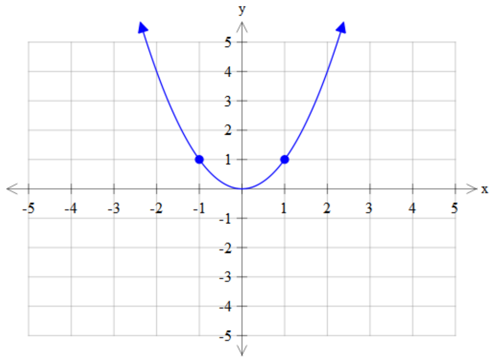
Notes Transformations of Functions

(Note: You must have the following 5 basic shape functions committed to memory)

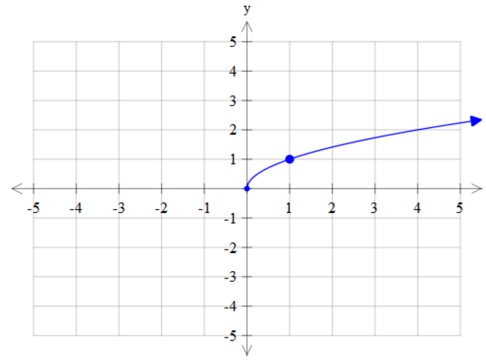
The Square Function: $f(x) = x^2$



Domain: $\Re$ Range: $[0, \infty)$

Vertex: (0, 0) Key Point: (1, 1) and (-1, 1)

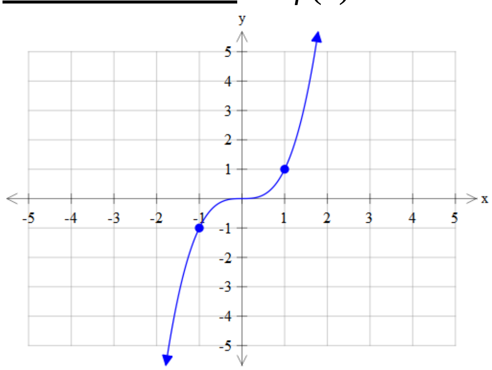
The Square Root Function: $f(x) = \sqrt{x}$



Domain: $[0, \infty)$ Range: $[0, \infty)$

Vertex: (0, 0) Key Point: (1, 1)

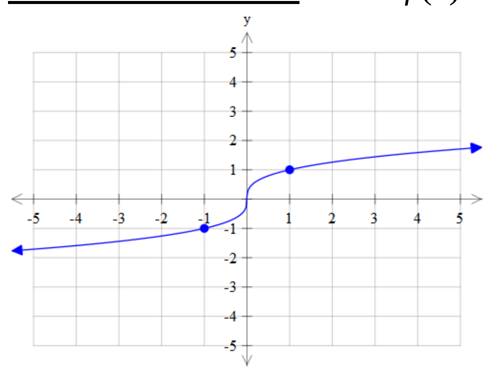
The Cube Function: $f(x) = x^3$



Domain: $\Re$ Range: $\Re$

Vertex: (0, 0) Key Point: (1, 1) and (-1, -1)

The Cube Root Function: $f(x) = \sqrt[3]{x}$



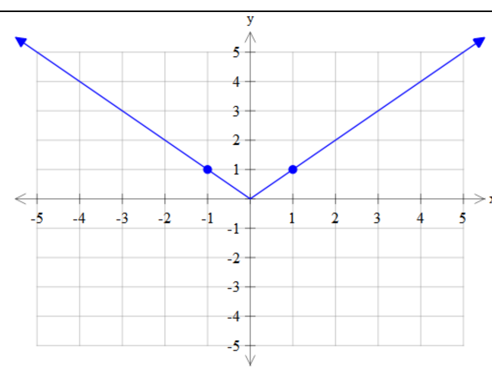
Domain: $\Re$ Range: $\Re$

Vertex: (0, 0) Key Point: (1, 1) and (-1, -1)

The Absolute Value Function: $f(x) = |x|$

Domain: $\Re$ Range: $[0, \infty)$

Vertex: (0, 0) Key Point: (1, 1) and (-1, 1)



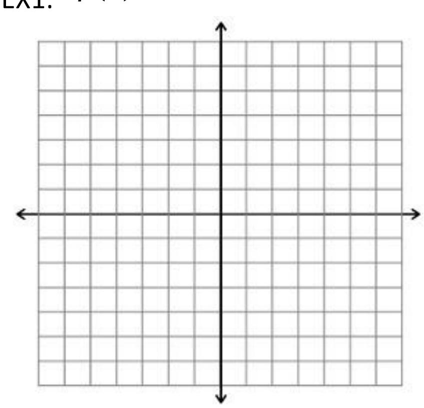
We will be sketching the graphs of the functions above by transformations. There are ***4 different types of transformations we will be looking at:***

- ***Vertical Shift:*** The number that is added/subtracted to the function. $y = f(x) + c$
  (where c is a constant and f(x) is one of the 5 functions above)
  - Vertical shifts will move the graph up or down c units
- ***Horizontal Shift:*** The number that is added/subtracted to x in the function. $y = f(x+b)$
  (where b is a constant and f(x) is one of the 5 functions above)
  - Horizontal shifts move the graph left or right b units (opposite the sign with x)
- ***Compression/Stretch:*** This is the number that is multiplied to the function. $y = a \cdot f(x)$
  (where a is a constant and f(x) is one of the 5 functions above)
  - Compression/Stretch will vertically stretch or shrink a graph a units
- ***Reflection:*** If there is a negative in front of the function. $y = -f(x)$
  (where f(x) is one of the 5 functions above)
  - Reflections reflect the original function about the y-axis.

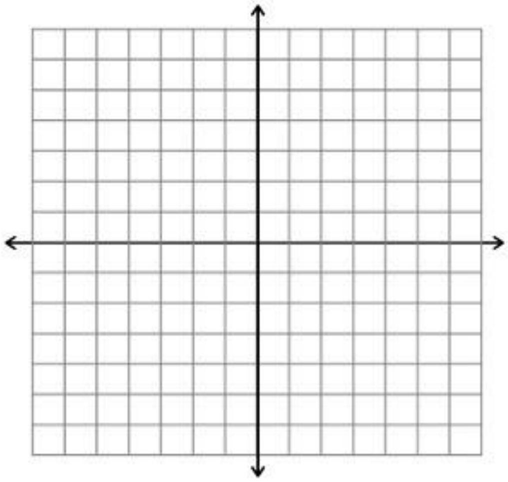
To graph functions using transformations:

1. Determine the basic shape function (this gives you an idea of what the graph should look like).
2. Determine the new vertex of your graph. Your new vertex will be an ordered pair; the x value is your horizontal shift, and the y value is your vertical shift. Plot this point on the graph.
3. Determine the compression/stretch, this will guide you in finding the location of the new key points. The key points will always be up/down the value of the compression stretch and over ONE. (a negative compression/stretch will reflect your basic shape about the y-axis)
4. Find the x-intercept(s) of the graph (if any) and plot them.
5. Connect all your points. Make sure the shape of your graph looks like the basic shape function

Use transformations of functions to graph each of the following functions. Identify for each the (a) Basic Shape, (b) vertical shift, (c) horizontal shift, (d) compression/stretch, (e) x-intercepts

EX1: $f(x) = x^2$ 	a) Basic Shape:_______________ b) vertical shift:_______________ c) horizontal shift:_____________ d) compression/stretch:_______________ e) x-intercepts: _______________

EX2: $f(x) = x^2 + 2$



a) Basic Shape:_______________

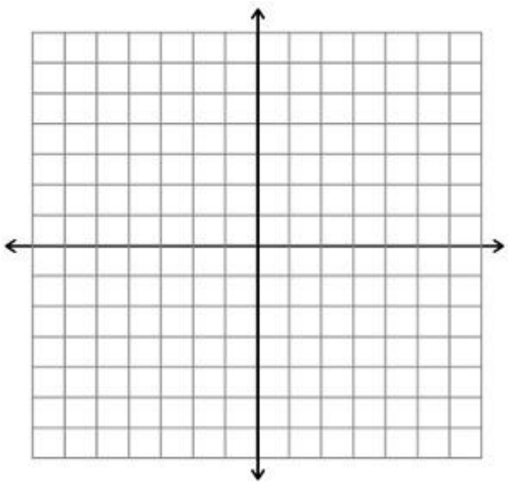
b) vertical shift:_______________

c) horizontal shift:_____________

d) compression/stretch:_______________

e) x-intercepts: _______________

Ex3: $f(x) = 2(x+2)^2 + 2$



a) Basic Shape:_______________

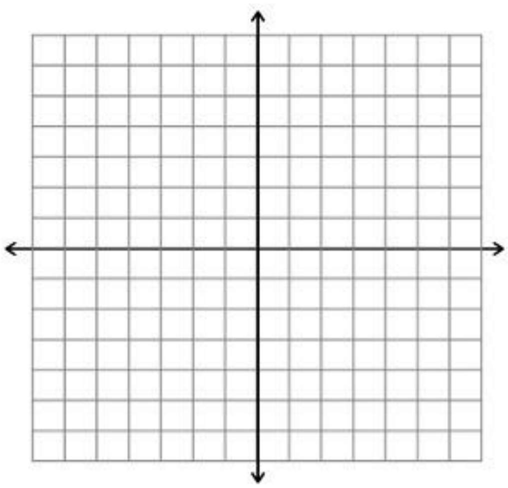
b) vertical shift:_______________

c) horizontal shift:_____________

d) compression/stretch:_______________

e) x-intercepts: _______________

EX4: $f(x) = -2(x-1)^2 - 3$



a) Basic Shape:_______________

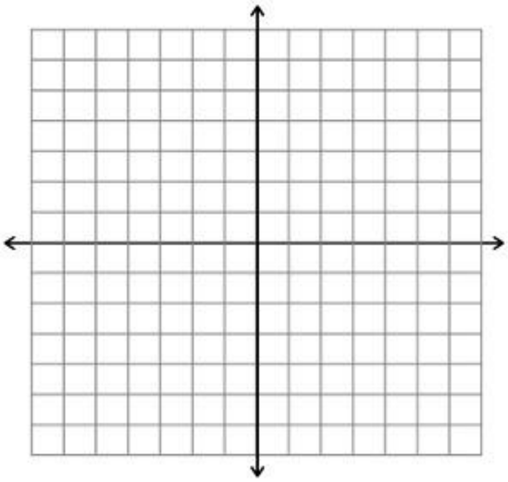
b) vertical shift:_______________

c) horizontal shift:_____________

d) compression/stretch:_______________

e) x-intercepts: _______________

EX5: $f(x) = -2(x-3)^2 - 5$



a) Basic Shape:________________

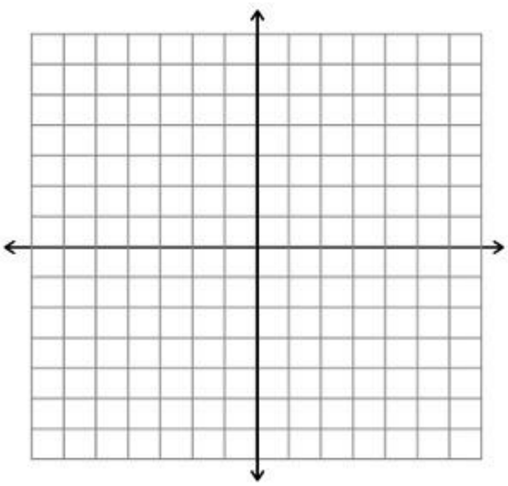
b) vertical shift:________________

c) horizontal shift:______________

d) compression/stretch:________________

e) x-intercepts: ________________

EX6: $f(x) = \frac{1}{2}\sqrt{x-3} + 3$



a) Basic Shape:________________

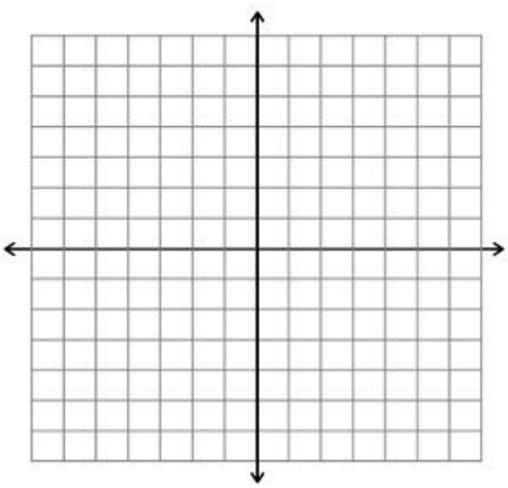
b) vertical shift:________________

c) horizontal shift:______________

d) compression/stretch:________________

e) x-intercepts: ________________

EX7: $f(x) = -(x-3)^3 + 1$



a) Basic Shape:________________

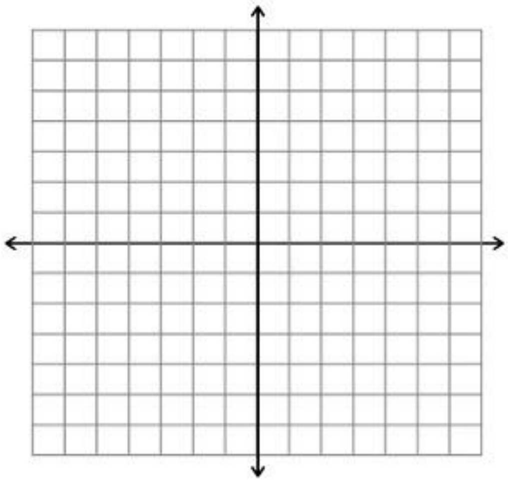
b) vertical shift:________________

c) horizontal shift:______________

d) compression/stretch:________________

e) x-intercepts: ________________

EX8: $f(x)=\sqrt[3]{x}+1$



a) Basic Shape:________________

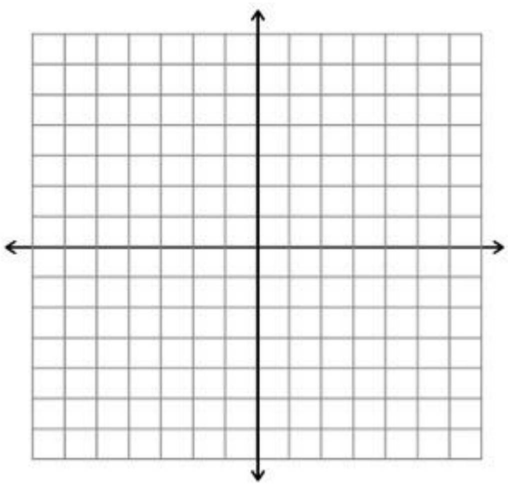
b) vertical shift:________________

c) horizontal shift:______________

d) compression/stretch:________________

e) x-intercepts: ________________

EX9: $f(x)=|x-4|+2$



a) Basic Shape:________________

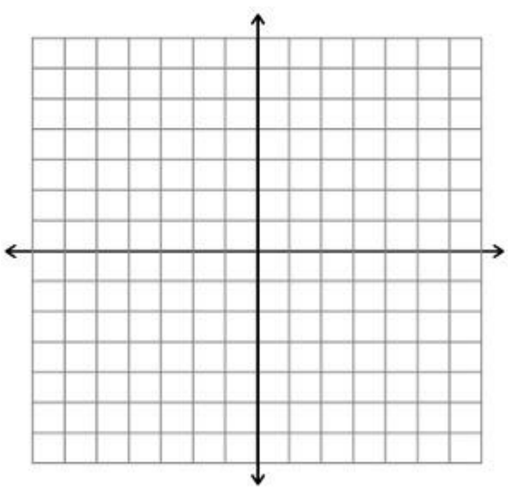
b) vertical shift:________________

c) horizontal shift:______________

d) compression/stretch:________________

e) x-intercepts: ________________

EX9: $f(x)=\frac{3}{2}\sqrt[3]{x+4}-3$



f) Basic Shape:________________

g) vertical shift:________________

h) horizontal shift:______________

i) compression/stretch:________________

j) x-intercepts: ________________

Even and Odd Functions

A function is considered EVEN if $f(-x) = f(x)$. A function is considered ODD if $f(-x) = -f(x)$

To determine if a function is even, odd or neither:

1. Plug –x in for x in the function and simplify your expression.
2. If the simplified expression from step 1 is equal to your function you are EVEN (you're done)
3. If you are not equal to your function, find –f(x) (negative 1 times your original function).
4. If the expression from step 3 is equal to the expression in step 2, your function is ODD (you're done).
5. If none of the functions match, your function is neither even nor odd.

Determine if the following functions are even or odd algebraically

EX10: $f(x) = 3x - x^3$	EX11: $f(x) = x^2 - 3$	EX12: $f(x) = x^2 + 2x$