Notes Rational Functions

A Rational Function is a function that can be expressed as a ratio of polynomial functions.

$$f(x)=\frac{p(x)}{q(x)} \text{ where } p(x) \text{ and } q(x) \text{ are polynomial functions and } q(x)\neq 0.$$

The **domain** of a rational function is all real numbers ***EXCEPT*** those for which the value of the denominator is equal to zero.

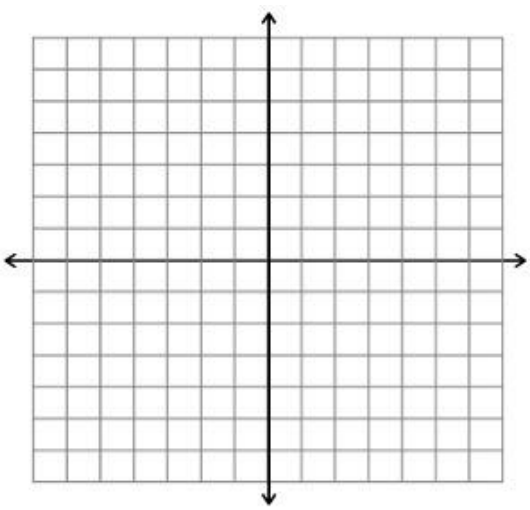
To determine the domain or a rational function:

- Set the denominator of the rational function equal to zero and solve for x.
- Any **REAL** number(s) obtained will need to be **EXCLUDED** from your domain.

Find the domain of the following functions.

EX1: $f(x)=\frac{6-x}{2x+1}$	EX2: $g(x)=\frac{x+2}{x^2-9}$
EX3: $h(x)=\frac{x+2}{x^2+9}$	

Sketch the graph of $f(x)=\frac{1}{x}$



x	f(x)

Domain:

Horizontal Asymptote:

Vertical Asymptote:

An asymptote is a line or curve that our graph wants to get close to (as it heads to infinity) but usually does touch or cross.

To find Vertical Asymptote(s):

1. Get the function into lowest terms

2. Set the denominator equal to zero and solve for x.

   a. Any REAL NUMBER(S) obtained is/are our vertical asymptote(s).

      We write: $V.A. \quad at \quad x = \underline{\hspace{2cm}}$

NOTE: You will NEVER cross a vertical asymptote.

To find the Horizontal Asymptote choose one of the following:

- Determine the degree of numerator and the degree of the denominator.

  A. If the degree of the numerator is SMALLER than the degree than the denominator, then you will have a horizontal asymptote: $H.A. \quad at \quad y = 0$

  B. If the degree of the numerator is EQUAL TO the degree of the denominator, then you will have a horizontal asymptote: $H.A. \quad at \quad y = \frac{\text{leading coefficient of the numerator}}{\text{leading coefficient of the denominator}}$

  C. If the degree of the numerator is LARGER than the degree than the denominator, then you DO NOT have a horizontal asymptote.

You **CAN** cross a horizontal asymptote. To find the point where you will cross the H.A.:

- Set the REDUCDED function equal to the horizontal asymptote and solve for x (this will give you the x value of the point). The y-value is the value of the H.A.

Identify the Vertical and Horizontal Asymptotes of each function

EX4: $f(x) = \frac{x^2 - 1}{x^2 + x}$

EX5: $f(x) = \dfrac{5x-2}{3x^2 - x - 2}$

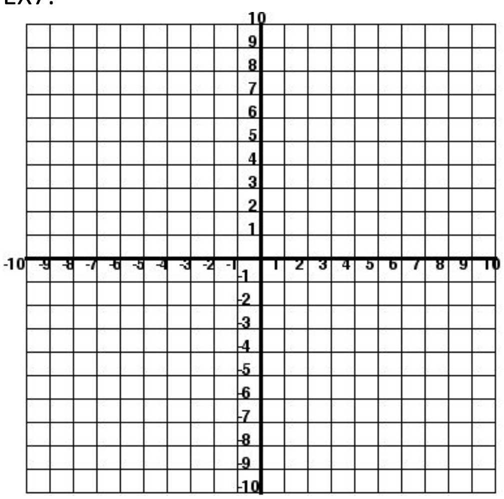
EX6: $f(x) = \dfrac{4x^2 + 3x}{x-5}$

<u>To graph a Rational Function:</u>

1. Find the x-intercept(s) of the function by setting the numerator equal to zero and solving for x.
2. Find the y-intercept of the function by finding $f(0)$.
3. Find ALL the asymptotes of the function.
4. Determine if the function will cross the horizontal asymptote
5. Plot all the points and asymptotes on your graph
6. Choose test points from each section that your points and asymptotes broke the x-axis into (from step 5) to determine the location/shape of the graph.
7. Connect the points with a smooth curve.

Graph the following functions

EX7: $f(x) = \dfrac{4}{x-2}$



a. x-intercepts:________________________________

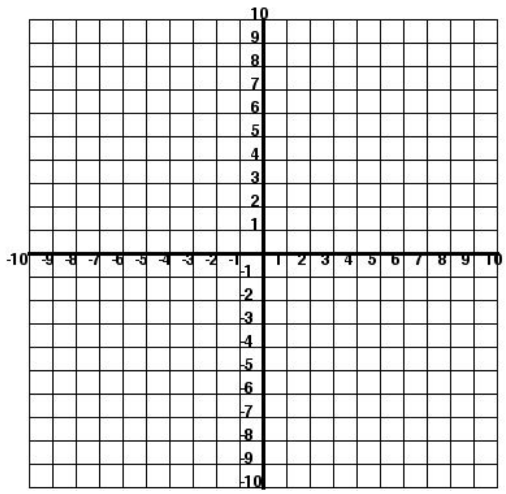
b. y-intercept:_________________________________

c. vertical asymptote:__________________________

d. horizontal asymptote:________________________

e. point where we cross H.A.:__________________

EX8: $g(x)=\dfrac{x+1}{x^2-9}$



a. x-intercepts:________________________________

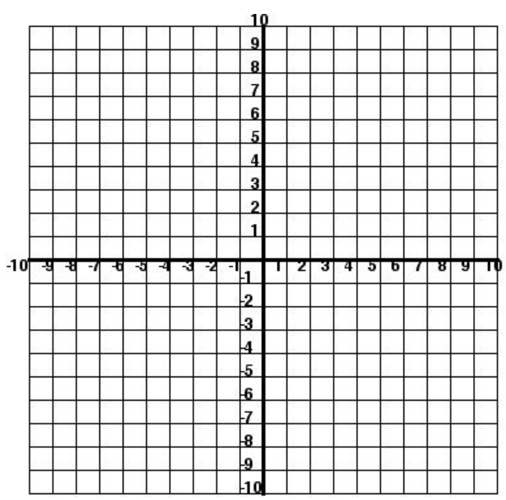
b. y-intercept:_________________________________

c. vertical asymptote:__________________________

d. horizontal asymptote:________________________

e. point where we cross H.A.:__________________

EX9: $h(x)=\dfrac{3x}{x-1}$



a. x-intercepts:________________________________

b. y-intercept:_________________________________

c. vertical asymptote:__________________________

d. horizontal asymptote:________________________

e. point where we cross H.A.:__________________