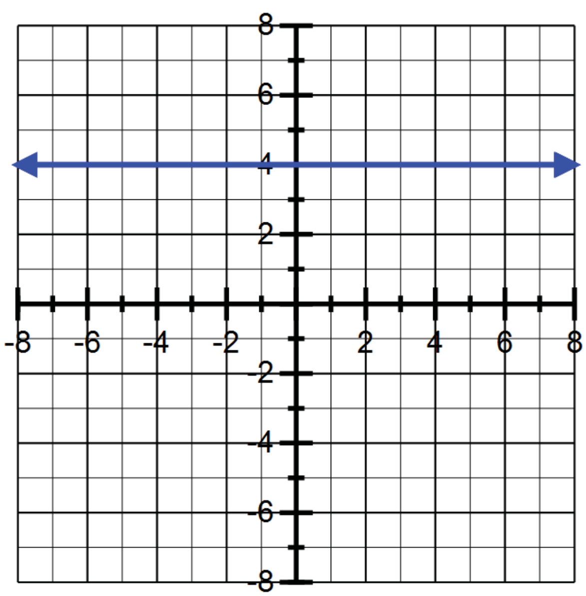
4
2
-8 -6 -4 -2 2 4 6 8
-2
-4
-6
-8

11.



8
6
4
2
-8 -6 -4 -2 2 4 6 8
-2
-4
-6
-8

12.



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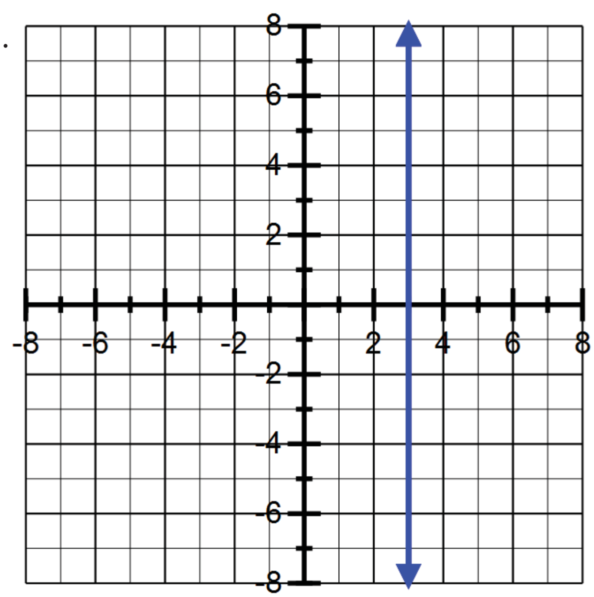
13.



8
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-8 -6 -4 -2 2 4 6 8
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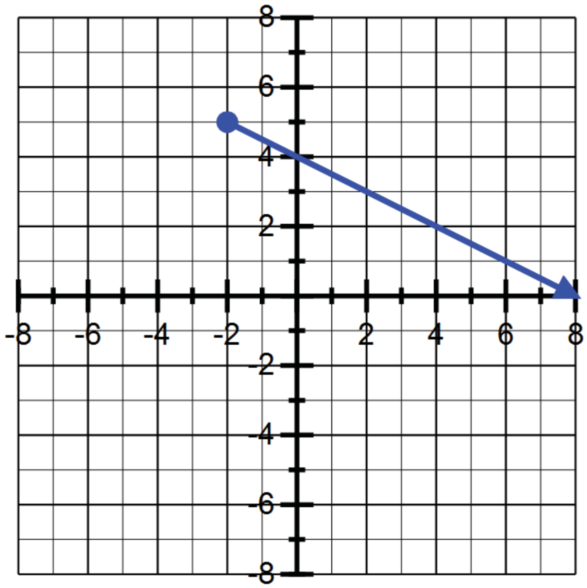
14



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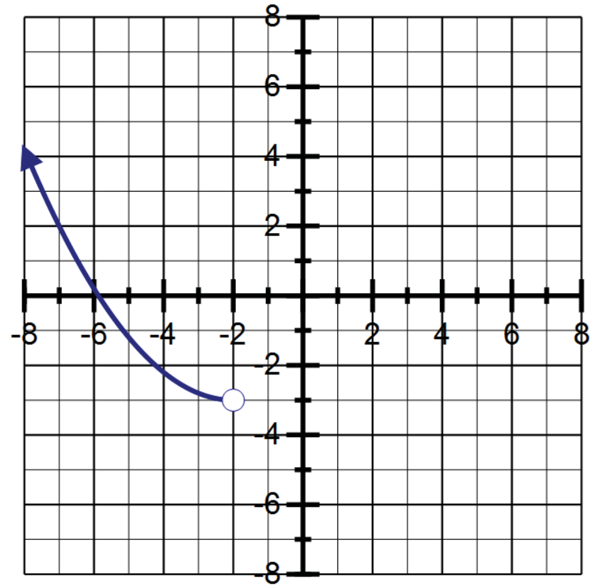

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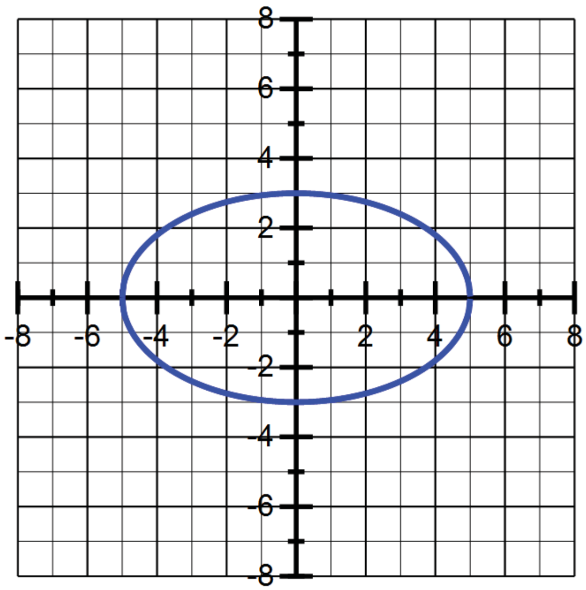

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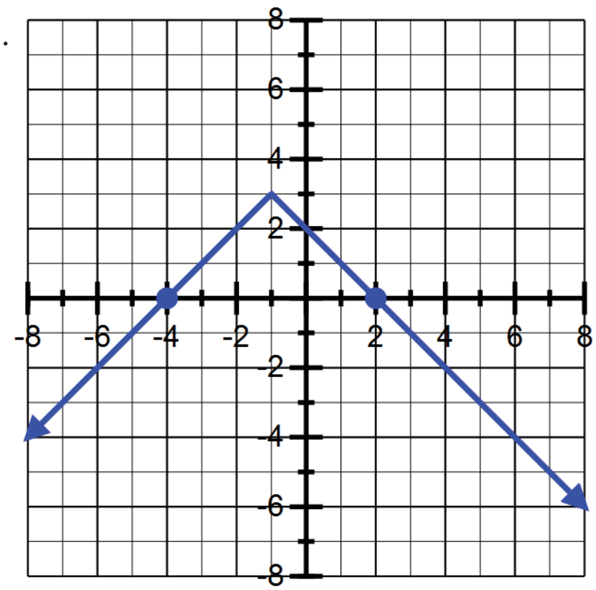
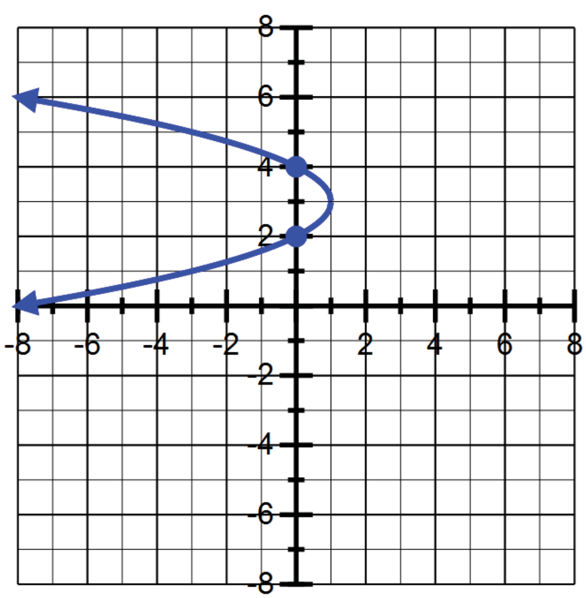
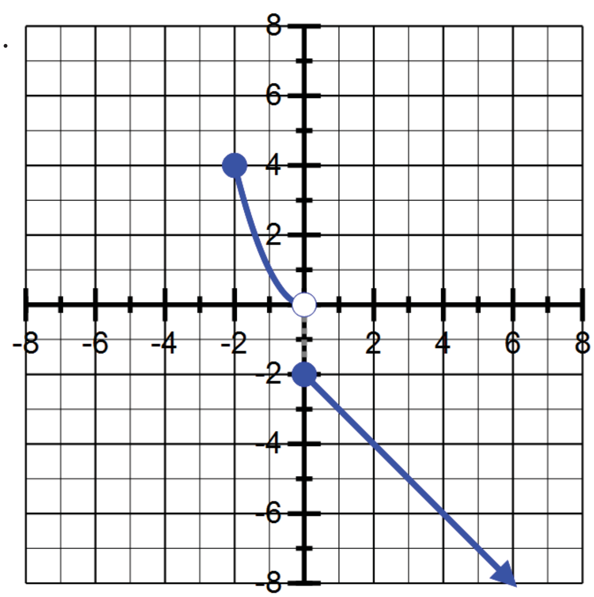


18.



19.



Relations and Functions-Answers

1. $V=\left\{(-5,17),(0,7),(2,3),(1,5),\left(\frac{3}{2},4\right)\right\}$ Domain= $=\left\{-5,0,2,1,\frac{3}{2}\right\}$ Range= $=\{17,7,3,5,4\}$

2. $Z=\{(-2,5),(0,3),(2,5)\}$ Domain= $=\{-2,0,2\}$ Range= $=\{3,5\}$

3. $X=\left\{(1,1),(3,\sqrt{5}),(5,3),\left(\frac{1}{2},0\right)\right\}$ Domain= $=\left\{1,3,5,\frac{1}{2}\right\}$ Range= $=\{1,\sqrt{5},3,0\}$

4. Domain= $=(-\infty,\infty)$ Range= $=(-\infty,\infty)$; Yes, it is a function

5. Domain= $=[-3,3]$ Range= $=[0,3]$; Yes, it is a function

6. Domain= $=(-\infty,\infty)$ Range= $=[0,\infty)$; Yes, it is a function

7. Domain= $=(-\infty,\infty)$ Range= $=(-\infty,2]$; Yes, it is a function

8. Domain= $=(-\infty,1]$ Range= $=(-\infty,0]$; Yes, it is a function

9. Domain= $=[-2,2]$ Range= $=[0,4]$; Not a function

10. Domain= $=[4,\infty)$ Range= $=(-\infty,\infty)$; Not a function

11. Domain= $=[-6,4]$ Range= $=[-2,4]$; Yes, it is a function

12. Domain= $=(-\infty,\infty)$ Range= $=[4]$; Yes, it is a function

13. Domain= $=[3]$ Range= $=(-\infty,\infty)$; Not a function

14. Domain= $=[-2,\infty)$ Range= $=(-\infty,5]$; Yes, it is a function

15. Domain= $=(-\infty,-2)$ Range= $=(-3,\infty)$; Yes, it is a function

16. Domain= $=[-5,5]$ Range= $=[-3,3]$; Not a function

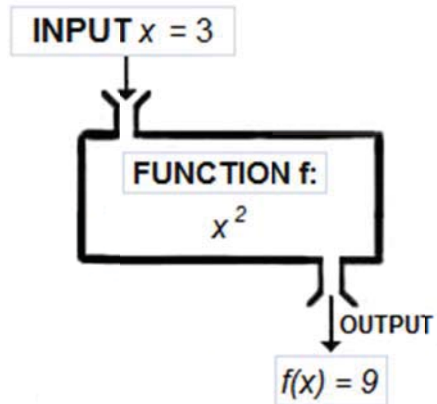
17. Domain= $=(-\infty,\infty)$ Range= $=(-\infty,3]$; Yes, it is a function

18. Domain= $=(-\infty,1]$ Range= $=(-\infty,\infty)$; Not a function

19. Domain= $=[-2,\infty)$ Range= $=(-\infty,-2]\cup(0,4]$; Yes, it is a function

Function Notation

In algebra, we use function notation:



Non-function notation: $y = x^2$

Re-written with function notation: $f(x) = x^2$

We read this as "f of x"

$f(3) = (3)^2$

$f(3) = 9$

$f(*) = *^2$ $\qquad f(\Theta) = \Theta^2$ $\qquad f(\forall) = \forall^2$

Let $f(x) = 2x + 1$. Find $f(-2)$, $f(0)$, $f(a)$, $f(a+h)$, $\dfrac{f(a+h) - f(a)}{h}$

Let $g(x) = 2 - x^2$. Find $g(-2)$, $g(0)$, $g(a)$, $g(a+h)$, $\dfrac{g(a+h) - g(a)}{h}$

Combinations of Functions

We can perform operation on functions:

Addition: $(f+g)(x) = f(x) + g(x)$

Subtraction: $(f-g)(x) = f(x) - g(x)$

Multiplication: $(f \cdot g)(x) = f(x) \cdot g(x)$

Division: $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}, \quad g(x) \neq 0$ **Why this restriction?**

Let $f(x) = 3x^2 + x + 4$ and $g(x) = x - 1$.

Find $(f+g)(x)$, $(f-g)(x)$, $(f \cdot g)(x)$, $\left(\dfrac{f}{g}\right)(x)$

Let $f(x) = x^2 - 3x + 1$ and $g(x) = x - 1$

Find $(f+g)(3)$

Find $(f-g)(-1)$

Find $(f \cdot g)(2)$

Find $\left(\dfrac{f}{g}\right)(-2)$

Find $2f(3) - 4g(2)$

Find $\left(\dfrac{g(-2)}{f(-1)}\right)^3$

Function Notation and Operations of Functions

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

For the given functions, find the following values:

$$f(-2),\ f(-1),\ f(0),\ f(1),\ f(a),\ f(a+h)$$

1. $f(x) = 2x - 3$
2. $f(x) = 3 - x$
3. $f(x) = x^2 - 2$
4. $f(x) = 5 - x^2$
5. $f(x) = 2x^2$

Find $(f+g)(x), (f-g)(x), (f \cdot g)(x), \left(\dfrac{f}{g}\right)(x)$.

6. $f(x) = 2x^2 + 5x - 1,\quad g(x) = x - 2$
7. $f(x) = x^2 + 5,\quad g(x) = x^2 - 9$
8. $f(x) = 2x + 3,\quad g(x) = x - 11$
9. $f(x) = x^3 + 3x - 5,\quad g(x) = 2x + 1$

Let $f(x) = 3x - 1$, $g(x) = 3x^2 + 5x - 1$, $m(x) = x^2 - 4$, **and** $p(x) = 2x + 1$.
Find the following:

10. $f(-2)$
11. $m(-3)$
12. $5f(-2) + 4m(-3)$
13. $[f(-2)]^3$
14. $g(3)$
15. $\left(\dfrac{m(-3)}{g(3)}\right)^2$
16. $(m \cdot p)(-1)$
17. $(g - f)(0)$
18. $g(x) - m(x)$
19. $\dfrac{m(x)}{p(x)}$
20. $g(2x)$
21. $p(3x)$
22. $f(a)$
23. $f(a+h)$
24. $\dfrac{f(a+h) - f(a)}{h}$
25. $g(a)$
26. $g(a+h)$
27. $\dfrac{g(a+h) - g(a)}{h}$

Function Notation and Operations of Functions-Answers

1. $f(-2)=-7, f(-1)=-5, f(0)=-3, f(1)=-1, f(a)=2a-3, f(a+h)=2a+2h-3$

2. $f(-2)=5, f(-1)=4, f(0)=3, f(1)=2, f(a)=3-a, f(a+h)=3-a-h$

3. $f(-2)=2, f(-1)=-1, f(0)=-2, f(1)=-1, f(a)=a^2-2, f(a+h)=a^2+2ah+h^2-2$

4. $f(-2)=1, f(-1)=4, f(0)=5, f(1)=4, f(a)=5-a^2, f(a+h)=5-a^2-2ah-h^2$

5. $f(-2)=8, f(-1)=2, f(0)=0, f(1)=2, f(a)=2a^2, f(a+h)=2a^2+4ah+2h^2$

6. $(f+g)(x)=2x^2+6x-3, (f-g)(x)=2x^2+4x+1$
   $(f\cdot g)(x)=2x^3+x^2-11x+2, \left(\frac{f}{g}\right)(x)=\frac{2x^2+5x-1}{x-2}, x\neq 2$

7. $(f+g)(x)=2x^2-4, (f-g)(x)=14$
   $(f\cdot g)(x)=x^4-4x^2-45, \left(\frac{f}{g}\right)(x)=\frac{x^2+5}{x^2-9}, x\neq \pm 3$

8. $(f+g)(x)=3x-8, (f-g)(x)=x+14$
   $(f\cdot g)(x)=2x^2-19x-33, \left(\frac{f}{g}\right)(x)=\frac{2x+3}{x-11}, x\neq 11$

9. $(f+g)(x)=x^3+5x-4, (f-g)(x)=x^3+x-6$
   $(f\cdot g)(x)=2x^4+x^3+6x^2-7x-5, \left(\frac{f}{g}\right)(x)=\frac{x^3+3x-5}{2x+1}, x\neq -\frac{1}{2}$

10. -7
11. 5
12. -15
13. -343
14. 41
15. $\frac{25}{1681}$
16. 3
17. 0
18. $2x^2+5x+3$
19. $\frac{x^2-4}{2x+1}$
20. $12x^2+10x-1$
21. $6x+1$
22. $3a-1$
23. $3a+3h-1$
24. 3
25. $3a^2+5a-1$
26. $3a^2+6ah+3h^2+5a+5h-1$
27. $6a+3h+5$