Descriptive Statistics:

Describes, organizes, and presents data(information) about populations and samples.

Inferential Statistics:

Draws conclusions about populations based upon samples.

This class focuses on descriptive statistics!

Frequency Distribution:

A table that lists all the values of a data set along with their respective frequencies(number of times they occur).

Data Set: $\{1,2,2,1,5,4,3,2,1,1\}$

Value	Frequency
1	**4**
2	3
3	1
4	1
5	1
Total	10

Grouped Frequency Distribution:

Disjoint classes(ranges) are established for the data values. The grouped frequency distribution lists all the classes along with their respective frequencies.

Data Set: $\{10, 21, 22, 12, 50, 42, 33, 20, 15, 17\}$

Class	Frequency
10-19	**4**
20-29	3
30-39	1
40-49	1
50-59	1
Total	10

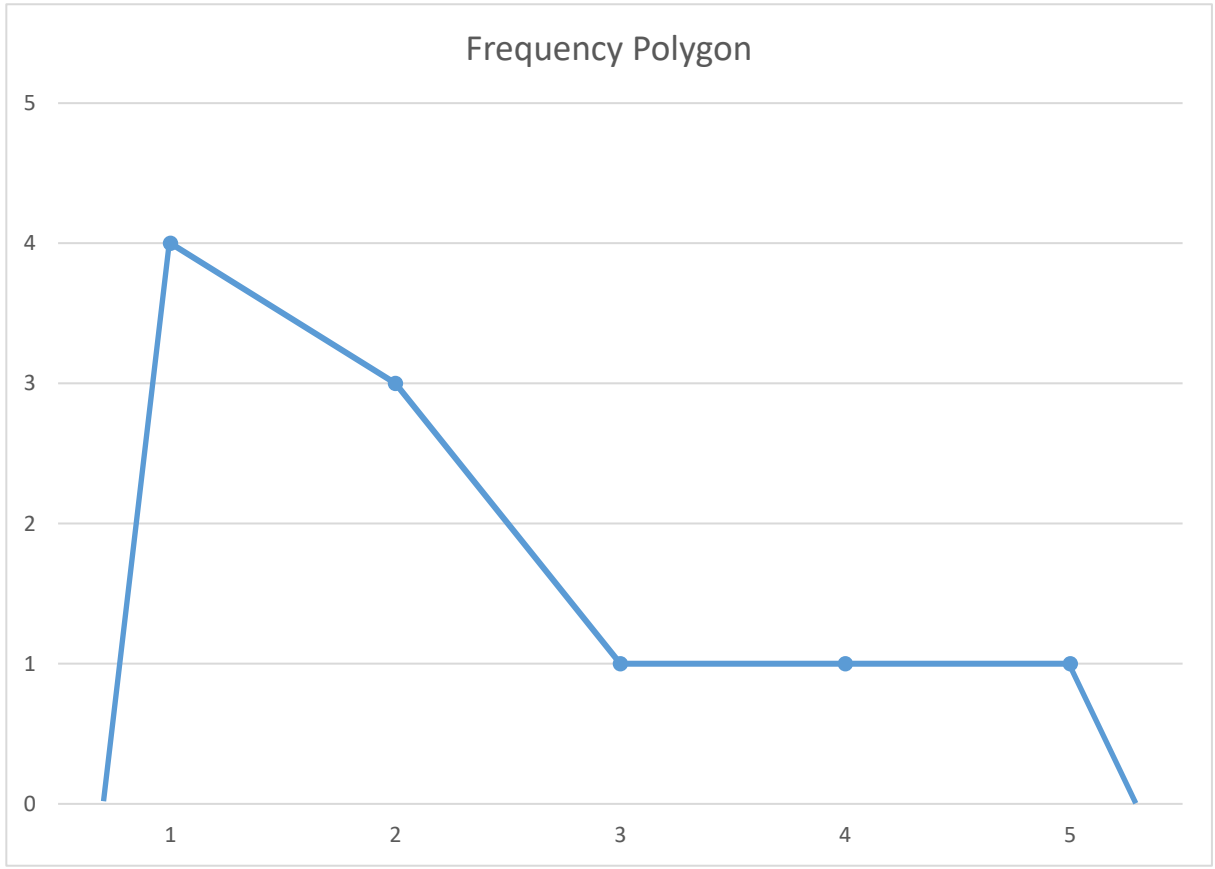
There is a loss of detail in a grouped frequency distribution-the actual data values are lost!

Graphs of frequency distributions:

Frequency Polygon:

The dots represent frequencies, and they're connected with segments.

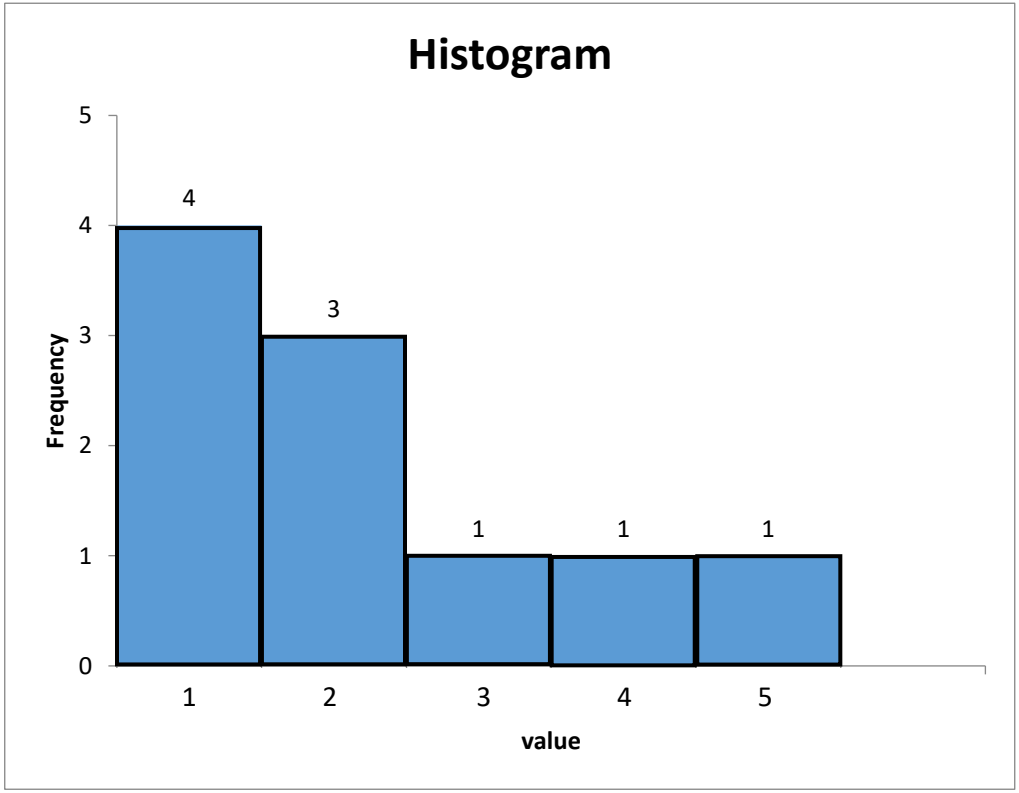
Value	Frequency
1	4
2	3
3	1
4	1
5	1
Total	10



Allows easy visual determination of the most and least frequently occurring values!

Histogram:

Special bar graph where the bars are centered at the values, touch adjacent bars, and the height is the frequency.

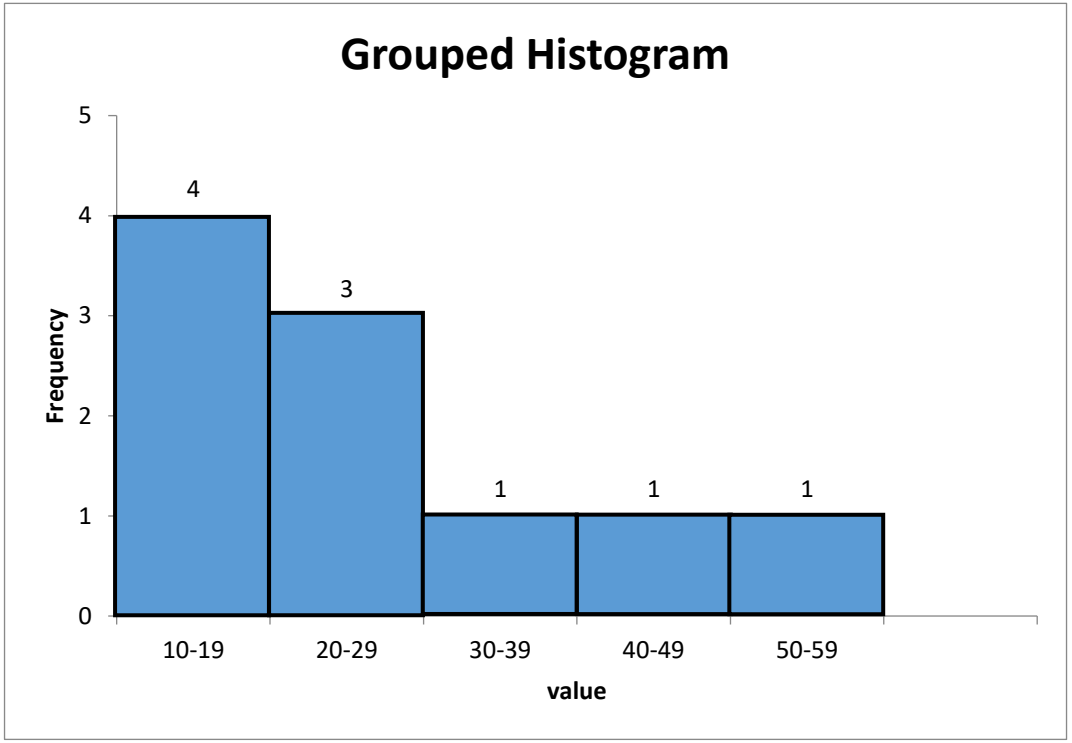


<u>Allows easy visual determination of the most and least frequently occurring values!</u>

Grouped Histogram:

Similar to a histogram

Class	Frequency
10-19	4
20-29	3
30-39	1
40-49	1
50-59	1
Total	10



Here is a data set consisting of 1,000 values from 0 to 9.

9	4	6	5	8	3	7	5	3	5
2	3	7	6	1	5	9	5	5	6
3	5	5	8	8	3	8	7	7	7
5	6	6	5	2	4	4	7	4	6
5	6	4	5	4	6	4	5	7	3
2	4	5	4	6	6	3	5	6	2
4	3	4	7	5	2	5	5	5	6
7	3	5	6	4	3	8	3	4	6
6	6	5	6	6	6	6	3	5	6
7	2	5	8	9	5	7	4	4	4
7	5	4	3	7	4	5	6	6	8
5	4	6	6	6	7	6	7	2	6
6	5	4	6	2	7	5	5	3	6
3	7	4	4	6	2	6	6	4	5
3	5	8	4	6	5	2	4	4	5
8	7	7	6	3	6	4	5	5	3
4	5	5	3	5	4	2	6	6	5
6	4	4	6	4	6	4	4	6	7
4	5	4	7	5	5	9	4	5	4
6	6	4	3	8	6	5	4	5	4
2	4	6	6	5	5	5	6	6	6
7	7	5	5	6	2	2	4	4	6
8	9	9	7	3	6	4	6	4	5
5	7	5	2	4	3	4	4	5	6
4	4	8	4	6	8	4	5	8	5
8	7	5	1	3	6	4	6	6	7

8	5	4	5	2	2	5	8	5	5
1	6	7	4	9	1	6	6	7	4
5	5	6	5	5	6	6	2	7	2
7	6	3	6	5	6	4	8	3	5
3	5	6	4	8	7	5	6	8	6
3	4	6	2	6	4	8	4	5	5
5	3	6	6	4	5	6	5	4	5
2	7	7	7	4	4	6	5	4	8
6	4	5	4	5	6	8	5	4	7
2	2	2	7	6	3	3	5	4	5
6	4	3	5	8	4	3	4	6	2
3	6	6	3	3	4	7	4	3	2
6	3	6	3	6	4	3	4	3	3
2	6	6	4	5	6	3	5	4	3
6	3	3	3	7	4	4	5	1	5
3	3	5	6	4	3	5	5	4	4
6	6	7	6	6	3	4	8	2	5
6	5	5	5	5	7	8	4	1	5
3	4	4	3	4	5	6	4	5	5
4	5	6	5	4	4	7	6	7	3
5	4	4	5	5	5	6	4	4	5
5	3	7	7	2	3	7	6	6	5
2	5	3	3	4	3	4	4	7	6
6	6	3	6	4	5	2	3	6	4
6	6	6	3	6	5	5	2	7	3
4	5	5	3	2	4	6	7	2	2
6	4	5	7	3	4	6	4	5	2
5	7	4	4	3	5	2	7	6	5
5	5	3	4	5	2	6	3	4	4
3	4	6	4	6	5	6	5	5	1

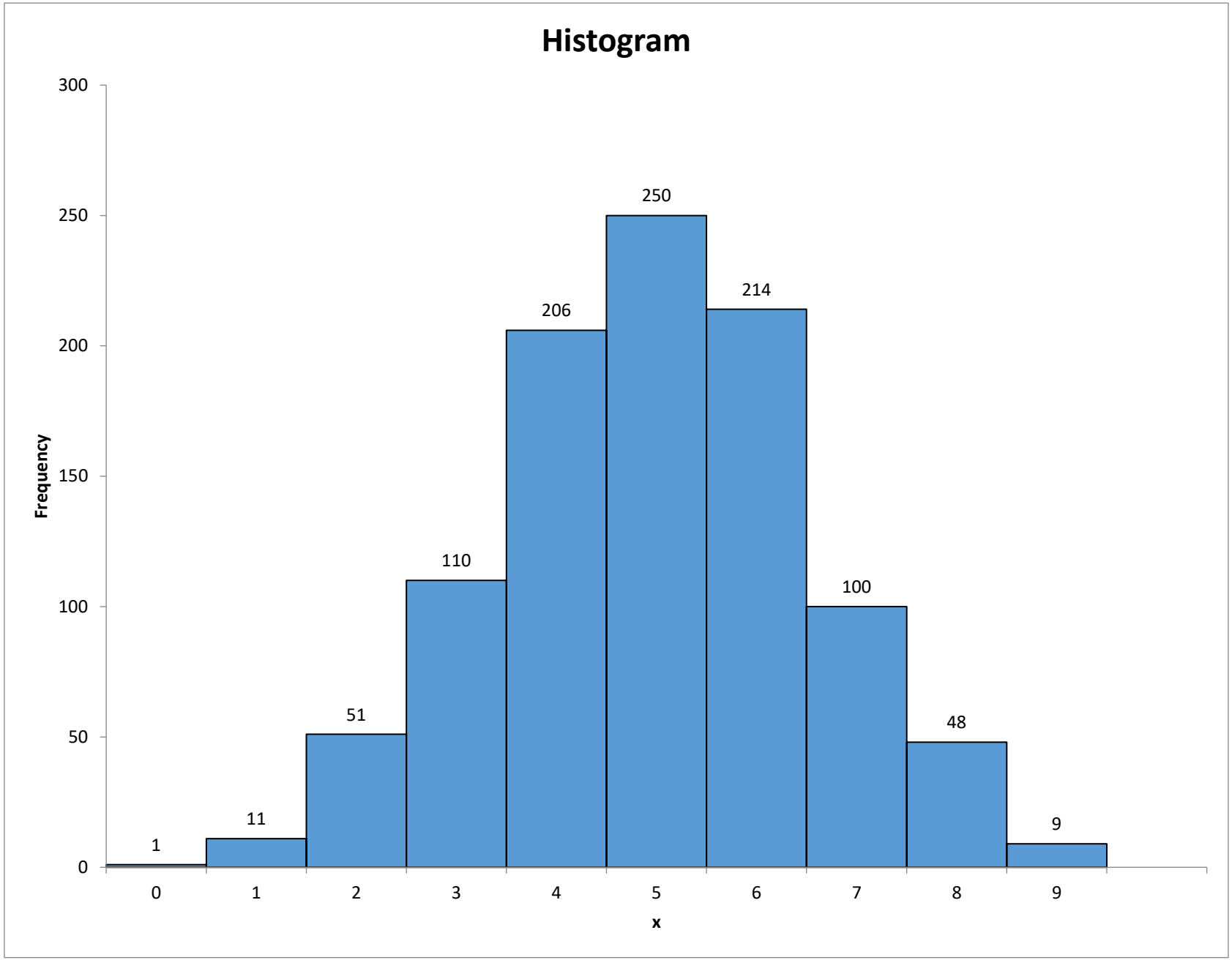
4	3	5	6	5	6	7	6	3	5
6	8	5	3	5	5	6	4	6	3
4	4	5	5	6	5	1	4	6	5
4	4	5	5	5	4	5	5	7	7
6	6	4	5	5	5	2	7	6	4
6	5	6	4	5	6	5	4	3	4
5	5	4	4	6	4	4	5	3	3
7	5	4	6	4	3	3	2	5	5
8	6	7	6	6	4	8	6	5	5
2	6	4	5	4	4	5	5	5	5
5	4	5	4	4	5	4	4	6	2
7	5	5	6	5	4	4	8	6	5
5	4	6	5	6	6	6	2	6	5
1	5	7	6	6	5	8	5	8	7
3	2	5	5	3	6	5	3	6	4
5	7	6	7	3	7	2	4	4	6
7	6	9	6	6	4	4	6	6	5
5	5	5	6	6	5	4	4	4	5
6	4	8	3	8	4	4	5	7	5
3	7	3	5	4	5	5	5	7	3
8	4	4	4	5	6	5	5	3	4
5	6	7	4	5	6	5	4	2	5
3	8	7	5	5	3	9	4	5	4
5	3	3	5	5	8	4	6	7	5
8	5	6	5	3	7	6	7	6	5
6	2	3	1	5	4	7	5	5	4
5	6	6	5	8	4	6	5	6	4
6	6	7	7	6	3	7	5	6	3
2	5	7	7	3	6	6	4	7	5
3	7	8	5	5	5	3	5	5	5

6	4	7	4	3	4	5	5	4	6
4	6	7	7	6	4	5	6	6	8
7	3	8	6	3	7	5	8	5	7
4	4	4	5	6	4	8	7	6	6
4	5	5	2	6	4	6	4	5	6
3	0	4	5	7	5	4	6	8	6
5	6	6	7	7	5	6	4	7	5
3	4	6	6	5	4	6	8	3	4
6	7	5	7	7	5	6	5	5	4
6	6	4	4	5	3	6	6	5	4
4	5	5	5	4	5	5	4	2	5
3	3	7	3	4	7	4	1	4	6
4	6	8	3	5	3	6	4	7	4
5	6	7	4	3	2	4	7	8	6

Excel can automatically construct a frequency distribution and histogram for this large data set.

x	*Frequency*
0	**1**
1	**11**
2	**51**
3	**110**
4	**206**
5	**250**
6	**214**
7	**100**
8	**48**
9	**9**
Total	**1,000**

A nice compact summary of this large data set!


Histogram
Frequency
300
250
200
150
100
50
0
1
11
51
110
206
250
214
100
48
9
0
1
2
3
4
5
6
7
8
9
x

Here is a data set consisting of 1,000 values from 36 to 92.

62	56	54	58	66	58	64	53	63	69
74	58	59	55	61	63	71	54	64	54
63	39	61	44	61	50	52	62	45	63
63	56	58	67	76	57	55	58	65	49
76	63	61	60	60	70	60	55	79	48
63	50	55	57	76	57	73	47	58	62
49	71	56	59	70	45	57	60	68	73
52	60	60	80	66	51	62	68	54	58
61	58	64	67	66	61	51	63	52	57
71	60	62	53	60	68	57	57	62	55
64	46	54	63	64	66	65	49	56	72
69	63	63	67	72	64	51	68	63	40
62	57	69	68	49	65	66	71	53	68
59	56	52	61	54	58	61	63	76	75
65	64	51	61	63	57	58	66	63	46
73	69	54	57	51	51	54	60	59	52
59	53	49	56	54	57	64	68	64	64
61	64	57	62	57	56	65	51	58	50
59	56	68	65	71	56	63	71	58	59
65	63	64	63	58	66	74	53	71	76
55	73	65	67	59	52	52	62	55	70
60	64	77	58	57	58	63	68	54	64
69	57	61	62	58	57	51	62	66	52
61	67	69	73	54	70	51	71	45	54
69	54	57	65	67	67	69	70	55	46
67	52	62	60	47	62	51	68	67	55

71	76	56	53	63	60	57	41	64	66
59	62	64	53	54	64	52	73	73	62
68	58	58	70	68	74	65	43	65	52
81	57	57	58	54	64	72	71	58	66
56	64	62	68	48	53	41	56	68	73
60	49	56	55	70	53	59	61	71	60
44	79	64	60	56	67	45	57	50	58
54	54	46	69	59	51	66	53	58	64
65	59	55	60	53	58	85	59	60	70
67	58	79	64	61	68	57	53	45	56
67	57	69	40	71	61	74	61	63	56
57	59	63	49	66	53	57	51	47	50
57	56	65	63	65	71	60	70	64	60
51	69	53	53	71	54	65	48	47	62
81	58	55	73	65	59	65	70	65	58
65	69	49	53	60	59	56	60	55	53
56	58	61	67	59	68	64	59	66	58
63	61	58	66	62	67	66	59	75	54
55	55	71	47	58	70	48	57	49	55
52	55	60	53	68	63	51	64	64	77
54	70	67	49	59	52	57	69	92	65
78	74	54	67	55	57	69	59	55	58
52	73	60	69	47	55	64	61	53	60
48	54	59	54	62	63	68	57	72	71
43	58	58	70	57	50	53	63	61	69
68	61	53	60	60	59	81	63	51	53
42	47	49	64	68	60	68	71	55	41
59	56	52	54	50	54	60	69	61	50
70	55	56	48	64	70	53	51	54	56
62	70	67	57	68	56	60	54	60	71

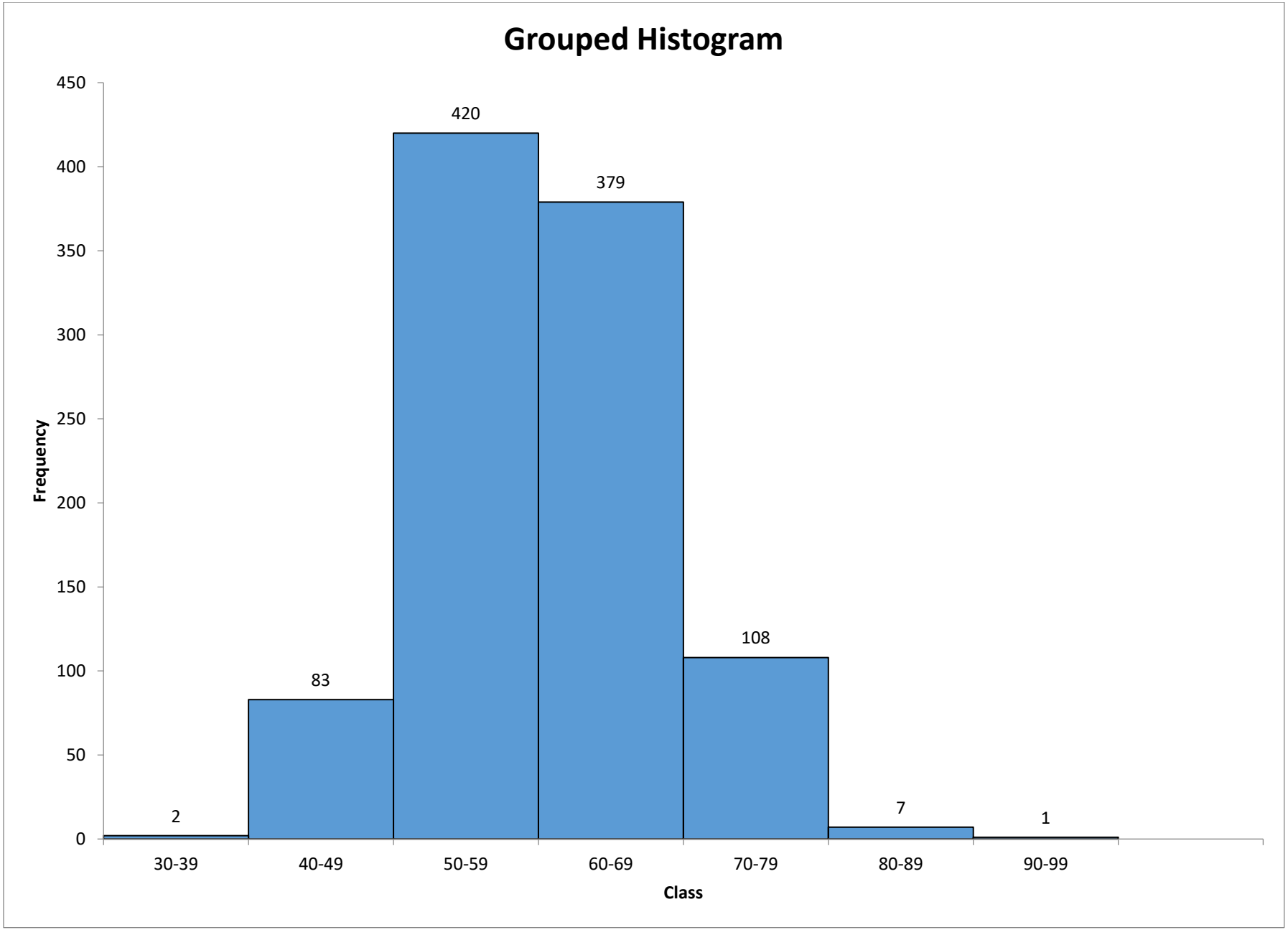
59	61	60	54	62	71	56	79	59	55
49	50	63	59	47	64	53	62	56	50
68	58	72	60	55	50	57	59	54	68
65	44	57	61	79	58	54	66	72	65
69	57	62	57	72	61	58	56	61	36
59	52	58	68	50	57	63	48	83	57
46	53	69	65	59	45	61	72	61	71
50	58	64	63	64	62	66	72	62	53
62	50	73	72	58	51	59	66	52	47
62	49	72	57	65	66	55	67	52	53
54	63	59	61	60	49	52	59	60	61
65	59	66	68	59	53	70	66	56	49
62	56	58	66	51	58	83	69	49	54
72	52	59	55	56	70	62	60	70	57
58	56	53	54	60	74	62	64	60	63
64	53	62	56	56	72	53	66	60	57
63	60	59	67	52	69	53	53	49	66
76	64	58	63	51	63	44	69	74	61
54	51	67	44	56	66	56	53	54	66
58	46	54	61	53	62	50	58	55	59
55	51	55	52	65	61	58	60	69	55
65	66	72	56	58	63	49	47	69	54
64	69	54	68	60	71	58	66	53	59
72	69	51	62	55	45	47	59	69	61
65	61	56	59	64	65	64	56	60	61
69	79	54	74	65	62	64	51	58	55
46	54	54	67	66	45	53	70	51	69
47	53	70	63	66	42	47	47	62	51
64	53	57	60	69	56	61	54	57	51
54	48	65	57	57	61	62	59	70	73

69	46	56	52	64	65	52	63	63	57
60	65	59	58	52	51	50	64	53	44
59	68	59	54	63	49	62	64	52	53
65	53	47	57	52	48	59	61	61	40
60	73	54	60	70	49	47	60	55	53
51	52	60	63	59	51	52	69	58	52
54	63	69	51	74	54	57	54	59	62
76	61	70	70	51	70	65	73	53	60
61	55	70	57	52	59	53	51	58	68
40	53	64	50	69	48	59	56	58	68
56	61	54	60	56	55	58	41	63	63
58	66	54	62	53	55	49	53	48	59
61	59	72	54	50	49	65	50	57	78
68	60	51	69	53	62	63	56	53	74

Excel can automatically construct a grouped frequency distribution and histogram for this large data set.

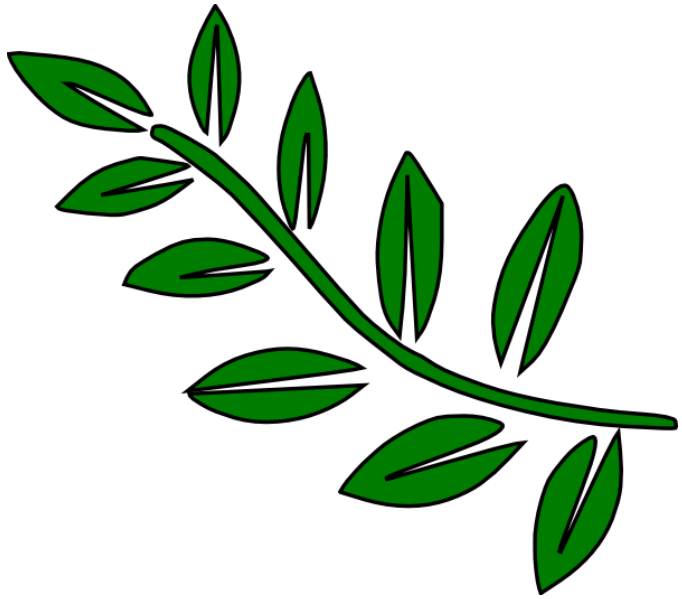
Class	*Frequency*
30-39	**2**
40-49	**83**
50-59	**420**
60-69	**379**
70-79	**108**
80-89	**7**
90-99	**1**
Total	**1,000**

A nice compact summary of this large data set!


Grouped Histogram
450
400
350
300
250
200
150
100
50
0
Frequency
420
379
108
83
7
2
1
30-39
40-49
50-59
60-69
70-79
80-89
90-99
Class

Stem and Leaf Plot:

The actual data values themselves are used to create a frequency distribution of the data set.



Data Set: $\{10, 22, 31, 11, 22, 36, 46, 45, 50, 42\}$

1	0	1	
2	2	2	
3	1	6	
4	6	5	2
5	0		

Allows easy visual determination of the most and least frequently occurring decades!

Ordered Stem and Leaf Plot:

The leaves are listed in ascending order.

1	0 1
2	2 2
3	1 6
4	2 5 6
5	0

Allows easy visual determination of the smallest and largest values!

Example:

3	2 3 5 8
4	0 3 9
5	1 1
6	2 2 7
7	0 0 0 5

How many values are in the data set? 16 **What's the smallest value?** 32

What's the largest value? 75 **What's the most frequently occurring value?** 70

How many values are in the 60's? 3 **Which decade has the fewest values?** 50's

Back-to-Back Stem-and-Leaf Plots:

Used to compare two data sets.

Data Set A	Stem	Data Set B
	1	6 8
	2	1 4 5 6 7 7
7	3	0 5 9 9 9
2	4	1 3 4 6 7 8
7 4 2 1	5	5
6 5 5 5 4 3 2	6	2
7 7 6 6 5 4 3 1	7	
9 9 8 8 8 7 7 3 2 2 0	8	
6 5 4 2 1 0 0	9	

The values of which data set are generally larger? Data Set A

Which data set has more values?

Data Set A

What value is common to both data sets?

62

Measures of Central Tendency:

One value will be used to characterize or summarize an entire data set. In the case of numerical data, it's thought to represent the center or middle of the values.

Some data sets are very large with a wide range of values, so trying to characterize all of the values with one value is a pretty bold undertaking!

Our textbook discusses four such measures of center:

Mean

Median

Mode

Midrange

The *Mean* of a list of numerical data $\{x_1, x_2, x_3, \ldots, x_n\}$ is defined to be $\frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$.

The formula is sometimes written in abbreviated form as $\frac{\sum x}{n}$, where the Greek letter $\sum$ is the abbreviation for add them up.

If the data set represents a sample, the mean value is abbreviated as $\bar{x}$ and called the sample mean.

If the data set represents an entire population, the mean value is abbreviated with the Greek letter μ and called the population mean.

Examples of Mean Calculations:

1. $\{1, 2, 2, 3\}$

$$\frac{1+2+2+3}{4} = \frac{8}{4} = \boxed{2}$$

Sometimes the value of the mean will be an actual value in the data set.

2. $\{1, 2, 2, 3, 4\}$

$$\frac{1+2+2+3+4}{5} = \frac{12}{5} = \boxed{2.4}$$

Sometimes the value of the mean won't be an actual value in the data set.

If you give a rounded value for the mean, always round to one more decimal place than the data values.

3.

0	2	3	
1	0	3	9
2	1	1	

$$\frac{2+3+10+13+19+21+21}{7}=\frac{89}{7}=\boxed{12.7}$$

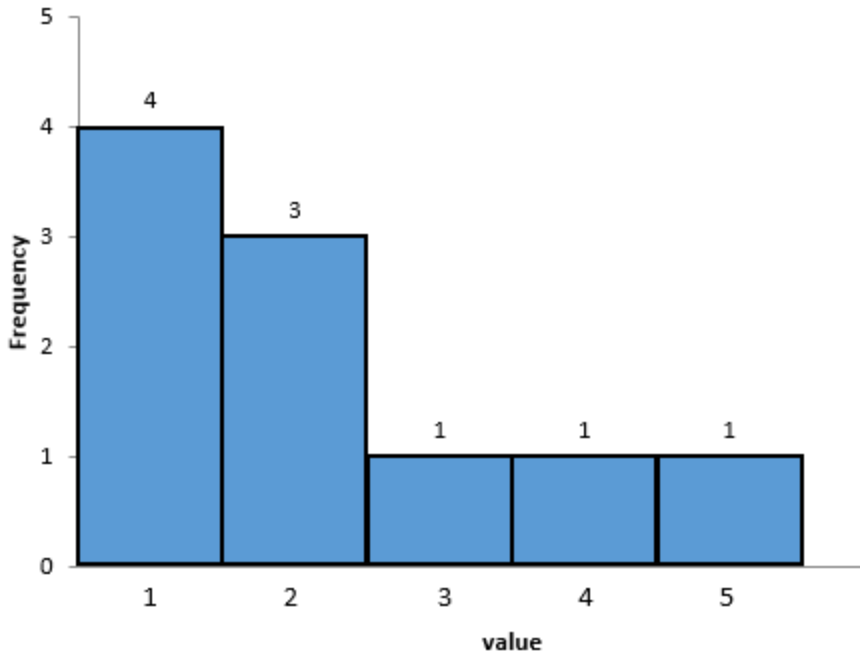
4.

Value	Frequency
1	**11**
2	**13**
3	**22**
Total	**46**

$$\frac{1+1+\cdots+1+2+2+\cdots+2+3+3+\cdots+3}{46}=\frac{1\cdot 11+2\cdot 13+3\cdot 22}{46}=\frac{11+26+66}{46}=\frac{103}{46}=\boxed{2.2}$$

Use multiplication to simplify repeated addition of data values!

5.



$$\frac{1\cdot 4+2\cdot 3+3\cdot 1+4\cdot 1+5\cdot 1}{4+3+1+1+1}=\frac{4+6+3+4+5}{10}=\frac{22}{10}=\boxed{2.2}$$

Use multiplication to simplify repeated addition of data values!

To determine the *Median* of a list of numerical data, you first arrange the values in order.

If there is an odd number of values in the data set, the median is the middle value.

If there is an even number of values in the data set, the median is the average of the middle two values.

Examples of Median Calculations:

1. $\{5, 2, 6, 3, 8\}$

2,3,5,6,8

The middle value is 5, so the median is 5.

When the number of values is odd, the median value will definitely be an actual value in the data set.

2. $\{5,2,6,3,8,10\}$

$2,3,5,6,8,10$

The median is the average of the middle two values, $\frac{5+6}{2} = \boxed{5.5}$.

When the number of values is even, the median value might not be an actual value in the data set.

<u>There is another method for finding the median that works well for larger data sets.</u>

If the data set has *n* values, put the values in order.

If *n* is odd, the median is the value in the $\frac{n+1}{2}$ position.

If *n* is even, the median is the average of the values in the $\frac{n}{2}$ and $\frac{n}{2}+1$ positions.

More Examples of Median Calculations:

3. $\{5,2,6,3,8\}$

$$n=5 \Rightarrow \frac{n+1}{2}=\frac{6}{2}=3$$

$$2,3,5,6,8$$

The third value is the median value, and it's 5.

4. $\{5,2,6,3,8,10\}$

$$n=6 \Rightarrow \frac{n}{2}=\frac{6}{2}=3, \frac{n}{2}+1=4$$

$$2,3,5,6,8,10$$

The average of the third and fourth values is the median, and it's $\frac{5+6}{2}=\boxed{5.5}$.

5.

3	2	3	5	8	9	9	
4	0	3	4	7	9	9	
5	1	1	2	2	8	9	
6	2	2	7	7	8	9	9

$$n = 25 \Rightarrow \frac{n+1}{2} = \frac{26}{2} = 13$$

The median is the thirteenth value, and it's 51.

6.

3	2	3	5	8	9	9
4	0	3	4	7	9	9
5	1	1	2	2	8	9
6	2	2	7	7	8	9

$$n = 24 \Rightarrow \frac{n}{2} = \frac{24}{2} = 12, \frac{n}{2} + 1 = 13$$

The median is the average of the twelfth and thirteenth values, and it's $\frac{49+51}{2} = \boxed{50}$.

7.

Value	Frequency
1	12
2	13
3	22
Total	47

$$n = 47 \Rightarrow \frac{n+1}{2} = \frac{48}{2} = 24$$

The median is the 24^{th} value, and it's 2.

8.

Value	Frequency
1	12
2	11
3	23
Total	46

$$n = 46 \Rightarrow \frac{n}{2} = \frac{46}{2} = 23, \frac{n}{2} + 1 = 24$$

The median is the average of the 23^{rd} and 24^{th} values, and it's $\frac{2+3}{2} = \boxed{2.5}$.

The *Mode* of a list of numerical or descriptive data is defined to be the most frequently occurring value.

Unlike the textbook, I don't allow for two modes(bimodal), or three modes(trimodal), etc.. For me either there is one most frequently occurring value or there isn't.

Mode Examples:

1. $\{1,2,2,3\}$

The mode is 2.

2. $\{1,1,2,2,3\}$

There is no mode.

Sometimes a data set won't have a mode.

3. $\{red, blue, green, green\}$

The mode is *green*.

Means and Medians only work for numerical data, but this isn't the case for Modes.

4.

3	2	3	5	8	9	9	
4	0	3	4	7	9	9	
5	1	1	2	2	8	9	
6	2	2	7	7	7	9	9

The longest horizontal repetition of digits is three 7's, so the mode is 67.

5.

Value	Frequency
1	12
2	11
3	23
Total	46

The largest frequency value is 23, so the mode is 3.

6.

Value	Frequency
Small	10
Medium	11
Large	40
X-Large	23
Total	84

The largest frequency value is 40, so the mode is Large.

The *Midrange* of a list of numerical data is defined to be the average of the smallest and largest data values.

$$\text{midrange} = \frac{\text{smallest value} + \text{largest value}}{2}$$

Examples of Midrange Calculations:

1. $\{1, 2, 2, 3\}$

$$\text{The midrange is } \frac{1+3}{2} = \boxed{2}.$$

Sometimes the value of the midrange will be an actual value in the data set.

2. $\{1, 2, 2, 3, 4\}$

$$\text{The midrange is } \frac{1+4}{2} = \boxed{2.5}.$$

Sometimes the value of the midrange won't be an actual value in the data set.

If you give a rounded value for the midrange, always round to one more decimal place than the data values.

3.

0	2	3	
1	0	3	9
2	1	2	

$$\text{The midrange is } \frac{2+22}{2} = \boxed{12}.$$

4.

Value	Frequency
1	11
2	13
3	22
Total	46

The midrange is $\frac{1+3}{2} = \boxed{2}$.

For the data set $\{2,2,15,16,17,18,19,20,21,22,23\}$, the mode is 2 and the median is 18. Which measure better represents this data set?

median

For the data set $\{38,39,42,42,43,156\}$, the mean is 60 and the median is 42. Which measure better represents this data set?

median