

Learning and practice has occurred using the student resources and graded assignments in this course. When studying math you should not study in order, practice using the eText and MLM HW assignments. If you can work the questions without your notes correctly then you are ready for an exam, otherwise seek help and ask questions. MLM exams have 12 to 20 questions.

*\*NOTE: This study sheet may reflect some exam questions, however, this study sheet may not reflect every type of question that may be included on the exam.*

**Find the unit vector having the same direction as  $\mathbf{v}$ .**

1.  $\mathbf{v} = -3\mathbf{i} + \mathbf{j}$

**Write the vector  $\mathbf{v}$  in the form  $a\mathbf{i} + b\mathbf{j}$ , given its magnitude  $\|\mathbf{v}\|$  and the angle  $\alpha$  it makes with the positive  $x$ -axis.**

2.  $\|\mathbf{v}\| = 13, \alpha = 225^\circ$

**Find the dot product  $\mathbf{v} \cdot \mathbf{w}$ .**

3.  $\mathbf{v} = -7\mathbf{i} + 8\mathbf{j}, \quad \mathbf{w} = -15\mathbf{i} - 9\mathbf{j}$

**The polar coordinates of a point are given. Find the rectangular coordinates of the point.**

4.  $\left(-9, \frac{3\pi}{4}\right)$

**Find the quantity if  $\mathbf{v} = 5\mathbf{i} - 7\mathbf{j}$  and  $\mathbf{w} = 3\mathbf{i} + 2\mathbf{j}$ .**

5.  $\|\mathbf{v} + \mathbf{w}\|$

**State whether the vectors are parallel, orthogonal, or neither.**

6.  $\mathbf{v} = 4\mathbf{i} + 3\mathbf{j}, \quad \mathbf{w} = 8\mathbf{i} + 6\mathbf{j}$

7.  $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j}, \quad \mathbf{w} = 2\mathbf{i} - 3\mathbf{j}$

8.  $\mathbf{v} = \mathbf{i} + \sqrt{2}\mathbf{j}, \quad \mathbf{w} = \mathbf{i} - 3\mathbf{j}$

**Express the product as a sum containing only sines or cosines.**

9.  $\sin(5\theta) \sin(9\theta)$

10.  $\sin(5\theta) \cos(6\theta)$

**Find the angle between  $\mathbf{v}$  and  $\mathbf{w}$ . Round your answer to one decimal place, if necessary.**

11.  $\mathbf{v} = -5\mathbf{i} + 7\mathbf{j}, \quad \mathbf{w} = -6\mathbf{i} - 4\mathbf{j}$

**Transform the polar equation to an equation in rectangular coordinates. Then identify and graph the equation.**

12.  $r \sin \theta = 2$

**Test the equation for symmetry with respect to the given axis, line, or pole.**

13.  $r = -4 \cos \theta$ ; the line  $\theta = \frac{\pi}{2}$

14.  $r = 2 + 4 \sin \theta$ ; the polar axis

**Solve the problem.**

15. A building 220 feet tall casts a 60 foot long shadow. If a person looks down from the top of the building, what is the measure of the angle between the end of the shadow and the vertical side of the building (to the nearest degree)? (Assume the person's eyes are level with the top of the building.)

16. A plane takes off from an airport on the bearing  $S29^\circ W$ . It continues for 20 minutes then changes to bearing  $S52^\circ W$  and flies for 2 hours 20 minutes on this course then lands at a second airport. If the plane's speed is 420 mph, how far from the first airport is the second airport? Round your answer correct to the nearest mile.

17. A pier 1250 meters long extends at an angle from the shoreline. A surveyor walks to a point 1500 meters down the shoreline from the pier and measures the angle formed by the ends of the pier. It is found to be  $53^\circ$ . What acute angle (correct to the nearest  $0.1^\circ$ ) does the pier form with the shoreline? Is there more than one possibility? If so, how can we know which is the correct one?

18. Two hikers on opposite sides of a canyon each stand precisely 525 meters above the canyon floor. They each sight a landmark on the canyon floor on a line directly between them. The angles of depression from each hiker to the landmark meter are  $37^\circ$  and  $21^\circ$ . How far apart are the hikers? Round your answer to the nearest whole meter.

19. A new homeowner has a triangular-shaped back yard. Two of the three sides measure 65 ft and 80 ft and form an included angle of  $125^\circ$ . The owner wants to approximate the area of the yard, so that he can determine the amount of fertilizer and grass seed to be purchased. Find the area of the yard rounded to the nearest square foot.

**Use the given vectors to find the indicated expression.**

20.  $\mathbf{v} = -5\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$ ,  $\mathbf{w} = -4\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ ,  $\mathbf{u} = 4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$  Find  $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$ .

21.  $\mathbf{v} = 4\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ ,  $\mathbf{w} = -5\mathbf{i} - 2\mathbf{j} - \mathbf{k}$  Find  $(-5\mathbf{v}) \times \mathbf{w}$ .

22.  $\mathbf{v} = -3\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$ ,  $\mathbf{w} = -4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ ,  $\mathbf{u} = 5\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$  Find  $\mathbf{w} \cdot (\mathbf{v} \times \mathbf{u})$ .

23.  $\mathbf{v} = 4\mathbf{i} - 4\mathbf{j} - 4\mathbf{k}$ ,  $\mathbf{u} = -3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$  Find  $\mathbf{v} \times (-5\mathbf{u})$ .

**The rectangular coordinates of a point are given. Find polar coordinates for the point.**

24.  $(-\sqrt{3}, 1)$

**Plot the point given in polar coordinates.**

25.  $\left(-4, \frac{-3\pi}{4}\right)$

**Express the sum or difference as a product of sines and/or cosines.**

26.  $\cos(4\theta) - \cos(6\theta)$

27.  $\sin(8\theta) + \sin(4\theta)$

**The letters x and y represent rectangular coordinates. Write the equation using polar coordinates (r,  $\theta$ ).**

28.  $y^2 = 16x$

**Plot the complex number in the complex plane.**

29.  $-5 + i$

**Write the complex number in polar form. Express the argument in degrees, rounded to the nearest tenth, if necessary.**

30.  $1 + \sqrt{3}i$

**Solve the problem. Leave your answer in polar form.**

31.  $z = \sqrt{3} \left( \cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)$      $w = \sqrt{6} \left( \cos \frac{9\pi}{4} + i \sin \frac{9\pi}{4} \right)$     Find  $\frac{z}{w}$ .

32.  $z = 6 \left( \cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right)$      $w = 12 \left( \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$     Find  $zw$ .

**The letters r and  $\theta$  represent polar coordinates. Write the equation using rectangular coordinates (x, y).**

33.  $r = 10 \sin \theta$

**Write the complex number in rectangular form.**

34.  $3 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$

**Perform the indicated operation.**

35.  $\mathbf{v} = -4\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$  and  $\mathbf{w} = -6\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$     Find  $2\mathbf{v} - 4\mathbf{w}$ .

36.  $\mathbf{v} = -4\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$  and  $\mathbf{w} = -3\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$     Find  $\|\mathbf{v}\| - \|\mathbf{w}\|$ .

37.  $\mathbf{v} = 3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}$  and  $\mathbf{w} = 2\mathbf{i} - 6\mathbf{j} + 5\mathbf{k}$     Find  $\|\mathbf{v} - \mathbf{w}\|$ .

**The vector v has initial position P and terminal point Q. Write v in the form  $a\mathbf{i} + b\mathbf{j}$ ; that is, find its position vector.**

38.  $P = (6, 2)$ ;  $Q = (-2, -4)$

**Write the expression in the standard form  $a + bi$ .**

39.  $\left[ \sqrt{3} \left( \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right) \right]^4$

**Solve the problem.**

40. A room in the shape of a triangle has sides of length 6 yd, 8 yd, and 11 yd. If carpeting costs \$19.50 a square yard and padding costs \$4.25 a square yard, how much to the nearest dollar will it cost to carpet the room, assuming that there is no waste?
41. If  $\mathbf{u} = -6\mathbf{i} - 2\mathbf{j}$  and  $\mathbf{v} = 2\mathbf{i} + 7\mathbf{j}$ , find  $\mathbf{u} - \mathbf{v}$ .
42. A ship sailing parallel to shore sights a lighthouse at an angle of  $12^\circ$  from its direction of travel. After traveling 3 miles farther, the angle is  $25^\circ$ . At that time, how far is the ship from the lighthouse?
43. Two surveyors 180 meters apart on the same side of a river measure their respective angles to a point between them on the other side of the river and obtain  $54^\circ$  and  $68^\circ$ . How far from the point (line-of-sight distance) is each surveyor? Round your answer to the nearest 0.1 meter.
44. Plot the point  $\left(4, \frac{\pi}{6}\right)$  and find other polar coordinates  $(r, \theta)$  of the point for which:  
(a)  $r > 0, -2\pi \leq \theta < 0$       (b)  $r < 0, 0 \leq \theta < 2\pi$       (c)  $r > 0, 2\pi \leq \theta < 4\pi$
45. A famous golfer tees off on a long, straight 480 yard par 4 and slices his drive  $20^\circ$  to the right of the line from tee to the hole. If the drive went 280 yards, how many yards will the golfer's second shot have to be to reach the hole?
46. Two points A and B are on opposite sides of a building. A surveyor selects a third point C to place a transit. Point C is 50 feet from point A and 70 feet from point B. The angle ACB is  $46^\circ$ . How far apart are points A and B?

**Find the dot product  $\mathbf{v} \cdot \mathbf{w}$ .**

47.  $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$  and  $\mathbf{w} = 3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$

**Find the position vector for the vector having initial point P and terminal point Q.**

48.  $P = (-3, 2, -1)$  and  $Q = (2, -4, 1)$

**Find the direction angles of the vector. Round to the nearest degree, if necessary.**

49.  $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$

# Answer Key

## Testname: MATH 1316 MLM FINAL EXAM STUDY SHEET

1.  $\mathbf{u} = -\frac{3\sqrt{10}}{10}\mathbf{i} + \frac{\sqrt{10}}{10}\mathbf{j}$

2.  $\mathbf{v} = 13\left(-\frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}\right)$

3. 33

4.  $\left(\frac{9\sqrt{2}}{2}, \frac{-9\sqrt{2}}{2}\right)$

5.  $\sqrt{89}$

6. Parallel

7. Orthogonal

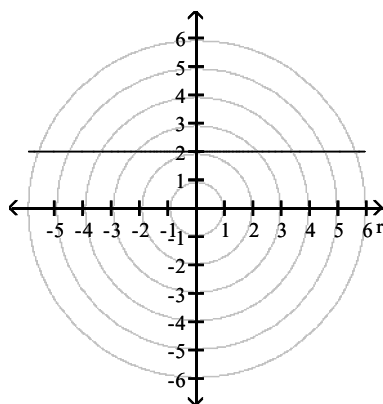
8. Neither

9.  $\frac{1}{2}[\cos(4\theta) - \cos(14\theta)]$

10.  $\frac{1}{2}[\sin(11\theta) - \sin\theta]$

11.  $88.2^\circ$

12.



$y = 2$ ; horizontal line 2 units  
above the pole

13. May or may not be symmetric with respect to the line  $\theta = \frac{\pi}{2}$

14. May or may not be symmetric with respect to the polar axis

15.  $15^\circ$

16. 1110 mi

17.  $53.6^\circ$  or  $20.4^\circ$ ; yes; direct observation

18. 2064 m

19. 2130 sq. ft

20. 132

21.  $5\mathbf{i} + 5\mathbf{j} - 35\mathbf{k}$

22. 21

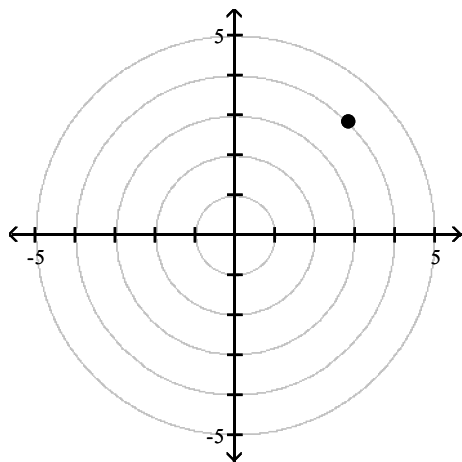
23.  $120\mathbf{i} - 20\mathbf{j} + 140\mathbf{k}$

24.  $\left(2, \frac{5\pi}{6}\right)$

# Answer Key

## Testname: MATH 1316 MLM FINAL EXAM STUDY SHEET

25.

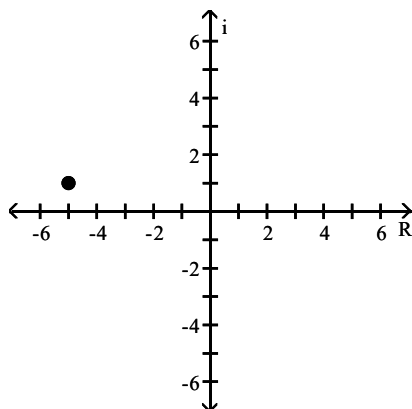


26.  $2 \sin(5\theta) \sin \theta$

27.  $2 \sin(6\theta) \cos(2\theta)$

28.  $r \sin^2 \theta = 16 \cos \theta$

29.



30.  $2(\cos 60^\circ + i \sin 60^\circ)$

31.  $\frac{\sqrt{2}}{2} \left( \cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right)$

32.  $72 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$

33.  $x^2 + y^2 = 10y$

34.  $\frac{3}{2} + \frac{3\sqrt{3}}{2}i$

35.  $16\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}$

36.  $\sqrt{61} - \sqrt{29}$

37.  $\sqrt{10}$

38.  $\mathbf{v} = -8\mathbf{i} - 6\mathbf{j}$

39.  $-\frac{9}{2} - \frac{9\sqrt{3}}{2}i$

40. \$556

41.  $-8\mathbf{i} - 9\mathbf{j}$

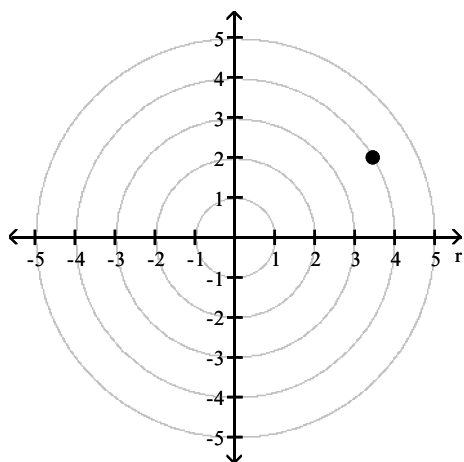
42. 2.77 mi

# Answer Key

Testname: MATH 1316 MLM FINAL EXAM STUDY SHEET

43. 196.8 m, 171.7 m

44.



- (a)  $\left(4, -\frac{11\pi}{6}\right)$
- (b)  $\left(-4, \frac{7\pi}{6}\right)$
- (c)  $\left(4, \frac{13\pi}{6}\right)$

45. 237.1 yd

46. 50.4 ft

47. 21

48.  $\mathbf{v} = 5\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}$

49.  $\alpha = 68^\circ, \beta = 56^\circ, \gamma = 138^\circ$