Ordinary Annuities:

You want to accumulate an amount of money by making *n* equal payments at the end of the compounding periods into an account paying compound interest.

Compounding Period:	**1**	**2**	**3**	**4**				***n* - 1**	***n***
	P →								$P(1+i)^{n-1}$
		P →							$P(1+i)^{n-2}$
			P →						$P(1+i)^{n-3}$
				P →					$P(1+i)^{n-4}$
					⋱				
						⋱			
							⋱		
								P →	$P(1+i)$
									P

The total amount of money you'll have after you make the *n*th payment is

$A = P + P(1+i) + P(1+i)^2 + \cdots + P(1+i)^{n-1}$. **If you multiply by** $(1+i)$**, and subtract, you get**

$$\begin{aligned} A &= P + P(1+i) + P(1+i)^2 + \cdots + P(1+i)^{n-1} \\ -(1+i)A &= \quad\; P(1+i) + P(1+i)^2 + \cdots + P(1+i)^{n-1} + P(1+i)^n \\ \hline -iA &= P - P(1+i)^n \\ A &= P\left[\frac{(1+i)^n - 1}{i}\right] \end{aligned}$$

Let's change notation, $FV = Pmt\left[\frac{(1+i)^n - 1}{i}\right]$**.**

Examples:

1. You will make monthly deposits of $500 into an account paying 3% compounded monthly for 10 years. How much money will you have? How much of it is interest?

$$FV = 500\left[\frac{\left(1+\frac{.03}{12}\right)^{120}-1}{\frac{.03}{12}}\right] = \boxed{\$69,870.71}$$, and the interest portion is $\boxed{\$9,870.71}$.

To get the interest portion, just subtract the 120 deposits of $500 from the future value of $69,870.71.

2. You will make quarterly deposits of $200 into an account paying 2.4% compounded quarterly for 6 years. How much money will you have? How much of it is interest?

$$FV = 200\left[\frac{\left(1+\frac{.024}{4}\right)^{24}-1}{\frac{.024}{4}}\right] = \boxed{\$5,146.24}$$, and the interest portion is $\boxed{\$346.24}$.

3. You will make weekly deposits of $50 into an account paying 2.7% compounded weekly for 3 years. How much money will you have? How much of it is interest?

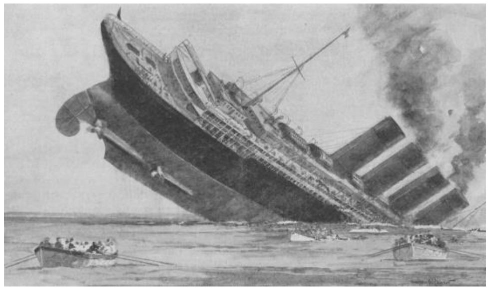
$$FV = 50\left[\frac{\left(1+\frac{.027}{52}\right)^{156}-1}{\frac{.027}{52}}\right] = \$8,122.41$$

$$\text{Interest} = \$8,122.41 - 156(\$50) = \$322.41$$

Planning For The Future(Sinking Fund Formula):

Let's solve the formula $FV = Pmt\left[\frac{(1+i)^n - 1}{i}\right]$ **for the payment amount.**

$$Pmt = FV\left[\frac{i}{(1+i)^n - 1}\right]$$



Examples:

1. **You would like to have $5,000 eight years from now by making monthly payments into an account paying 3.2% compounded monthly. What should your payment be?**

$$Pmt = 5{,}000\left[\frac{\frac{.032}{12}}{\left(1+\frac{.032}{12}\right)^{96} - 1}\right] = \boxed{\$45.77}$$

2. **You would like to have $10,000 ten years from now by making semi-annual payments into an account paying 2.1% compounded semi-annually. What should your payment be?**

$$Pmt = 10{,}000\left[\frac{\frac{.021}{2}}{\left(1+\frac{.021}{2}\right)^{20} - 1}\right] = \boxed{\$451.95}$$

3. You would like to have $25,000 four years from now by making quarterly payments into an account paying 1.9% compounded quarterly. What should your payment be?

$$Pmt = 25000\left[\frac{\frac{.019}{4}}{\left(1+\frac{.019}{4}\right)^{16}-1}\right] = \$1,507.58$$

Scientific Calculator Advice:

1. You will make quarterly deposits of $200 into an account paying 2.4% compounded quarterly for 6 years. How much money will you have? How much of it is interest?

The formula that we will use to find the amount of money is $FV = Pmt\left[\frac{\left(1+\frac{r}{m}\right)^n - 1}{\frac{r}{m}}\right]$. We need to plug in the correct values and get the calculator to cooperate. Let's start with plugging in the correct values: The payment amount is $200. The annual interest rate as a decimal is .024. The number of payments/compounding-periods in one year is 4. The total number of payments/compounding-periods is 24. Therefore, $FV = 200\left[\frac{\left(1+\frac{.024}{4}\right)^{24} - 1}{\frac{.024}{4}}\right]$.

Now for the calculator. Using a standard scientific calculator, I'd start by typing in .024, pressing $\div$, typing 4 and pressing =. Then I would press +, type 1 and press =. We have to raise this to the 24th power, so I'd press either ^ or the x^y key(depending on the calculator), type in 24 and press =. Next, I'd press -, type 1 and press =. Then I'd press $\div$, type .024 and press =. Then I'd press $\times$, type 4 and press =. Finally, I'd press $\times$, type 200 and press =. You should be looking at 5146.243073 on your calculator display. This needs to be converted into dollar and cent format rounded to the nearest penny, which would be $5,146.24.

To get the amount of interest, we must subtract the amount of money contributed through the 24 payments of $200 from the amount of money in the account, $5,146.24. This leads to $5,146.24 – 24($200) = $5,146.24 - $4800 = $346.24.

2. You would like to have \$10,000 ten years from now by making semi-annual payments into an account paying 2.1% compounded semi-annually. What should your payment be?

The formula that we will use to find the payment amount is $Pmt = FV\left[\frac{\frac{r}{m}}{\left(1+\frac{r}{m}\right)^n - 1}\right]$. We need to plug in the correct values and get the calculator to cooperate. Let's start with plugging in the correct values: The future value amount is \$10,000. The annual interest rate as a decimal is .021. The number of payments/compounding-periods in one year is 2. The total number of payments/compounding-periods is 20. Therefore, $Pmt = 10{,}000\left[\frac{\frac{.021}{2}}{\left(1+\frac{.021}{2}\right)^{20} - 1}\right]$. Now for the calculator. Using a standard scientific calculator, I'd start by typing in .021, pressing $\div$, typing 2 and pressing =. Then I would press +, type 1 and press =. We have to raise this to the 20th power, so I'd press either ^ or the x^y key(depending on the calculator), type in 20 and press =. Next, I'd press -, type 1 and press =. So far we've calculated the value of the denominator, but the calculator doesn't know that value is the denominator. To move this value into the denominator, we need to press the either the $\frac{1}{x}$ or the x^{-1} key(depending on the calculator). Then I'd press $\times$, type .021 and press =. Then I'd press $\div$, type 2 and press =. Finally, I'd press $\times$, type 10,000 and press =. You should be looking at 451.9470222 on your calculator display. This needs to be converted into dollar and cent format rounded to the nearest penny, which would be \$451.95.