Math 1332 Review 3(answers)

1. Find the standard deviation of the data set $\{3,3,5,8,10,13\}$ by completing the following table:

		(, -			
х	\overline{x}	$x-\overline{x}$	$(x-\overline{x})^2$			
3	7	-4	16			
3	7	-4	16			
5	7	-2	4			
8	7	1	1			
10	7	3	9			
13	7	6	36			
$\sum (x - \overline{x})^2 = 82$						
$\frac{\sum (x-\overline{x})^2}{n-1} = \frac{16.4}{}$						
$\sqrt{\frac{\sum (x-\overline{x})^2}{n-1}} = \boxed{4.05}$						

2. For a particular data set the mean is 10 and the standard deviation is 2. According to Chebyshev's Theorem, at least what percentage of the data values are between

a) 6 and 14?

$$10-6=4$$

 $14-10=4$
 $k=4 \div 2=2$

 $=\frac{3}{4}=\boxed{75\%}$

$$k = 6 \div 2 = 3$$

$$1 - \frac{1}{k^2} = 1 - \frac{1}{3^2} = 1 - \frac{1}{9}$$

$$= \frac{8}{9} = \boxed{88.\overline{8}\%}$$

b) 4 and 16?

10 - 4 = 6

16 - 10 = 6

$$\begin{array}{rcl}
 & 10-6=4 & 10-4=6 & 10-3=7 \\
 & 14-10=4 & 16-10=6 & 17-10=7 \\
 & k=4\div 2=2 & k=6\div 2=3 & k=7\div 2=3.5 \\
 & 1-\frac{1}{k^2}=1-\frac{1}{2^2}=1-\frac{1}{4} & 1-\frac{1}{k^2}=1-\frac{1}{3^2}=1-\frac{1}{9} & 1-\frac{1}{k^2}=1-\frac{1}{3.5^2}=1-\frac{1}{12.25} \\
 & =\frac{3}{4}=\boxed{75\%} & =\frac{8}{9}=\boxed{88.\overline{8}\%}
 \end{array}$$

c) 3 and 17?

3. Find the simple interest for a loan of \$4,902 at 6.5% for 11 months.

$$I = \Pr t$$

$$I = 4902 \cdot .065 \cdot \frac{11}{12}$$
$$= \boxed{\$292.08}$$

4. Find the present value using simple interest of \$80,612 in 128 days at 6.77%. Use a 360 day year.

$$P = \frac{A}{1+rt}$$

$$= \frac{80,612}{1+.0677 \cdot \frac{128}{360}}$$

$$= \boxed{\$78,717.19}$$

5. Find the future value at the end of the 3 years for the following investment: \$4,677.23 at 4.57% compounded monthly for 3 years. How much is interest?

$$A = P \left(1 + \frac{r}{m}\right)^{mt}$$

$$= 4,677.23 \left(1 + \frac{.0457}{12}\right)^{12\cdot3}$$

$$= \left[\$5,363.12\right]$$

$$I = A - P$$

$$= \$5,363.12 - \$4,677.23$$

$$= \left[\$685.89\right]$$

6. Find the present value of \$1,347 in 3.5 years at 6.2% compounded semi-annually.

$$P = \frac{A}{\left(1 + \frac{r}{m}\right)^{mt}}$$
$$= \frac{1,347}{\left(1 + \frac{.062}{2}\right)^{2\cdot3.5}}$$
$$= \boxed{\$1,087.82}$$

7. Find the future value of an annuity with deposits of \$250 at the end of each month into an account paying 3% compounded monthly after 5 years. How much is interest?

$$A = P \left[\frac{\left(1 + \frac{r}{m}\right)^{mt} - 1}{\frac{r}{m}} \right]$$

$$= 250 \left[\frac{\left(1 + \frac{.03}{12}\right)^{12.5} - 1}{\frac{.03}{12}} \right]$$

$$= \left[\$16, 161.68 \right]$$

$$I = A - 60 \cdot P$$

$$= \$16, 161.68 - 60 \cdot (\$250)$$

$$= \left[\$1, 161.68 \right]$$

8. Find the present value of an annuity that pays \$500 at the end of each month from an account earning 4% compounded monthly for the next 70 months.

$$PV = P \left[\frac{1 - \left(1 + \frac{r}{m}\right)^{-n}}{\frac{r}{m}} \right]$$
$$= 500 \left[\frac{1 - \left(1 + \frac{.04}{12}\right)^{-70}}{\frac{.04}{12}} \right]$$
$$= \left[\$31,170.46 \right]$$

9. Find the periodic payment needed to amortize a loan of \$32,000 at 8.4% compounded quarterly for 10 quarters.

$$PMT = A \left[\frac{\frac{r}{m}}{1 - \left(1 + \frac{r}{m}\right)^{-n}} \right]$$
$$= 32,000 \left[\frac{\frac{.084}{4}}{1 - \left(1 + \frac{.084}{4}\right)^{-10}} \right]$$
$$= \left[\$3,581.11 \right]$$

10. Use the effective rate to rank the following compound interest schemes in ascending order:

Scheme #1: 6% compounded monthly

Scheme #2: 5.9% compounded weekly

Scheme #3: 5.8% compounded daily(Use a 360 day year.)

$$r_1 = \left(1 + \frac{.06}{12}\right)^{12} - 1 = .0616778$$

$$r_2 = \left(1 + \frac{.059}{52}\right)^{52} - 1 = .0607397$$

$$r_3 = \left(1 + \frac{.058}{360}\right)^{360} - 1 = .0597100$$

So in ascending order, it's Scheme #3, Scheme #2, and Scheme #1.

11. Your class is given the option of choosing a day for the final exam. The results of the election are shown in the preference table:

Votes	9	5	4	2	2	1
First	Tu	Tu	M	Tu	W	W
Second	W	W	Tu	M	F	M
Third	F	M	F	W	Tu	F
Fourth	M	F	W	F	M	Tu

a) How many students voted in the election?

$$9+5+4+2+2+1=23$$

b) How many students selected the days in the order: M, Tu, F, W?

4

c) How many students selected Tuesday as their first choice for the final exam?

16

d) How many students selected Wednesday as their second choice for the final exam?

14

e) Determine the winning day using the Plurality method.

Tuesday

f) Determine the winning day using the Borda Count method.

Tuesday

g) Determine the winning day using the Plurality with Elimination method.

Tuesday

h) Determine the winning day using the Pairwise Comparison method.

Tuesday

12. Voters are considering four proposals. The results of the election are shown in the following preference table:

Votes	1500	600	300
First	A	В	C
Second	В	D	В
Third	C	С	D
Fourth	D	A	A

a) Which proposal wins using the Borda Count method?

В

b) Which proposal has a majority of first-place votes?

A

c) In this case, does the Borda Count method satisfy the Majority Criterion?

No

d) Which proposal wins in a head to head comparison?

A

e) In this case, does the Borda Count method satisfy the Head to Head Criterion?

No