INTERMEDIATE ALGEBRA

Math 0310

Turnell

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LONE STAR COLLEGE

Notes-- Absolute Value Equations

Absolute value of a real number is the _____ from zero on a number line. Distance is always positive.

$|x| = 2$

$|x| = -3$

$|x| = 0$

Process:

1. Isolate the absolute value expression. $|\text{expression}| = \textit{number}$
2. Determine the type of number the absolute value expression is equal to
   a. If it is equal to a NEGATIVE NUMBER the answer is NO SOLUTION.
   b. If it is equal to a POSITIVE NUMBER you will split into 2 equations (without absolute value bars)—(2 solutions)
      $\text{expression} = \textit{number}$ or $\text{expression} = -(\textit{number})$
   c. If it is equal to ZERO rewrite the equation without absolute value bars and solve for the variable. (one solution)

Example 1: $|2x + 5| = 13$

Example 2: $|3x-4|-3=11$

Example 3: $2|2x-5|+5=11$

Example 4: $|2x+1|+4=4$

Example 5: $2|2x-5|+9=9$

Example 6: $4|x-1|+7=3$

Example 7: $5-|2x-3|=7$

Absolute Value Equations

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Solve the equations.

1. $|x| = 8$

2. $2|x| = 12$

3. $|2x - 1| = 7$

4. $|3 - 4x| = 11$

5. $|3x - 8| = -7$

6. $\left|\frac{3}{4}x - 2\right| = 4$

7. $|2x + 5| = 0$

8. $3|5x + 1| - 1 = 5$

9. $3|3x + 8| + 2 = 8$

10. $2|3x - 1| + 7 = 7$

11. $5|x - 4| + 6 = 1$

12. $5 - |2x - 3| = -7$

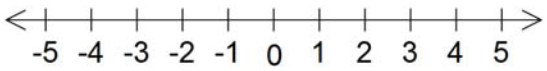
Absolute Value Equations-Answers

1. $\{8,-8\}$
2. $\{6,-6\}$
3. $\{4,-3\}$
4. $\left\{\frac{7}{2},-2\right\}$
5. $\varnothing$
6. $\left\{\frac{-8}{3},8\right\}$
7. $\left\{\frac{-5}{2}\right\}$
8. $\left\{\frac{1}{5},\frac{-3}{5}\right\}$
9. $\left\{\frac{-10}{3},-2\right\}$
10. $\left\{\frac{1}{3}\right\}$
11. $\varnothing$
12. $\left\{\frac{-9}{2},\frac{15}{2}\right\}$

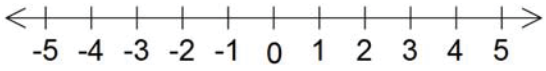
Notes-- Absolute Value Inequalities

Recall that absolute value is the distance away from zero. The distance is ALWAYS positive.

$|x| < 4$



$|x| \geq 3$



Less Than: ($\leq, <$)

1. Isolate the absolute value: $|\text{expression}| < \textit{number}$
2. RE-write without the absolute value sign. Use a "sandwich" inequality:
   $-\textit{number} < \text{expression} < \textit{number}$
3. Solve.
4. Graph answer on a number line.
5. Write answer in interval notation.

Greater Than: ($\geq, >$)

1. Isolate the absolute value: $|\text{expression}| > \textit{number}$
2. RE-write without the absolute value sign. You must separate into 2 inequalities:
   $\text{expression} > \textit{number}$ or $\text{expression} < -\textit{number}$
3. Solve.
4. Graph answers on a number line.
5. Write answer in interval notation.

Example 1: $|x-4| \leq 8$

Example 2: $|x+2| > 9$

Example 3: $|3x-1|-2\geq 9$

Example 4: $2|x+3|+5<11$

Example 5: $|5x-13|+7\leq 6$

Example 6: $|3x+1|+5>1$

Absolute Value Inequalities

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Solve the inequalities. Graph solutions on a number line. Write the answers in interval notation.

1. $|x| < 3$

2. $|x| \geq 2$

3. $|x| + 2 \leq 6$

4. $|x| - 9 > 5$

5. $|x| \leq -11$

6. $|x| \geq -10$

7. $|x - 5| < 1$

8. $|2x - 5| > 3$

9. $|2x + 2| - 2 < 6$

10. $|2x - 1| + 1 \geq 8$

11. $|4x - 7| < 2$

12. $|3x - 5| + 12 < 6$

13. $|4x + 1| + 6 > 3$

14. $4|x - 1| - 7 > 13$

15. $3|x - 1| + 7 \leq 10$

Absolute Value Inequalities-Answers

1. $(-3,3)$
2. $(-\infty,-2]\cup[2,\infty)$
3. $[-4,4]$
4. $(-\infty,-14)\cup(14,\infty)$
5. $\varnothing$
6. $(-\infty,\infty)$
7. $(4,6)$
8. $(-\infty,1)\cup(4,\infty)$
9. $(-5,3)$
10. $(-\infty,-3]\cup[4,\infty)$
11. $\left(\frac{5}{4},\frac{9}{4}\right)$
12. $\varnothing$
13. $(-\infty,\infty)$
14. $(-\infty,-4)\cup(6,\infty)$
15. $[0,2]$

Notes-- Greatest Common Factor

GCF: The largest number that will divide into all the terms AND the variable(s) that occur in all the terms (raised to the smallest power).

Find the GCF of the following polynomials:

1. $14x^3 - 21x^2 + 28x^4$

2. $-16a + 24b + 8a^2$

Factor the following:

3. $8x^2 + 18xy + 2x$

4. $-7x^4y - 35x^2y^2 - 14x^2y^5$

5. $7y(3x+9) - 14z(3x+9)$

6. $15x(y-3) - 5z(3-y)$

Notes-- Factor by Grouping

GROUPING: This factoring technique is used when you have 4 or more terms.

Try grouping the first two terms and the last terms.
Factor out the GCF of each group.
If the GCF matches exactly, then "take out" that GCF from both groups.
Factor further, if necessary.

Factor the following:

1. $10hn - 3km + 2hm - 15kn$

2. $-42x^2z^3 + 6x^2z^2 - 2xz^3 + 14xz^4$

3. $7xy^3 - 20xy + 2y^3 - 70x^2y$

GCF and Grouping

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Factor completely.

1. $10x^2 - 35$
2. $8x^3 + 18xy + 2x$
3. $-6x^5 - 18x^3 - 10x^2$
4. $4x(x-2y) - 6y(x-2y)$
5. $2x(x-3) + (x-3)$
6. $8y^2z(1-5x) - 6yz^3(1-5x)$
7. $15x(y-3) - 5z(3-y)$
8. $2ax + 3bx + 2ay + 3by$
9. $6cf - 2cg + 6df - 2dg$
10. $4x + 15xy - 3x^2 - 20y$
11. $3x^2y - 5x^2y^2 - 12xy^2 + 20xy^3$
12. $3y^3 + 7y^2 + 18y + 42$
13. $8x^6 + 12x^4 + 40x^3 + 60x$
14. $54y^3 + 6x^4 - 27x^3y - 12xy^2$

GCF and Grouping-Answers

1. $5(2x^2-7)$
2. $2x(4x^2+9y+1)$
3. $-2x^2(3x^3+9x+5)$ or $2x^2(-3x^3-9x-5)$
4. $2(x-2y)(2x-3y)$
5. $(2x+1)(x-3)$
6. $2yz(1-5x)(4y-3z^2)$
7. $5(y-3)(3x+z)$
8. $(x+y)(2a+3b)$
9. $2(3f-g)(c+d)$
10. $(3x-4)(5y-x)$ or $(4-3x)(x-5y)$
11. $xy(3-5y)(x-4y)$
12. $(3y+7)(y^2+6)$
13. $4x(2x^2+3)(x^3+5)$
14. $3(2y^2-x^3)(9y-2x)$

Notes-- Factoring: Difference of 2 Squares

ALWAYS look for a GCF first.

Difference of 2 Squares: $a^2 - b^2 = (a+b)(a-b)$

1. $25x^2 - 81y^2$

2. $16x^4 - 1$

3. $2x^2 - 50$

4. $(x+2)^2 - 9$

5. $(3x-2y)^2 - 49z^2$

6. $x^2(5x+4) - 9(5x+4)$

Notes-- Factoring: Cubes

Cubes: 1, 8, 27, 64, 125, 216, 1000

Sum of 2 cubes: $a^3+b^3=(a+b)(a^2-ab+b^2)$

Difference of 2 cubes: $a^3-b^3=(a-b)(a^2+ab+b^2)$

Sum of 2 cubes: $x^3+8=$

Difference of 2 cubes: $x^3-125=$

ALWAYS look for a GCF first.

1. $125x^3-27y^3=$

2. $40x^3+135y^3=$

3. $y^4-2y^3-64y+128=$

4. $12x^5y-75x^3y-12x^3y^3+75xy^3=$

Factoring Binomials

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Factor completely.

1. $x^2 - 9$
2. $36x^2 - 49y^2$
3. $27x^3y - 12xy^3$
4. $3x^2 + 48$
5. $81x^4 - 16y^4$
6. $(x-3)^2 - 49$
7. $(2x+3y)^2 - 9z^2$
8. $x^3 - 64$
9. $8x^3 + 1$
10. $64x^3 - 27y^3$
11. $54x^3 + 128y^3$
12. $375x^3 - 3$
13. $x^2(2x-3) - (2x-3)$
14. $x^3(x-6) - 8(x-6)$
15. $5x^3 + 2x^2 - 45x - 18$
16. $3x^3 + 4y^3 - 4x^2y - 3xy^2$
17. $3x^4 - 5x^3 - 3x + 5$
18. $4x^4 - 3x^3 + 32x - 24$
19. $4x^5 - x^3 - 32x^2 + 8$
20. $7x^5 - 7x^3 - 28x^3y^2 + 28xy^2$

Factoring Binomials-Answers

1. $(x+3)(x-3)$
2. $(6x+7y)(6x-7y)$
3. $3xy(3x+2y)(3x-2y)$
4. $3(x^2+16)$
5. $(9x^2+4y^2)(3x+2y)(3x-2y)$
6. $(x+4)(x-10)$
7. $(2x+3y+3z)(2x+3y-3z)$
8. $(x-4)(x^2+4x+16)$
9. $(2x+1)(4x^2-2x+1)$
10. $(4x-3y)(16x^2+12xy+9y^2)$
11. $2(3x+4y)(9x^2-12xy+16y^2)$
12. $3(5x-1)(25x^2+5x+1)$
13. $(2x-3)(x+1)(x-1)$
14. $(x-6)(x-2)(x^2+2x+4)$
15. $(5x+2)(x+3)(x-3)$
16. $(x+y)(x-y)(3x-4y)$
17. $(3x-5)(x-1)(x^2+x+1)$
18. $(4x-3)(x+2)(x^2-2x+4)$
19. $(2x+1)(2x-1)(x-2)(x^2+2x+4)$
20. $7x(x+2y)(x-2y)(x+1)(x-1)$

Notes-- Factoring Trinomials

TRINOMIAL: A polynomial with 3 terms.

ALWAYS look for a GCF first.
It is a good idea to make leading coefficient positive.
Guess and check method is one of many methods, but not the only one.

Factor the following:

1. $x^2 - 14x + 45$

2. $x^2 + 6x + 10$

3. $-8x^2 + 16x - 40$

4. $6x^2 + 7x - 20$

5. $2x^2 - 5x - 12$

6. $24 - 22x + 3x^2$

7. $8x^2 + 6xy - 5y^2$

8. $x^4 + 4x^2 - 5$

Factoring Trinomials

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Factor completely.

1. $x^2 - 9x + 14$
2. $x^2 - 9x + 8$
3. $x^2 - x + 7$
4. $x^2 - 6x + 9$
5. $3x^2 + 24x + 21$
6. $-2x^3 - 16x^2 - 32x$
7. $x^2 + 6xy + 5y^2$
8. $6x^2 - 13x - 5$
9. $12x^2 + 25x - 7$
10. $9x^2 - 15x + 4$
11. $9x^2 + 30x + 25$
12. $-2 + 17x + 9x^2$
13. $6x^2 - 13x - 15$
14. $20x^2 - 60x - 35$
15. $15x^3 + 42x^2 - 9x$
16. $9x^2 - 16xy - 4y^2$
17. $x^4 + 2x^2 - 3$
18. $4x^4 + 7x^2 - 2$
19. $6x^2 + 19x + 15$
20. $10x^2 - 19x + 6$

Factoring Trinomials-Answers

1. $(x-7)(x-2)$
2. $(x-8)(x-1)$
3. *PRIME*
4. $(x-3)(x-3)$ or $(x-3)^2$
5. $3(x+7)(x+1)$
6. $-2x(x+4)(x+4)$
7. $(x+5y)(x+y)$
8. $(3x+1)(2x-5)$
9. $(4x-1)(3x+7)$
10. $(3x-4)(3x-1)$
11. $(3x+5)(3x+5)$ or $(3x+5)^2$
12. $(9x-1)(x+2)$
13. $(x-3)(6x+5)$
14. $5(2x+1)(2x-7)$
15. $3x(5x-1)(x+3)$
16. $(9x+2y)(x-2y)$
17. $(x+1)(x-1)(x^2+3)$
18. $(2x+1)(2x-1)(x^2+2)$
19. $(3x+5)(2x+3)$
20. $(5x-2)(2x-3)$

Notes-- Solve Quadratic Equations by Factoring

A quadratic equation has the form of $ax^2 + bx + c = 0$

Steps to solve by factoring:

1. Write in standard form: $ax^2 + bx + c = 0$
2. Factor.
3. Set each factor that has a variable equal to zero.
4. Solve each resulting linear equation.

1. $-14x^2 + 21x = 0$

2. $12x^2 - 26x - 10 = 0$

3. $5x^2 + 8x - 1 = x(x + 8)$

4. $7x^2 + 23x + 4 = (x + 4)(x - 4)$

Quadratic Equations

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Solve by factoring.

1. $x^2+6x+8=0$
2. $2x^2+12x-32=0$
3. $6x^2-2x=0$
4. $4x^2-9=0$
5. $2x^2-7x-4=0$
6. $6x^2-41x-7=0$
7. $12x^2-31x+9=0$
8. $x^2+5x-4=10$
9. $8x^2-13x-9=5x-4$
10. $3x^2+5x+20=2x^2-3x+4$
11. $9x^2+7x-7=3x(x-4)$
12. $3x^2-8x-31=(2x+3)(x-7)$
13. $(2x+5)(x-4)=(x+7)(x-4)$

Quadratic Equations-Answers

1. $\{-2,-4\}$

2. $\{-8,2\}$

3. $\left\{0,\frac{1}{3}\right\}$

4. $\left\{-\frac{3}{2},\frac{3}{2}\right\}$

5. $\left\{-\frac{1}{2},4\right\}$

6. $\left\{-\frac{1}{6},7\right\}$

7. $\left\{\frac{9}{4},\frac{1}{3}\right\}$

8. $\{-7,2\}$

9. $\left\{-\frac{1}{4},\frac{5}{2}\right\}$

10. $\{-4\}$

11. $\left\{-\frac{7}{2},\frac{1}{3}\right\}$

12. $\{-5,2\}$

13. $\{2,4\}$

Notes-- Evaluating Integer Exponents

5^2

Negative Exponents: 2^{-3}

Take the reciprocal of base and change the sign of the exponent.

$\frac{x^{-2}}{y^3} =$ $\frac{x^{-3}}{y^{-2}} =$

Zero Exponent: $5^0 =$ $x^0 =$ $(10x^2y^4)^0 =$

ANYTHING (except zero) raised to the power of zero equals 1.

Evaluate each of the following.

1. $2^5 \cdot 2^{-2}$
2. -5^{-4}
3. $(-2)^{-4}$

4. $(-5)^{-2}$ compared to -5^{-2}

5. $3^{-1} - 2^{-2}$
6. $3^{-2} - 3^{-1}$

7. $\left(3^{-2} - 3^{-1}\right)^{-2}$
8. $-5^{-1} + 5^0 + 5$

Notes-- Simplifying Integer Exponents

Review Negative Exponents:

Take the reciprocal of base and change the sign of the exponent.

$\frac{x^{-2}}{y^{3}}=$ $\frac{x^{-3}}{y^{-2}}=$

Zero Exponent: $x^{0}=$ $(10x^{2}y^{4})^{0}=$

Simplify each of the following. Your answers should have no NEGATIVE exponents.

1. Multiply the exponents to get rid of parentheses.
2. Make all exponents positive.
3. Clean up.

1. $\left(3x^{-2}y^{-3}\right)^{2}\left(5xy^{-1}\right)^{-3}$

2. $\dfrac{\left(3x^{5}y^{-4}\right)^{-2}}{\left(2x^{-2}y^{-3}\right)^{3}}$

3. $\left(\dfrac{5x^{2}y^{-5}}{8x^{6}y^{-12}}\right)^{2}$

4. $\left(\dfrac{6x^{3}y^{-2}}{9^{0}x^{-5}y^{-6}}\right)^{-3}$

Integer Exponents

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Evaluate each of the following. Your answers should have NO exponents.

1. $2^2 \cdot 2^3$
2. $\dfrac{(-7)^4}{(-7)^5}$
3. $(4^{-1})^{-2}$
4. -8^2
5. $(-8)^2$
6. -2^3
7. $(-2)^3$
8. $(-3)^{-4}$
9. -3^{-4}
10. $(-3)^{-3}$
11. -3^{-3}
12. $2^{-3} + 4^{-1}$
13. $4^{-2} + 3^{-1}$
14. $2^{-2} + 4^{-1}$
15. $8^0 + 8^{-1} + 8$
16. $5^0 + 5^{-1} + 5$
17. $\left(\dfrac{2}{3}\right)^3$
18. $\left(\dfrac{5}{8}\right)^{-1}$
19. $\left(\dfrac{4}{3}\right)^{-2}$
20. $\dfrac{5^{-2}}{3^{-1}}$

Simplify each of the following. Your answers should have no NEGATIVE exponents.

21. $\left(\dfrac{8x^9y^3}{4x^3y^2}\right)^2$
22. $(3x^{-2}y^2)^{-1}(4xy^3)^{-2}$
23. $(8x^0y^{-4})^0$
24. $\left(\dfrac{2x^4}{5y^{-2}}\right)^3$
25. $\left(\dfrac{5x^{-3}y}{3x^2y^{-2}}\right)^3$
26. $\dfrac{8x^3y^{-2}}{10x^{-6}y^5}$
27. $\dfrac{2^{-2}x^{-3}y^{-4}}{8^{-1}x^{-1}y^{-2}}$
28. $\dfrac{\left(2x^3y^{-4}\right)^4}{\left(5x^0y^7\right)^2}$
29. $\dfrac{\left(-3x^4y^7\right)^3}{\left(2x^{-4}y^3\right)^{-4}}$
30. $\left(\dfrac{4^0x^{-2}y^6}{8x^3y^4}\right)^{-2}$

Integer Exponents-Answers

1. 32
2. $\frac{-1}{7}$
3. 16
4. –64
5. 64
6. –8
7. –8
8. $\frac{1}{81}$
9. $\frac{-1}{81}$
10. $\frac{-1}{27}$
11. $\frac{-1}{27}$
12. $\frac{3}{8}$
13. $\frac{19}{48}$
14. $\frac{1}{2}$
15. $\frac{73}{8}$ or $9\frac{1}{8}$
16. $\frac{31}{5}$ or $6\frac{1}{5}$
17. $\frac{8}{27}$
18. $\frac{8}{5}$
19. $\frac{9}{16}$
20. $\frac{3}{25}$
21. $4x^{12}y^{2}$
22. $\frac{1}{48y^{8}}$
23. 1
24. $\frac{8x^{12}y^{6}}{125}$
25. $\frac{125y^{9}}{27x^{15}}$
26. $\frac{4x^{9}}{5y^{7}}$
27. $\frac{2}{x^{2}y^{2}}$
28. $\frac{16x^{12}}{25y^{30}}$
29. $\frac{-432y^{33}}{x^{4}}$
30. $\frac{64x^{10}}{y^{4}}$

Notes-- Reducing Rational Expressions

Review Reducing Fractions:

$\frac{25}{30} =$ $\frac{7}{9} =$ $\frac{x+5}{x^2+25} =$

1. Factor.
2. "Cancel" common FACTORS.

1. $\frac{8xy^5z^3}{-2x^3y^4z}$

2. $\frac{2x^2y-14xy}{x^2y+3xy^2}$

3. $\frac{x^2-6x-16}{64-x^2}$

4. $\frac{8x^3+27}{2x^2-3x-9}$

Reducing Rational Expressions

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Reduce the following expressions to lowest terms.

1. $\frac{8x^4y^2}{6xy^3}$

2. $\frac{-9ab^2c^3}{4ab^3c}$

3. $\frac{3x^3+x^2y}{x^2y^2-2xy^3}$

4. $\frac{3x+6y}{3x}$

5. $\frac{4x^3y-4x^2y}{2xy^2-2x^2y^2}$

6. $\frac{x^2+5x+6}{x^2-4x-21}$

7. $\frac{6x^2+7x-3}{4x^2-9}$

8. $\frac{6x^2+7x-5}{2x^2-x}$

9. $\frac{x^2+3x-10}{4-x^2}$

10. $\frac{4x^2-12x+9}{2x^2-11x+12}$

11. $\frac{3x^2+9x-30}{6x^2+30x}$

12. $\frac{2x^2+5x-12}{9-4x^2}$

13. $\frac{x^3+8}{3x^2+2x-8}$

14. $\frac{27x^3-1}{6x^2+4x-2}$

Reducing Rational Expressions-Answers

1. $\frac{4x^3}{3y}$
2. $\frac{-9c^2}{4b}$
3. $\frac{x(3x+y)}{y^2(x-2y)}$
4. $\frac{x+2y}{x}$
5. $\frac{-2x}{y}$
6. $\frac{x+2}{x-7}$
7. $\frac{3x-1}{2x-3}$
8. $\frac{3x+5}{x}$
9. $\frac{-(x+5)}{(2+x)}$
10. $\frac{(2x-3)}{(x-4)}$
11. $\frac{(x-2)}{2x}$
12. $\frac{-(x+4)}{(3+2x)}$
13. $\frac{x^2-2x+4}{3x-4}$
14. $\frac{9x^2+3x+1}{2(x+1)}$

Notes--

Multiplying and Dividing Rational Expressions

Directions:

Perform the indicated operations and reduce to lowest terms.

Even though the "indicated operations" are multiplication and division, what we need to do is FACTOR and REDUCE.

1. $\frac{10m^6p^7}{9x^3y^2} \cdot 18x^5y^3 \div \frac{4x^2y}{3m^2p}$

2. $\frac{4-x}{6x+9} \div \frac{5x-20}{2xz+3z}$

1. Factor
2. Reduce

3. $\frac{2x^2-x-15}{2x^2+9x+10} \cdot \frac{x^2-3x-10}{21x-7x^2}$

Multiplying and Dividing Rational Expressions

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Perform the indicated operations and reduce to lowest terms.

1. $\frac{5xy^2}{7a^3b^4} \div \frac{3x^4y^7}{6a^2b^6} \cdot \frac{14x^5y^2}{15a^4b}$

2. $\frac{5x^3y^5}{8a^3b^2} \cdot \frac{14ab^7}{9xy^8} \div \frac{a^2b^2}{12xy}$

3. $\frac{2x+2}{5x-15} \cdot \frac{x-3}{xy+y}$

4. $\frac{10x^2y^2+5xy^2}{8-4x} \div \frac{6x+3}{12x-24}$

5. $\frac{10x^2+17x+3}{2x^2+7x+6} \div \frac{5x^2+41x+8}{2x^2+x-6}$

6. $\frac{x^2-1}{3x^2-x-4} \cdot \frac{3x^2+2x-8}{5x-5x^2}$

7. $\frac{6-23x-4x^2}{3x^2+10x-48} \cdot \frac{3x^2+x-24}{8x^2-6x+1}$

8. $\frac{3x^2-10x-8}{2x^2-3x-20} \cdot (2x+5)$

9. $\frac{x^3-8}{x^2+2x-3} \cdot \frac{3x^2-2x-1}{3x^2-5x-2}$

10. $\frac{x^3-1}{2x^2-11x+12} \cdot \frac{2x^2-3x-20}{8x^3+12x^2} \div \frac{2x^2+3x-5}{4x^2-9}$

Multiplying and Dividing Rational Expressions-Answers

1. $\frac{4bx^2}{3a^5y^3}$

2. $\frac{35b^3x^3}{3a^4y^2}$

3. $\frac{2}{5y}$

4. $-5xy^2$

5. $\frac{2x-3}{x+8}$

6. $\frac{-(x+2)}{5x}$

7. $\frac{-(x+3)}{2x-1}$

8. $3x+2$

9. $\frac{x^2+2x+4}{x+3}$

10. $\frac{x^2+x+1}{4x^2}$

Notes--

Adding and Subtracting Rational Expressions—Part 1

When adding or subtracting fractions that have the same denominator:

1. Keep the denominator and collect like terms in the numerators.
2. Try to reduce the expression by factoring.

Directions:

Perform the indicated operations and reduce to lowest terms.

1. $\frac{25}{3x}-\frac{4}{3x}$

2. $\frac{3x+1}{5x-2}+\frac{4x-6}{5x-2}$

3. $\frac{x^2}{2x+3}-\frac{x+6}{2x+3}$

OPPOSITE DENOMINATORS

4. $\frac{5}{x-2}-\frac{6}{2-x}$

5. $\frac{x}{y-3}+\frac{5x}{3-y}$

6. $\frac{3x^2-7}{x-4}+\frac{x^2-2x+12}{4-x}$

Notes--

Adding and Subtracting Rational Expressions—Part 2

When adding or subtracting fractions that have **different** denominators, one must find the least common denominator (**LCD**) before adding or subtracting the fractions.

Process to find the LCD:

1. Factor each denominator.
2. Write down **one** of every kind of factor.
3. Raise each factor to its highest power.

Find the LCD:

$\frac{5}{a^2b^3c} - \frac{7}{ab^4c^5d}$ $\qquad$ $\frac{5}{6x^3y} + \frac{7}{4x^2y^5}$

Perform the indicated operations and reduce to lowest terms:

1. Find the LCD
2. Re-write each fraction with the LCD
3. Collect like terms of numerators
4. Reduce, if possible.

1. $\frac{5}{x^3} - \frac{1}{8x}$ $\qquad$ 2. $\frac{3}{x-1} + \frac{6}{x+4}$

3. $\frac{3x+1}{x+3} - \frac{x-4}{3x-4}$ $\qquad$ 4. $\frac{2x-7}{x^2+3x-4} + \frac{x+31}{x^2-x-20}$ $\qquad$ 5. $\frac{3x-2}{2x^2-9x+10} - \frac{x+6}{x^2-6x+8}$

Adding and Subtracting Rational Expressions

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Perform the indicated operations and reduce to lowest terms.

1. $\frac{3}{2x}+\frac{7}{2x}$

2. $\frac{2n}{xy}-\frac{9n}{xy}$

3. $\frac{x}{4x-7}-\frac{3x+5}{4x-7}$

4. $\frac{5x+1}{2x^2+x-11}-\frac{2x+9}{2x^2+x-11}$

5. $\frac{x^2+2x}{x+1}+\frac{1}{x+1}$

6. $\frac{7x}{y-2}-\frac{3x}{2-y}$

7. $\frac{2x^2-8}{x-5}+\frac{x^2+x+12}{5-x}$

8. $\frac{2x^2-5}{x-1}-\frac{x^2+2}{1-x}$

9. $\frac{8}{5x}+\frac{3}{2y}$

10. $\frac{3}{4xy^2}+\frac{7}{3x^2y}$

11. $\frac{5}{x+2}+\frac{2}{x+3}$

12. $\frac{3}{2x+5}-\frac{5}{x-3}$

13. $\frac{x+2}{2x-3}-\frac{x-3}{x+6}$

14. $\frac{x-1}{2x+5}+\frac{3x-2}{2x-3}$

15. $\frac{2}{x^2-4x-5}+\frac{5}{x^2-2x-15}$

16. $\frac{4x}{3x^2-5x-2}-\frac{1}{3x^2+13x+4}$

17. $\frac{2x+3}{x^2-1}+\frac{x-2}{x^2-6x+5}$

18. $\frac{3x-5}{x^2-x-12}-\frac{x+1}{x^2+5x+6}$

19. $\frac{x-1}{6x^2-7x+2}+\frac{x+2}{2x^2-7x+3}$

20. $\frac{x+1}{4x^2+4x-15}-\frac{4x+5}{8x^2-10x-3}$

Adding and Subtracting Rational Expressions--Answers

1. $\frac{5}{x}$

2. $-\frac{7n}{xy}$

3. $\frac{-2x-5}{4x-7}$

4. $\frac{3x-8}{2x^2+x-11}$

5. $x+1$

6. $\frac{10x}{y-2}$

7. $x+4$

8. $3(x+1)$

9. $\frac{16y+15x}{10xy}$

10. $\frac{9x+28y}{12x^2y^2}$

11. $\frac{7x+19}{(x+2)(x+3)}$

12. $\frac{-7x-34}{(2x+5)(x-3)}$

13. $\frac{-x^2+17x+3}{(2x-3)(x+6)}$

14. $\frac{8x^2+6x-7}{(2x+5)(2x-3)}$

15. $\frac{7x+11}{(x+1)(x+3)(x-5)}$

16. $\frac{4x^2+15x+2}{(3x+1)(x+4)(x-2)}$

17. $\frac{3x^2-8x-17}{(x+1)(x-1)(x-5)}$

18. $\frac{2(x-1)}{(x-4)(x+2)}$

19. $\frac{2x+1}{(3x-2)(x-3)}$

20. $\frac{-4x^2-25x-24}{(4x+1)(2x-3)(2x+5)}$

Notes-- Synthetic Division

Synthetic Division is a condensed method of long division. It is quick and easy. Unfortunately, it can only be used when the divisor is in the form of $(x \pm a)$

Review long division: $\dfrac{9x^2 - 5x + 1}{x - 1}$

Synthetic division:

$\dfrac{9x^2 - 5x + 1}{x - 1}$

$\dfrac{2x^3 - 5x^2 - 3}{x + 1}$

$\dfrac{x^4 - 5x^2 + 10x}{x + 3}$

$\dfrac{x^3 + 125}{x + 5}$

Reminders:
1. Write both polynomials in standard form.
2. Fill in all missing terms with a place holder of zero.
3. Write your answer as a polynomial that is **<u>one degree less</u>** than the dividend (numerator).

Division of Polynomials

This is an optional topic. Long division is covered in 0308. Synthetic is a good tool for 1314

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Perform the indicated divisions using LONG division method.

1. $\dfrac{24x^2-62x+36}{4x-7}$

2. $\dfrac{40x^2-44x+17}{5x-3}$

3. $\dfrac{2x^3-13x^2-10x+19}{2x+3}$

4. $\dfrac{-19-8x+21x^2+12x^3}{4x+7}$

5. $\dfrac{4x^2-7}{2x-3}$

6. $\dfrac{4x^3-x-7}{2x-3}$

7. $\dfrac{6x^3+20x^2-19}{2x+6}$

8. $\dfrac{2x^4+x^3-3x^2+9x-13}{x^2+x-3}$

Perform the indicated divisions using SYNTHETIC division.

9. $\dfrac{x^2-3x+5}{x-2}$

10. $\dfrac{2x^3-7x^2-x+6}{x-3}$

11. $\dfrac{x^3-7x-4}{x+2}$

12. $\dfrac{3x^3+11x^2-5x}{x+4}$ (Hint: need a place holder for the constant)

13. $\dfrac{x^4-2x^3-5x^2+x+5}{x+2}$

14. $\dfrac{2x^4-3x^3+x^2+7x-5}{x-1}$

15. $\dfrac{6x^4-7x^3-11x^2+2x+3}{x-\frac{1}{6}}$

Division of Polynomials-Answers

1. $6x-5+\frac{1}{4x-7}$
2. $8x-4+\frac{5}{5x-3}$
3. $x^2-8x+7-\frac{2}{2x+3}$
4. $3x^2-2-\frac{5}{4x+7}$
5. $2x+3+\frac{2}{2x-3}$
6. $2x^2+3x+4+\frac{5}{2x-3}$
7. $3x^2+x-3-\frac{1}{2x+6}$
8. $2x^2-x+4+\frac{2x-1}{x^2+x-3}$
9. $x-1+\frac{3}{x-2}$
10. $2x^2-x-4-\frac{6}{x-3}$
11. $x^2-2x-3+\frac{2}{x+2}$
12. $3x^2-x-1+\frac{4}{x+4}$
13. $x^3-4x^2+3x-5+\frac{15}{x+2}$
14. $2x^3-x^2+7+\frac{2}{x-1}$
15. $6x^3-6x^2-12x+\frac{3}{x-\frac{1}{6}}$

Notes-- Complex Fractions

A complex fraction is a fraction that has fractions in the numerator and/or the denominator.

Single Fractions:

Change the division to multiplication. Reduce.

Directions: Simplify the following fraction.

1. $\dfrac{\dfrac{21x^4}{8y}}{\dfrac{7x^3}{16y^2}}$

2. $\dfrac{\dfrac{x^2-5x+6}{10x+5}}{\dfrac{4-x^2}{6x+3}}$

3. $\dfrac{\dfrac{\frac{3}{x}-\frac{1}{y}}{6}}{5x^3y}$

Multiple Fractions:

1. Find the LCD of **all** the fractions.

2. Multiply **every** term by the LCD.

3. Reduce

4. $\dfrac{\frac{1}{x}-\frac{1}{2}}{\frac{5}{7x}-\frac{5}{14xy}}$

5. $\dfrac{2-\dfrac{7}{3x}-\dfrac{10}{3x^2}}{4-\dfrac{8}{3x}-\dfrac{5}{x^2}}$

Multiple Fractions:

1. Find the LCD of **all** the fractions.

2. Multiply **every** term by the LCD.

3. Reduce

6. $\dfrac{\dfrac{2}{x}+\dfrac{1}{y}}{x+3}$

7. $\dfrac{\dfrac{5+h}{3+h}-\dfrac{5}{3}}{h}$

Complex Fractions

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Simplify the following fractions.

1. $\dfrac{\frac{5}{7}}{\frac{45}{16}}$

2. $\dfrac{\frac{3x}{2y^2}}{\frac{5x^4}{4y^3}}$

3. $\dfrac{\frac{2x^2+3x}{y^2-y}}{\frac{8x^4+12x^3}{3y^3-3y^2}}$

4. $\dfrac{\frac{8xy-4y}{6xy^2}}{\frac{12x^2y-6xy}{9x^2y^2}}$

5. $\dfrac{2+\frac{3}{x}}{1-\frac{7}{x}}$

6. $\dfrac{\frac{2}{x}+\frac{2}{y}}{\frac{4}{x^2y^2}}$

7. $\dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x^2}-\frac{1}{y^2}}$

8. $\dfrac{1-\frac{5}{2x}-\frac{3}{2x^2}}{2-\frac{15}{2x}+\frac{9}{2x^2}}$

9. $\dfrac{2x-y}{\frac{2}{y}-\frac{1}{x}}$

10. $\dfrac{1-\frac{1}{9x^2}}{1-3x}$

11. $\dfrac{\frac{3}{x^2}-\frac{1}{3}}{x-3}$

12. $\dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{x-y}{5}}$

13. $\dfrac{\frac{4}{2+h}-2}{h}$

14. $\dfrac{\frac{4}{2x+1}+3}{\frac{5}{2x+1}-2}$

Complex Fractions--Answers

1. $\frac{16}{63}$

2. $\frac{6y}{5x^3}$

3. $\frac{3y}{4x^2}$

4. 1

5. $\frac{2x+3}{x-7}$

6. $\frac{xy(x+y)}{2}$

7. $\frac{xy}{x+y}$

8. $\frac{2x+1}{4x-3}$

9. xy

10. $\frac{-(3x+1)}{9x^2}$

11. $\frac{-(x+3)}{3x^2}$

12. $\frac{-5}{xy}$

13. $\frac{-2}{2+h}$

14. $\frac{6x+7}{-4x+3}$

Notes-- Rational Equations

1. Find the L.C.D.
2. Multiply EVERY term by the LCD to get rid of all the fractions.
(OR Cross-multiply, if you can)**
3. Solve the resulting equation.
4. Check for extraneous solutions. (Substitute answers into the denominator to see if this would cause division by zero. You must throw out any solutions that cause division by zero because it is undefined.)

1. $\frac{7}{6x} - \frac{5}{8x} = \frac{13}{12}$

2. $\frac{1}{x^2} - \frac{1}{12x} = \frac{1}{12}$

3. $\frac{3}{x-1} + \frac{4}{x+2} = \frac{16}{x^2+x-2}$

4. $\frac{3}{x-2} - \frac{2}{x} = \frac{6}{x^2-2x}$

5. $\frac{3}{x+1} = \frac{2x+5}{x^2+x}$ **

6. $\frac{5}{5x+3} = \frac{5x-2}{x-1}$ **

7. $\frac{2x+4}{x+2} - \frac{x+3}{x-2} = \frac{1}{2}$

Rational Equations

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Solve the following equations. Do not forget to check solutions.

1. $\frac{-3}{2x}=\frac{9}{8}$

2. $\frac{3}{2x}+\frac{1}{3x}=\frac{11}{12}$

3. $\frac{x+1}{2}=\frac{x-5}{4}$

4. $\frac{2}{2x-1}+\frac{3}{x-4}=\frac{5}{2x-1}$

5. $\frac{2}{x+1}-\frac{1}{3x-2}=\frac{3}{3x-2}$

6. $\frac{5}{2x+1}-\frac{1}{2x-1}=\frac{6}{4x^2-1}$

7. $\frac{1}{x}-\frac{2}{4x+1}=\frac{1}{8x^2+2x}$

8. $\frac{2x+1}{x-2}=\frac{2x+1}{x+2}+\frac{10}{x^2-4}$

9. $\frac{1}{x^2}-\frac{1}{6x}=\frac{1}{6}$

10. $\frac{5}{x^2}-\frac{1}{3x}=2$

11. $\frac{2}{2x-1}+\frac{x}{x+4}=\frac{36}{2x^2+7x-4}$

12. $\frac{4x}{x-1}-\frac{x}{x+3}=\frac{-12}{x^2+2x-3}$

13. $\frac{3x+6}{2x+5}=\frac{-1}{x}$

14. $\frac{x+3}{9x+1}=\frac{x+1}{7}$

15. $\frac{9x+3}{10x-30}=\frac{x}{x-3}$

16. $\frac{2}{x+3}+\frac{4}{x+1}=\frac{4}{3}$

17. $\frac{1}{x-5}-\frac{5}{x-2}=\frac{3}{2}$

18. $\frac{x+2}{2x-1}+\frac{x+5}{x+3}=\frac{5}{3}$

Rational Equations-Answers

1. $\left\{\frac{-4}{3}\right\}$
2. $\{2\}$
3. $\{-7\}$
4. $\{-3\}$
5. $\{4\}$
6. $\left\{\frac{3}{2}\right\}$
7. $\varnothing$
8. $\left\{\frac{3}{4}\right\}$
9. $\{-3,2\}$
10. $\left\{-\frac{5}{3},\frac{3}{2}\right\}$
11. $\left\{\frac{7}{2}\right\}$
12. $\left\{-\frac{4}{3}\right\}$
13. $\left\{-\frac{5}{3},-1\right\}$
14. $\left\{-\frac{5}{3},\frac{4}{3}\right\}$
15. $\varnothing$
16. $\left\{-\frac{5}{2},3\right\}$
17. $\left\{-1,\frac{16}{3}\right\}$
18. $\{-1,18\}$

Notes-- Applications—Rational Equations

Consecutive integers: Remember that integers are only negative and positive "whole numbers". They do not include any decimals or fractions.

Two consecutive integers: $x, x+1$

Two consecutive **odd** integers: $x, x+2$

Two consecutive **even** integers: $x, x+2$

Reciprocals: If the number is x, then its reciprocal would be $\frac{1}{x}$.

Define the variable. Write an equation. Solve.

1. The sum of a number and its reciprocal is $\frac{29}{10}$. What is the number?

2. The difference of the reciprocals of two consecutive integers is $\frac{1}{2}$. What are the integers?

3. The sum of the reciprocals of two consecutive even integers is $\frac{9}{40}$. What are the integers?

Work:

$$\frac{1}{\text{time alone}} + \frac{1}{\text{time alone}} = \frac{1}{\text{time together}}$$

4. Paris can wash her car in 4½ hours. Her friend, Celia, can wash the same car in 7 hours. Working together, how long will it take them to wash the car?

5. Working together, Joseph and Dylan can write the computer program in 11 hours. Working alone, Joseph can write the computer program in 15 hours. How long does it take Dylan to write the program by himself?

Applications-Rational Equations

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Find the solutions to the following problems.

1. The sum of a number and its reciprocal is $\frac{25}{12}$. What is the number?
2. The sum of the reciprocals of two consecutive integers is $\frac{7}{12}$. What are the integers?
3. The sum of the reciprocals of two consecutive even integers is $\frac{7}{24}$. What are the integers?
4. Tameka can mow her yard in 2 hours. Her brother Dante can mow the yard in 3 hours. Working together, how long will it take them to mow the yard?
5. Brittany can prepare her report in $1\frac{1}{2}$ hours. Her co-worker, Firza, can prepare the report in 6 hours. Working together, how long will it take them to prepare the report?
6. Working together Debbie and Jim can clean the house in 4 hours. Working alone, Debbie can clean the house in 5 hours. How long does it take Jim to clean the house?
7. Working together, Elmer and his son can paint their house in 3 days. Working alone, Elmer can spray paint the house in 4 days. How long will it take his son, using a brush, to paint the house by himself?

Applications-Rational Equations-Answers

1. $\frac{3}{4}$ or $\frac{4}{3}$
2. 3 and 4
3. 6 and 8
4. $1\frac{1}{5}$ hours
5. $1\frac{1}{5}$ hours
6. 20 hours
7. 12 days

Radicals

What does a radical sign look like?

Here are some examples: $\sqrt{}$, $\sqrt[3]{}$, $\sqrt[4]{}$, $\sqrt[5]{}$

Square root: $\sqrt{49}$ $\sqrt{\frac{1}{81}}$ $\sqrt{-16}$

Cube root: $\sqrt[3]{8}$ $\sqrt[3]{-27}$ $\sqrt[3]{\frac{1}{64}}$

Fourth root: $\sqrt[4]{16}$ $\sqrt[4]{(-2)^4}$ $\sqrt[4]{-16}$

Fifth root: $\sqrt[5]{32}$ $\sqrt[5]{-32}$ $\sqrt[5]{(-3)^5}$

Even root of a negative number is NOT real.	**Odd** root of a negative number is a negative number.

Convert rational exponents to radicals:

$8^{4/9}$ $(5x^2y)^{2/5}$

Convert radicals to exponents. Simplify where possible.

$\sqrt{49}$ $\sqrt[3]{8}$ $\sqrt[5]{32}$

$\sqrt[4]{7^3}$ $\left(\sqrt[3]{x}\right)^5$

$\sqrt[5]{p^{20}}$ $\sqrt[3]{(5x^2y^2)^{12}}$

Radicals

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Evaluate each of the following, if possible.

1. $\sqrt{100}$
2. $\sqrt{\frac{121}{4}}$
3. $\sqrt[3]{27}$
4. $\sqrt[3]{-64}$
5. $\sqrt{-49}$
6. $\sqrt[3]{\frac{8}{27}}$
7. $\sqrt[3]{4^3}$
8. $\sqrt{13^2}$
9. $\sqrt[4]{\frac{1}{16}}$
10. $\sqrt[5]{\frac{32}{243}}$
11. $\sqrt[4]{-\frac{1}{81}}$
12. $\sqrt[5]{-32}$
13. $\frac{\sqrt{100}}{\sqrt{49}}$
14. $\frac{\sqrt[3]{27}}{\sqrt[3]{64}}$
15. $\sqrt{\sqrt{81}}$

Convert the following expressions to radicals.

16. $7^{2/3}$
17. $(-5)^{1/5}$
18. $(4xy)^{3/4}$
19. $(2x)^{1/2}$

Convert the following radicals to expressions with rational exponents. Simplify where possible.

20. $\sqrt{7}$
21. $\left(\sqrt[3]{x}\right)^8$
22. $\sqrt[4]{k^4}$
23. $\sqrt{\left(7x^3y\right)^6}$
24. $\sqrt[3]{a^{15}}$

Radicals-Answers

1. 10
2. $\frac{11}{2}$
3. 3
4. –4
5. not real
6. $\frac{2}{3}$
7. 4
8. 13
9. $\frac{1}{2}$
10. $\frac{2}{3}$
11. not real
12. –2
13. $\frac{10}{7}$
14. $\frac{3}{4}$
15. 3
16. $\sqrt[3]{49}$
17. $\sqrt[5]{-5}$
18. $\sqrt[4]{64x^3y^3}$
19. $\sqrt{2x}$
20. $7^{1/2}$
21. $x^{8/3}$
22. k
23. $\left(7x^3y\right)^3 = 343x^9y^3$
24. a^5

Notes-- Rational Exponents

Evaluate each of the following, if possible.

1. **Make any negative exponents positive.**
2. **Change to radicals**
3. **Simplify.**

1. $100^{1/2}$

2. $(-25)^{1/2}$

3. $-25^{1/2}$

4. $-64^{5/3}$

5. $\left(\frac{81}{49}\right)^{3/2}$

6. $-\left(\frac{1}{32}\right)^{2/5}$

7. $25^{-1/2}$

8. $(-25)^{-1/2}$

9. $(-32)^{-3/5}$

10. $8^{1/2} \cdot 8^{1/6}$

11. $\frac{8^{1/2}}{8^{1/6}}$

Simplify. All answers should have only POSITIVE exponents.

12. $x^{1/5} \cdot x^{3/4}$

13. $\left(25x^5\right)^{1/2} \cdot \left(3x^{1/4}\right)$

14. $\frac{-5x^{-2}y}{10x^{1/4}y^{2/3}}$

Rational Exponents

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Evaluate each of the following, if possible.

1. $36^{1/2}$
2. $\left(\frac{25}{49}\right)^{1/2}$
3. $(-9)^{1/2}$
4. $-4^{1/2}$
5. $(-16)^{1/4}$
6. $-16^{1/4}$
7. $64^{1/3}$
8. $\left(\frac{-8}{125}\right)^{1/3}$
9. $-\left(\frac{27}{8}\right)^{1/3}$
10. $\left(\frac{-1}{32}\right)^{1/5}$
11. $4^{5/2}$
12. $-8^{2/3}$
13. $-\left(\frac{1}{16}\right)^{3/2}$
14. $49^{-1/2}$
15. $25^{-3/2}$
16. $(-49)^{-3/2}$
17. $-100^{-1/2}$
18. $\left(\frac{16}{81}\right)^{3/4}$
19. $25^{1/3} \cdot 25^{1/6}$
20. $\frac{27^{1/2}}{27^{1/6}}$

Simplify each of the following. Your answers should have no NEGATIVE exponents.

21. $x^{3/4} \cdot x^{1/12}$
22. $\left(4x^3\right)^{1/2} \cdot \left(8x^{1/3}\right)$
23. $\frac{2x^{-2/3}}{3x^{1/4}}$
24. $\frac{-6x^3 y^{-1/2}}{4x^{1/3} y^2}$
25. $2x^{1/3} \cdot 5x^{1/6}$
26. $\left(\frac{-27x^{-1}}{8y^{-3/2}}\right)^{2/3}$
27. $\left(\frac{25x^3}{9y^{1/3}}\right)^{-1/2}$
28. $-5x^{1/4} \cdot 7x^{-2/3}$

Rational Exponents-Answers

1. 6
2. $\frac{5}{7}$
3. not real
4. –2
5. not real
6. –2
7. 4
8. $-\frac{2}{5}$
9. $-\frac{3}{2}$
10. $-\frac{1}{2}$
11. 32
12. –4
13. $-\frac{1}{64}$
14. $\frac{1}{7}$
15. $\frac{1}{125}$
16. not real
17. $-\frac{1}{10}$
18. $\left(\frac{16}{81}\right)^{3/4} = \frac{8}{27}$
19. 5
20. 3
21. $x^{5/6}$
22. $16x^{11/6}$
23. $\frac{2}{3x^{11/12}}$
24. $\frac{-3x^{8/3}}{2y^{5/2}}$
25. $10x^{1/2}$
26. $\frac{9y}{4x^{2/3}}$
27. $\frac{3y^{1/6}}{5x^{3/2}}$
28. $\frac{-35}{x^{5/12}}$

Notes-- Simplify Radical Expressions—Part 1

1. Prime factor the number.
2. For square root:
   Look for pairs.
   For cube root:
   Look for 3 of a kind.
   For 4^{th} roots:
   Look for 4 of a kind. etc.
3. "Take out" the pairs, 3 of kind, etc.

Simplify the following:

$\sqrt{12}$ $\sqrt{98}$ $\sqrt{150}$

$\sqrt{162}$ $\sqrt{72}$

$\sqrt[3]{40}$ $\sqrt[3]{54}$ $\sqrt[3]{240}$

$\sqrt[4]{48}$ $\sqrt[4]{240}$

$\sqrt{x^3y^2}$ $\sqrt[3]{x^3y^2}$

$\sqrt[4]{x^4y^5z^6}$ $\sqrt[4]{x^{100}y^{52}z^{31}}$

Notes-- Simplify Radical Expressions—Part 2 Rationalize the Denominator

Rationalize the denominator means to eliminate any radicals in the denominator.

A process to follow is:

1. **Reduce** the fraction, if possible.
2. **Simplify** the radicals
3. **Rationalize** by multiplying by "what you need".
4. **Reduce** again if necessary.

Simplify the following:

SQUARE ROOTS:

1. $\frac{2}{\sqrt{3}}$

2. $\frac{\sqrt{50}}{\sqrt{32}}$

3. $\frac{3}{\sqrt{2y}}$

4. $\frac{\sqrt{5x}}{\sqrt{20x^2}}$

5. $\sqrt{\frac{1}{12x^3}}$

6. $\sqrt{\frac{49x^3}{9y^3}}$

CUBE ROOTS:

7. $\sqrt[3]{\frac{5}{2y}}$

8. $\sqrt[3]{\frac{5}{9y}}$

9. $\sqrt[3]{\frac{2y}{9x^5}}$

Simplify Radical Expressions

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Simplify the following radicals.

1. $\sqrt{48}$
2. $\sqrt{80}$
3. $\sqrt[3]{16}$
4. $\sqrt[4]{80}$
5. $\sqrt[3]{128x^7}$
6. $\sqrt{x^2y^2}$
7. $\sqrt{32x^7y^4}$
8. $\sqrt{144x^5y^7z^8}$
9. $\sqrt[3]{81x^9y^2z^4}$
10. $\frac{1}{\sqrt{3}}$
11. $\frac{\sqrt{3}}{\sqrt{12}}$
12. $\sqrt{\frac{3}{5}}$
13. $\frac{3}{\sqrt{6}}$
14. $\frac{5}{\sqrt{x}}$
15. $\frac{5}{\sqrt{8x}}$
16. $\sqrt[3]{\frac{1}{4}}$
17. $\sqrt[3]{\frac{1}{2}}$
18. $\sqrt{\frac{2}{9x}}$
19. $\sqrt{\frac{3}{7x}}$
20. $\sqrt{\frac{2x}{3y^3}}$
21. $\sqrt{\frac{8x^4}{5y^2}}$
22. $\sqrt[3]{\frac{7}{8x}}$
23. $\sqrt[3]{\frac{16}{3x^2}}$
24. $\sqrt[3]{\frac{3y^2}{5x^4}}$
25. $\sqrt{\frac{49x^3}{9y^3}}$

Simplify Radical Expressions-Answers

1. $4\sqrt{3}$
2. $4\sqrt{5}$
3. $2\sqrt[3]{2}$
4. $2\sqrt[4]{5}$
5. $4x^2\sqrt[3]{2x}$
6. xy
7. $4x^3y^2\sqrt{2x}$
8. $12x^2y^3z^4\sqrt{xy}$
9. $3x^3z\sqrt[3]{3y^2z}$
10. $\frac{\sqrt{3}}{3}$
11. $\frac{1}{2}$
12. $\frac{\sqrt{15}}{5}$
13. $\frac{\sqrt{6}}{2}$
14. $\frac{5\sqrt{x}}{x}$
15. $\frac{5\sqrt{2x}}{4x}$
16. $\frac{\sqrt[3]{2}}{2}$
17. $\frac{\sqrt[3]{4}}{2}$
18. $\frac{\sqrt{2x}}{3x}$
19. $\frac{\sqrt{21x}}{7x}$
20. $\frac{\sqrt{6xy}}{3y^2}$
21. $\frac{2x^2\sqrt{10}}{5y}$
22. $\frac{\sqrt[3]{7x^2}}{2x}$
23. $\frac{2\sqrt[3]{18x}}{3x}$
24. $\frac{\sqrt[3]{75x^2y^2}}{5x^2}$
25. $\frac{7x\sqrt{xy}}{3y^2}$

Notes--

Adding & Subtracting Radical Expressions—Part 1

Review of collecting like terms:

$$2x + 5x$$

$$2x^2 + 5x$$

Perform the indicated operations:

1. $4\sqrt{2} + 3\sqrt{2} - \sqrt{2}$

1. Simplify all radicals

2. Add the coefficients of "**like**" radicals.

2. $-\sqrt{40} - \sqrt{90} - \sqrt{160}$

3. $x^2\sqrt{18y} - 2x\sqrt{2x^2y} + 4y\sqrt{12x} - \sqrt{3xy^2}$

4. $\sqrt[3]{250xy^2} - \sqrt[3]{54xy^2}$

Notes--

Adding & Subtracting Radical Expressions—Part 2

Perform the indicated operations:

1. Simplify radicals.
2. Rationalize all denominators.
3. Find the LCD
4. Re-write all terms with LCD.
5. Combine like terms.

1. $\frac{4}{\sqrt{3}} - 7\sqrt{3}$

2. $\frac{5}{\sqrt{6}} + \sqrt{24}$

3. $\frac{2}{\sqrt[3]{3}} - \sqrt[3]{72}$

4. $\sqrt{\frac{2}{3}} + 5\sqrt{6} - \sqrt{\frac{3}{2}}$

Adding and Subtracting Radical Expressions

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Perform the indicated operations.

1. $3\sqrt{7}-5\sqrt{7}-8\sqrt{7}$
2. $2\sqrt{15x}-7\sqrt{15x}+5\sqrt{15x}$
3. $x\sqrt[3]{10y}-8x\sqrt[3]{10y}$
4. $5\sqrt{12}-2\sqrt{27}+3\sqrt{3}$
5. $5\sqrt{8}-\sqrt{18}+2\sqrt{32}$
6. $\sqrt[3]{32}+\sqrt[3]{108}$
7. $\sqrt{27x}-2\sqrt{12x}+2\sqrt{48x}$
8. $x\sqrt{18x}+5\sqrt{2x^3}+2x\sqrt{8x}$
9. $\sqrt{27x}+2\sqrt{12x}-\sqrt{150y}-4\sqrt{24y}$
10. $x\sqrt{20x^2}-x\sqrt{45}+2\sqrt{500}$
11. $\dfrac{1}{\sqrt{2}}+5\sqrt{2}$
12. $\dfrac{3}{\sqrt{5}}+\sqrt{80}$
13. $\dfrac{1}{\sqrt[3]{2}}-\sqrt[3]{108}$
14. $\sqrt{3}+\dfrac{5}{\sqrt{3}}+\dfrac{2\sqrt{27}}{3}$
15. $\sqrt{2}-\dfrac{4}{\sqrt{2}}-\dfrac{\sqrt{50}}{2}$

Adding and Subtracting Radical Expressions-Answers

1. $-10\sqrt{7}$
2. 0
3. $-7x\sqrt[3]{10y}$
4. $7\sqrt{3}$
5. $15\sqrt{2}$
6. $5\sqrt[3]{4}$
7. $7\sqrt{3x}$
8. $12x\sqrt{2x}$
9. $7\sqrt{3x}-13\sqrt{6y}$
10. $2x^2\sqrt{5}-3x\sqrt{5}+20\sqrt{5}$
11. $\frac{11\sqrt{2}}{2}$
12. $\frac{23\sqrt{5}}{5}$
13. $\frac{-5\sqrt[3]{4}}{2}$
14. $\frac{14\sqrt{3}}{3}$
15. $\frac{-7\sqrt{2}}{2}$

Notes-- Multiply Radicals

Multiplying a monomial by a monomial:
Multiply the "outsides"
Multiply the "insides"
Simplify, if possible.

$(2\sqrt{3})(4\sqrt{15})$ $(2\sqrt{3y})(5\sqrt{2x})$

Multiply a square root by the SAME square root:

$(\sqrt{3})(\sqrt{3})$ $(\sqrt{7y})(\sqrt{7y})$ $(\sqrt{5-y})(\sqrt{5-y})$

Perform the indicated operations and simplify your answers:

1. F.O.I.L.
2. Simplify
3. Combine like terms.

1. $3\sqrt{5y}\left(\sqrt{5y} + \sqrt{x} - 2\sqrt{3}\right)$

2. $(4\sqrt{2} - 2\sqrt{3})(5\sqrt{3} - 8\sqrt{6})$

3. $\left(3\sqrt{x} - 2\sqrt{y}\right)^2$

4. $(5+\sqrt{x+3})^2$

5. $(2-3\sqrt{5x-1})^2$

Notes-- Divide Radicals

Review: $(x+5)(x-5)$ $(\sqrt{x}+5)(\sqrt{x}-5)$ $(\sqrt{3}+\sqrt{5})(\sqrt{3}-\sqrt{5})$

The "*conjugate*" of $\sqrt{x}-5$ is $\sqrt{x}+5$ The "*conjugate*" of $\sqrt{3}+\sqrt{5}$ is $\sqrt{3}-\sqrt{5}$

As you can see, the conjugate is found by changing the **middle** sign.
When you multiply conjugates, you just need to square each term and then subtract.
We use the conjugate to rationalize the binomial denominators.

Rationalize the denominator of the following.

1. $\dfrac{1}{\sqrt{5}-1}$

2. $\dfrac{\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

3. $\dfrac{\sqrt{8}-\sqrt{2}}{\sqrt{6}-\sqrt{8}}$

4. $\dfrac{\sqrt{2x}-\sqrt{y}}{\sqrt{3x}+\sqrt{5y}}$

Operations with Radical Expressions

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Perform the indicated multiplications and simplify your answers.

1. $\sqrt{3}\left(2+\sqrt{5}\right)$
2. $\sqrt{x}\left(3+\sqrt{x}\right)$
3. $\left(2+\sqrt{7}\right)\left(\sqrt{8}-3\right)$
4. $\left(\sqrt{5}+\sqrt{7}\right)\left(\sqrt{10}+\sqrt{2}\right)$
5. $\left(2\sqrt{3}+4\sqrt{5}\right)\left(3\sqrt{2}-6\sqrt{7}\right)$
6. $\left(2\sqrt{x}+3\sqrt{y}\right)\left(4\sqrt{x}+5\sqrt{y}\right)$
7. $\left(3\sqrt{2}-6\sqrt{3}\right)^2$
8. $\left(2\sqrt{x}+7\right)^2$
9. $\left(3-\sqrt{x+2}\right)^2$
10. $\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)$
11. $\left(5\sqrt{3}+\sqrt{6}\right)\left(5\sqrt{3}-\sqrt{6}\right)$
12. $\left(3+4\sqrt{2x-6}\right)^2$

Rationalize the denominator of the following radicals.

13. $\dfrac{1}{\sqrt{3}+2}$
14. $\dfrac{2}{\sqrt{3}+\sqrt{5}}$
15. $\dfrac{x}{\sqrt{x}+\sqrt{y}}$
16. $\dfrac{2+\sqrt{3}}{2-\sqrt{3}}$
17. $\dfrac{\sqrt{x}+5}{\sqrt{x}-4}$
18. $\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{6}-\sqrt{3}}$
19. $\dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}$
20. $\dfrac{5\sqrt{3}+7}{2\sqrt{3}-4}$

Operations with Radical Expressions-Answers

1. $2\sqrt{3}+\sqrt{15}$
2. $3\sqrt{x}+x$
3. $4\sqrt{2}-6+2\sqrt{14}-3\sqrt{7}$
4. $5\sqrt{2}+\sqrt{70}+\sqrt{10}+\sqrt{14}$
5. $6\sqrt{6}-12\sqrt{21}+12\sqrt{10}-24\sqrt{35}$
6. $8x+22\sqrt{xy}+15y$
7. $126-36\sqrt{6}$
8. $4x+28\sqrt{x}+49$
9. $x+11-6\sqrt{x+2}$
10. 2
11. 69
12. $32x-87+24\sqrt{2x-6}$

.....

13. $2-\sqrt{3}$
14. $\sqrt{5}-\sqrt{3}$
15. $\dfrac{x\sqrt{x}-x\sqrt{y}}{x-y}$
16. $7+4\sqrt{3}$
17. $\dfrac{x+9\sqrt{x}+20}{x-16}$
18. $\dfrac{3\sqrt{2}+3+2\sqrt{3}+\sqrt{6}}{3}$
19. $\dfrac{x+2\sqrt{xy}+y}{x-y}$
20. $\dfrac{-\left(29+17\sqrt{3}\right)}{2}$

Notes-- Radical Equations

Process:

1. **Isolate the radical.**
2. **Get rid of the radical by raising both sides to the appropriate power.**

$$\left(\sqrt{x}\right)^2 = x \qquad \left(\sqrt[3]{x}\right)^3 = x \qquad \left(\sqrt[4]{x}\right)^4 = x$$

3. **Solve the resulting equation.**
4. **Check for extraneous solutions.**

1. $\sqrt{4x+1}-5=0$

2. $\sqrt[3]{x^2+4}-1=4$

3. $\sqrt{x^2+16}+6=1$

4. $\sqrt[4]{x^2+x-4}=2$

5. $2x=\sqrt{4x+15}$

6. $\sqrt{3x+4}-2=x$

Extra Example: $\sqrt{30-2x}+x=3$

Remember that a fractional exponent can be written in radical form.

$x^{3/2}=\sqrt{x^3}$ or $\left(\sqrt{x}\right)^3$ $\qquad$ $x^{2/5}=\sqrt[5]{x^2}$ or $\left(\sqrt[5]{x}\right)^2$

If you encounter an equation that has a variable raised to a fractional exponent, you solve it just like a radical equation.

Get rid of the radical by raising both sides to the appropriate power.

$\left(x^{3/2}\right)^{2/3}=x$ $\qquad$ $\left(x^{2/5}\right)^{5/2}=x$

7. $\left(x^2+6x-7\right)^{3/2}=27$

8. $\left(x-2\right)^{2/3}=9$

Radical Equations

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Find the solutions(s) of the following radical equations.

1. $\sqrt{2x+1}=3$
2. $\sqrt{x^2+7}-4=0$
3. $\sqrt{3x-7}+2=0$
4. $\sqrt[3]{3x+1}=4$
5. $\sqrt[3]{3x^2-10x}+2=0$
6. $\sqrt[4]{3x+5}=-3$
7. $\sqrt{3x+40}=x$
8. $\sqrt{2x^2+2x-3}=x$
9. $\sqrt{4x^2-x-1}+1=2x$
10. $\sqrt{x^2-8x+26}+5=x$
11. $\sqrt{2x+5}-x=3$
12. $\sqrt{x+8}-x=-4$
13. $\sqrt{3x+5}+1=3x$
14. $\sqrt{3-x}-x=3$
15. $\sqrt{x-4}+x=6$
16. $\left(x^2+4x-5\right)^{3/2}=64$
17. $\left(2x^2-5x+6\right)^{3/2}=8$
18. $\left(x-7\right)^{2/3}=4$
19. $\left(2x+5\right)^{2/3}=9$
20. $\left(x-1\right)^{2/5}=4$

Radical Equations-Answers

1. $\{4\}$
2. $\{3,-3\}$
3. $\varnothing$
4. $\{21\}$
5. $\left\{\frac{4}{3},2\right\}$
6. $\varnothing$
7. $\{8\}$
8. $\{1\}$
9. $\left\{\frac{2}{3}\right\}$
10. $\varnothing$
11. $\{-2\}$
12. $\{8\}$
13. $\left\{\frac{4}{3}\right\}$
14. $\{-1\}$
15. $\{5\}$
16. $\{-7,3\}$
17. $\left\{\frac{1}{2},2\right\}$
18. $\{-1,15\}$
19. $\{-16,11\}$
20. $\{-31,33\}$

Notes-- Complex Numbers

What happens when we want to solve the equation: $x^2 = -4$?

In order to solve this equation, we must introduce $\sqrt{-1}$ and the set of *imaginary* numbers.

We will represent $\sqrt{-1}$ with ***i***.

This leads to $\left(\sqrt{-1}\right)^2 = i^2$

$\left(\sqrt{-1}\right)^2 = -1 = i^2$

Therefore, $-1 = i^2$

Any square root of a negative number can be written in terms of ***i.***

$\sqrt{-2} =$ $\sqrt{-4} =$

$\sqrt{-9} =$ $\sqrt{-12} =$

A ***complex*** number has a real part and an imaginary part.

$3 + 5i$

We can add, subtract, multiply, and divide complex numbers.

Perform the indicated operations:

1. $(-4 + 7i) + (3 + 2i)$

2. $(3 - 5i) - (7 + 4i)$

3. $(2 + 3i)(4 + 5i)$

4. $(4 - i)^2$

Replace i^2 with -1

Rationalize the denominators. (Divide)

Review: $\dfrac{3}{5\sqrt{2}}$

5. $\dfrac{3-5i}{2i}$

Review: $\dfrac{3}{4+\sqrt{5}}$

The "***complex conjugate***" of $3+5i$ is $3-5i$

Multiply: $(3+5i)(3-5i)$

6. $\dfrac{2i}{3-5i}$

7. $\dfrac{3+2i}{-5+4i}$

Complex Numbers

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Perform the indicated operations.

1. $(2+3i)+(5-6i)$
2. $(-5-2i)-(3+i)$
3. $7(3+9i)$
4. $2i(4-7i)$
5. $(6+i)(3-4i)$
6. $(5-3i)(2-4i)$
7. $(2+5i)(2-5i)$
8. $(\sqrt{3}+i)(\sqrt{3}-i)$
9. $(2+3i)^2$
10. $(\sqrt{3}+i)^2$

Rationalize the denominators.

11. $\dfrac{5}{3i}$
12. $\dfrac{5+3i}{i}$
13. $\dfrac{4+i}{2i}$
14. $\dfrac{1}{4+2i}$
15. $\dfrac{3}{2-3i}$
16. $\dfrac{2i}{1+3i}$
17. $\dfrac{-4i}{-3-5i}$
18. $\dfrac{3-4i}{3+i}$
19. $\dfrac{9-17i}{2-i}$
20. $\dfrac{-2-5i}{1-4i}$

Complex Numbers-Answers

1. $7-3i$
2. $-8-3i$
3. $21+63i$
4. $14+8i$
5. $22-21i$
6. $-2-26i$
7. 29
8. 4
9. $-5+12i$
10. $2+2i\sqrt{3}$

.....

11. $-\frac{5}{3}i$
12. $3-5i$
13. $\frac{1}{2}-2i$ or $\frac{1-4i}{2}$
14. $\frac{2-i}{10}$ or $\frac{1}{5}-\frac{i}{10}$
15. $\frac{6+9i}{13}$ or $\frac{6}{13}+\frac{9i}{13}$
16. $\frac{3+i}{5}$ or $\frac{3}{5}+\frac{i}{5}$
17. $\frac{10+6i}{17}$ or $\frac{10}{17}+\frac{6i}{17}$
18. $\frac{1-3i}{2}$ or $\frac{1}{2}-\frac{3i}{2}$
19. $7-5i$
20. $\frac{18-13i}{17}$ or $\frac{18}{17}-\frac{13i}{17}$

Notes-- Quadratic Formula

This is another method to solve quadratic equations. If the quadratic cannot be factored we have to have something else that will allow us to solve the equation. There are 2 such methods—completing the square and the quadratic formula. The quadratic formula is derived from completing the square on the general equation: $ax^2+bx+c=0$

You MUST memorize the formula: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Process:

1. Write the equation in standard form: $ax^2+bx+c=0$
2. Identify $a, b,$ and c.
3. Substitute numbers into formula.
4. Carefully do the arithmetic under the square root sign.
5. If possible, simplify the radical.
6. If possible, reduce the fraction.

1. $x^2-4x-1=0$

2. $9x^2-18x+7=0$

3. $x^2+9x+11=3x-2$

4. $x(x+2)=6x-11$

5. $(3x-2)(x+4)=-7$

Quadratic Formula

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Use the quadratic formula to find the solutions of the following quadratic equations.

1. $x^2 + 9x - 9 = 0$

2. $9x^2 + 2x - 2 = 0$

3. $8x^2 + 10x + 1 = 0$

4. $6x^2 + 2x + 5 = 0$

5. $x^2 + 6x - 8 = 0$

6. $8x^2 - x + 2 = 0$

7. $4x^2 - 6x - 9 = 0$

8. $2x^2 + 8x + 7 = 0$

9. $4x^2 + 4x + 3 = 0$

10. $8x^2 - 4x - 5 = 0$

11. $x(2x + 3) = 7$

12. $2x(x + 3) = -1$

13. $(x + 5)(3x - 1) = 4$

14. $\frac{1}{2} - \frac{1}{x} - \frac{1}{x^2} = 0$

Quadratic Formula-Answers

1. $x = \frac{-9 \pm 3\sqrt{13}}{2}$

2. $x = \frac{-1 \pm \sqrt{19}}{9}$

3. $x = \frac{-5 \pm \sqrt{17}}{8}$

4. $x = \frac{-1 \pm i\sqrt{29}}{6}$

5. $x = -3 \pm \sqrt{17}$

6. $x = \frac{1 \pm 3i\sqrt{7}}{16}$

7. $x = \frac{3 \pm 3\sqrt{5}}{4}$

8. $x = \frac{-4 \pm \sqrt{2}}{2}$

9. $x = \frac{-1 \pm i\sqrt{2}}{2}$

10. $x = \frac{1 \pm \sqrt{11}}{4}$

11. $x = \frac{-3 \pm \sqrt{65}}{4}$

12. $x = \frac{-3 \pm \sqrt{7}}{2}$

13. $x = \frac{-7 \pm 2\sqrt{19}}{3}$

14. $x = 1 \pm \sqrt{3}$

Notes-- Extraction of Roots

Another method of solving quadratic equations is the extraction of roots. This is very convenient if the equation has a squared term that can be isolated. Some examples are:

$x^2 = 81$ $9x^2 - 23 = 0$ $(x-1)^2 = 25$ $(2x-10)^2 = 12$

Process:

1. Isolate the squared term.
2. Take the square root of both sides of the equation.
   Don't forget the "$\pm$" sign.
   Simplify all radicals. Rationalize all denominators.
3. Solve the equation.

1. $x^2 = 81$

2. $9x^2 - 23 = 0$

3. $7x^2 - 4 = 0$

4. $(x-1)^2 = 25$

5. $(2x-10)^2 = 12$

6. $(5x+3)^2 = -28$

Notes-- Completing the Square

This is another method to solve quadratic equations. If the quadratic cannot be factored we have to have something else that will allow us to solve the equation. There are 2 such methods—completing the square and the quadratic formula. Completing the Square is also used for other applications.

Process:

1. Write the equation in standard form: $ax^2+bx+c=0$
2. Move c to the left hand side of the equation.
$$x^2+bx+\_\_\_=-c+\_\_\_$$
3. If a is NOT = 1, divide all terms by a. Reduce any fractions.
4. Take $\frac{1}{2}$ of the coefficient of x.
5. Square this and add to both sides of the equation.
6. Re-write left hand side as a squared binomial.
7. Solve the equation by the extraction of roots method.

1. $x^2+8x-11=0$

2. $x^2-6x+18=0$

3. $x^2+3x-13=0$

4. $2x^2-2x+10=0$

5. $3x^2+5x+7=0$

Completing the Square

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Use the Extraction of Roots method to find the solutions of the following quadratic equations.

1. $x^2 = 25$
2. $x^2 = 80$
3. $x^2 = -121$
4. $x^2 - 64 = 0$
5. $4x^2 + 81 = 0$
6. $9x^2 - 49 = 0$
7. $5x^2 - 8 = 0$
8. $7x^2 - 2 = 0$
9. $2x^2 - 3 = 8$
10. $(2x - 7)^2 = 6$
11. $(4x - 5)^2 = 9$
12. $(x - 1)^2 = -75$
13. $(3x + 5)^2 = 27$
14. $(5x + 1)^2 = 50$

Find the solutions of the following quadratic equations by completing the square.

15. $x^2 + 4x - 1 = 0$
16. $x^2 - 2x - 7 = 0$
17. $x^2 + 4x + 29 = 0$
18. $x^2 - 2x + 37 = 0$
19. $x^2 + 5x - 2 = 0$
20. $4x^2 + 4x - 2 = 0$
21. $2x^2 - 5x - 4 = 0$
22. $4x^2 - 4x + 17 = 0$
23. $4x^2 + 12x + 7 = 0$

Completing the Square-Answers

1. $\{\pm 5\}$
2. $\{\pm 4\sqrt{5}\}$
3. $\{\pm 11i\}$
4. $\{\pm 8\}$
5. $\left\{\pm\frac{9}{2}i\right\}$
6. $\left\{\pm\frac{7}{3}\right\}$
7. $\left\{\pm\frac{2\sqrt{10}}{5}\right\}$
8. $\left\{\pm\frac{\sqrt{14}}{7}\right\}$
9. $\left\{\pm\frac{\sqrt{22}}{2}\right\}$
10. $\left\{\frac{7\pm\sqrt{6}}{2}\right\}$
11. $\left\{2,\frac{1}{2}\right\}$
12. $\{1\pm 5i\sqrt{3}\}$
13. $\left\{\frac{-5\pm 3\sqrt{3}}{3}\right\}$
14. $\left\{\frac{-1\pm 5\sqrt{2}}{5}\right\}$

.....

15. $\{-2\pm\sqrt{5}\}$
16. $\{1\pm 2\sqrt{2}\}$
17. $\{-2\pm 5i\}$
18. $\{1\pm 6i\}$
19. $\left\{\frac{-5\pm\sqrt{33}}{2}\right\}$
20. $\left\{\frac{-1\pm\sqrt{3}}{2}\right\}$
21. $\left\{\frac{5\pm\sqrt{57}}{4}\right\}$
22. $\left\{\frac{1\pm 4i}{2}\right\}$
23. $\left\{\frac{-3\pm\sqrt{2}}{2}\right\}$

Notes-- Applications—Quadratic Equations

Consecutive integers: Remember that integers are only negative and positive "whole numbers". They do not include any decimals or fractions.

Two consecutive integers: $x, x+1$

Two consecutive **odd** integers: $x, x+2$

Two consecutive **even** integers: $x, x+2$

Reciprocals: If the number is x, then its reciprocal would be $\frac{1}{x}$.

Define the variable. Write an equation. Solve.

1. Find two consecutive positive integers whose product is 132.	2. Find two consecutive odd integers whose product is 35.
3. The sum of the squares of two even consecutive integers is 340. Find the integers.	4. The sum of a number and its reciprocal is 6. What is the number?

5. Three times the square of a number is 6 more than twice the number. What is the number?

Area:

Rectangle: Area = *base x height* or Area = *length x width*.

Triangle: Area = $\frac{1}{2}$*base* x *height* or **2** ⁎ Area = *base* **x** *height*

6. The area of a rectangle is 35 sq. ft. The length is 1 ft. less than twice the width. What are the dimensions of the rectangle?

7. The height of a triangle is 2 in. more than three times the base. Find the base and the height if the area of the triangle is 5 sq. in.

Work:

$$\frac{1}{\text{time alone}} + \frac{1}{\text{time alone}} = \frac{1}{\text{time together}}$$

8. One pipe can fill a reservoir 2 hours faster than another pipe can. Together they fill the reservoir in 5 hours. How long does it take each pipe to fill the reservoir?

Applications-Quadratics Equations

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Find the solutions to the following problems.

1. Find two consecutive integers whose product is 72.
2. Find two consecutive odd integers whose product is 63.
3. The sum of a number and its reciprocal is 4. What is the number?
4. Find two consecutive even integers whose product is 48.
5. The sum of the squares of two consecutive integers is 25. Find the integers.
6. The sum of the squares of two consecutive integers is 61. Find the integers.
7. One number is 1 more than twice another number. The product of the two numbers is 21. What are the numbers?
8. The square of a number minus 2 times the number is 14. What is the number?
9. The area of a rectangle is 42 sq. ft. The length is 2 ft. more than 4 times the width. What are the dimensions of the rectangle?
10. The base of a triangle is 1 inch more than twice the height. Find the base and the height if the area of the triangle is 14 sq. in.?
11. The base of a triangle is 1 inch less than twice the height. Find the base and the height if the area of the triangle is 6 sq. in.?**
12. One pipe can fill a reservoir 1 hour faster than another pipe. Together they fill the reservoir in 4 hours. How long does it take for each pipe to fill the reservoir?**
13. Gary can process his job 2 days faster than his co-worker, Bob. Working together, they can process the job in 6 days. Working alone, how long does it take each of the men to process the job?**

**Determine a decimal approximation.

Applications-Quadratics Equations-Answers

1. 8 and 9; -8 and -9
2. 7 and 9; -7 and -9
3. $2 \pm \sqrt{3}$
4. 6 and 8; -6 and -8
5. 3 and 4; -3 and -4
6. 5 and 6; -5 and -6
7. 3 and 7; -6 and $-\frac{7}{2}$
8. $1 \pm \sqrt{15}$
9. 3 ft. by 14 ft.
10. base =8 in.; height = $\frac{7}{2}$ in.
11. height = $\frac{1+\sqrt{97}}{4} \approx 2.7$ inches; base = $\frac{-1+\sqrt{97}}{2} \approx 4.4$ inches
12. Pipe #1 = $\frac{7+\sqrt{65}}{2} \approx 7.53$ hours; Pipe #2 = $\frac{9+\sqrt{65}}{2} \approx 8.52$ hours
13. Gary $5+\sqrt{37} \approx 11.1$ days; Bob $7+\sqrt{37} \approx 13.1$ days