Notes Zeros of a Polynomial Function

Rational Zeros Theorem:

If $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_1 x^1 + a_0$ then all the POSSIBLE RATIONAL ZEROS of $f(x)$ are:

Possible Rational Zeros = $\dfrac{\text{Factors of } a_0}{\text{Factors of } a_n}$

When we are solving a polynomial, we will use the Rational Zeros Theorem to determine what our POSSIBLE rational zero will be.

Use the Rational Zeros Theorem to list all the Possible Rational Zeros of the following polynomials

EX1: $f(x) = x^3 - 4x^2 + 15$

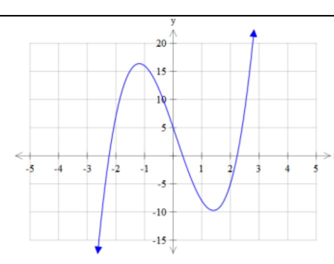
EX2: $f(x) = 10x^3 + 7x^2 - 2x - 6$

<u>To find the zeros of any polynomial:</u>

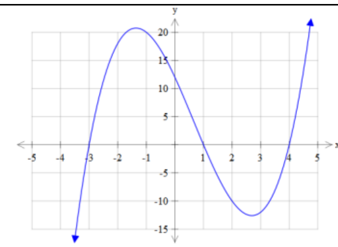
1. List all the possible rational zeros.
2. Use synthetic division to determine if any possible rational zeros are an ACTUAL zero of the polynomial. (we will have a graph of the function that will help us with this).
3. Once you find a zero:
   a. If the new polynomial is factorable or quadratic, finish solving using methods previously learned.
   b. If the new polynomial is NOT factorable or quadratic, repeat steps 2 – 3.

Find all the real zeros of each polynomial function, and factor it completely.

EX3: $f(x) = 3x^3 - x^2 - 15x + 5$



Ex4: $f(x) = x^3 - 2x^2 - 11x + 12$



Fundamental Theorem of Algebra:
Every polynomial function of degree 1 or more has at least 1 complex zero (or root)

Zeros of a Polynomial:
A polynomial function of degree $n \geq 1$ has exactly ***n*** complex roots, including multiplicity (n=degree)

How many zeros (solutions) will the following polynomials have?

EX5: $f(x) = x + 5$	EX6: $f(x) = x^2$
EX7: $f(x) = 13x^5 + 7x^3 - 2x^2 - 10$	EX8: $f(x) = 2x^2 + 7x - 5$

Linear Factors of a Polynomial:
A polynomial function of degree $n \geq 1$ can be expressed as a product of n linear (to 1st power) factors

$$f(x) = a(x - c_1) \cdot (x - c_2) \cdot \ldots \cdot (x - c_n)$$

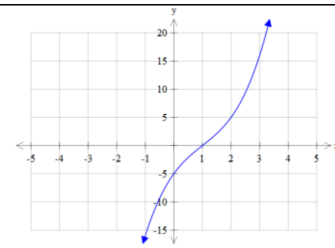
Express as Linear Factors:

EX9: $f(x) = x^2 - 25$	EX10: $f(x) = x^2 + 4$

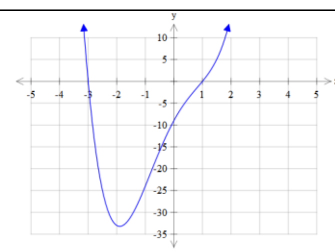
Conjugate Zeros Theorem:
If f is a polynomial function and $a + bi$ is a zero of f, then $a - bi$ is also a zero of f.

Find all the complex zeros of the following polynomials and express as a product of linear factors.

EX11: $f(x)=x^3-3x^2+7x-5$



EX12: $f(x)=x^4-4x^2+12x-9$



A complex root is given. Find all remaining roots & express the polynomial as a product of linear factors

EX13: $x^4 - 4x^2 - 5 = 0 \quad x = i \quad is \quad a \quad root$

Find a polynomial of minimal degree with real coefficients with the given zeros

EX14: $-1, \quad 4i, \quad -4i$