Fairness Criteria for Determining a Winner in an Election:

The Majority Criterion:

If a candidate receives a majority of first-place votes, then that candidate should win the election.

Example:

Number of votes	27	24	2
1st	A	B	C
2nd	C	C	B
3rd	B	A	A

Find the Borda count winner:

Candidate	3rd –place points	2nd –place points	1st –place points	Total
A	$26 \cdot 1 = 26$	$0 \cdot 2 = 0$	$27 \cdot 3 = 81$	107
B	$27 \cdot 1 = 27$	$2 \cdot 2 = 4$	$24 \cdot 3 = 72$	103
C	$0 \cdot 1 = 0$	$51 \cdot 2 = 102$	$2 \cdot 3 = 6$	108

C is the Borda count winner.

Which candidate received a majority of first-place votes?

Candidate A received 27 first-place votes which is a majority.

In this election, did the Borda count method satisfy or violate the majority criterion?

A received a majority of first-place votes but was not the Borda count winner; instead C is the Borda count winner, so in this election, the Borda count method violated the majority criterion.

Conclusion about the Borda count method:

The Borda count method is not a fair method because it doesn't always satisfy the majority criterion.

The Head-to-Head Criterion:

If one candidate is favored over all the other candidates in head-to-head comparisons, then that candidate should win the election.

Example:

Number of votes	20	19	5
1st	A	B	C
2nd	B	C	B
3rd	C	A	A

Find the plurality winner.

A received the most first-place votes with 20, so A is the plurality winner.

Head-to-head comparison	Result
A vs. B	B is preferred.
A vs. C	C is preferred.
B vs. C	B is preferred.

Which candidate is preferred over all the other candidates?

B is preferred over A and C, so B is preferred over all the other candidates.

In this election, did the plurality method satisfy or violate the head-to-head criterion?

B is favored over all the other candidates, but B is not the plurality winner; instead, A is the plurality winner, so in this case, the plurality method violated the head-to-head criterion.

Conclusion about the plurality method:

The plurality method is not a fair method because it doesn't always satisfy the head-to-head criterion.

The Monotonicity Criterion:

If a candidate wins an election, and in a re-election, the only changes are changes that favor that candidate, then that candidate should win the re-election.

Examples:

1.

Number of votes	**14**	**12**	**10**	**6**
1st	**C**	**B**	**A**	**A**
2nd	**A**	**C**	**B**	**C**
3rd	**B**	**A**	**C**	**B**

Find the plurality-with-elimination winner.

A majority is 22 or more. B must be eliminated.

Number of votes	14	12	10	6
1st	C	C	A	A
2nd	A	A	C	C

C is the plurality-with-elimination winner.

It was determined that an error occurred in the voting process, and the 6 ballots for the order A,C,B should have been for C,A,B. This is a change that favors candidate C.

Number of votes	20	12	10
1st	C	B	A
2nd	A	C	B
3rd	B	A	C

Find the plurality-with-elimination winner of the re-election.

This time A is eliminated.

Number of votes	20	12	10
1st	C	B	B
2nd	B	C	C

B is the plurality-with-elimination winner.

In this case, did the plurality-with-elimination method satisfy or violate the monotonicity criterion?

C won the original election, the change was favorable to C, but C didn't win the re-election; instead B won the re-election, so in this case, the plurality-with-elimination method violated the monotonicity criterion.

Conclusion about the plurality-with-elimination method:

The plurality-with-elimination method is not a fair method because it doesn't always satisfy the monotonicity criterion.

2.

Number of Votes	50	40	20	5
1st choice	A	C	C	B
2nd choice	B	A	B	C
3rd choice	C	B	A	A

Candidate	1st place count	2nd place count	3rd place count	Borda count
A	$3 \cdot 50 = 150$	$2 \cdot 40 = 80$	$1 \cdot 25 = 25$	**255**
B	$3 \cdot 5 = 15$	$2 \cdot 70 = 140$	$1 \cdot 40 = 40$	**195**
C	$3 \cdot 60 = 180$	$2 \cdot 5 = 10$	$1 \cdot 50 = 50$	**240**

So the Borda count winner is _____A_____.

The 5 ballots for the order B,C,A, are changed to the order A,B,C, resulting in the following new preference table for the re-election*(This change favors candidate A.)*

Number of Votes	55	40	20
1st choice	A	C	C
2nd choice	B	A	B
3rd choice	C	B	A

And the new Borda counts are in the table below.

Candidate	1st place count	2nd place count	3rd place count	Borda count
A	$3 \cdot 55 = 165$	$2 \cdot 40 = 80$	$1 \cdot 20 = 20$	**265**
B	$3 \cdot 0 = 0$	$2 \cdot 75 = 150$	$1 \cdot 40 = 40$	**190**
C	$3 \cdot 60 = 180$	$2 \cdot 0 = 0$	$1 \cdot 55 = 55$	**235**

So the Borda Count winner of the re-election is _____A_____ .
In this case, did the Borda count method satisfy or violate the monotonicity criterion?

Explain.

A won the election and the re-election, so the Borda count method satisfied the monotonicity criterion, in this case.

The Irrelevant Alternatives Criterion:

If a candidate wins an election, and in a re-election, the only change is that one or more of the other candidates are removed from the ballot, then that same candidate should win the re-election.

Example:

Number of votes	7	6	2
1st	A	B	C
2nd	B	C	B
3rd	C	A	A

Find the plurality winner.

A received the most first-place votes with 7, so A is the plurality winner.

Suppose that candidate C shouldn't have been on the ballot.

Number of votes	7	6	2
1st	A	B	B
2nd	B	A	A

Find the plurality winner of the re-election.

B received the most first-place votes with 8, so B is the plurality winner.

In this case, did the plurality method satisfy or violate the irrelevant alternatives criterion?

A won the original election, C was removed from the ballot, but A didn't win the re-election; instead, B won the re-election, so in this case the plurality method violated the irrelevant alternatives criterion.

Conclusion about the plurality method:

The plurality method is not a fair method because it doesn't always satisfy the irrelevant alternatives criterion.

A procedure for determining a winner in an election is considered fair if it is impossible for it to violate any of the four fairness criteria.

In 1951, economist Kenneth Arrow proved the following result(Nobel Prize 1972):

Arrow's Impossibility Theorem:

It is impossible for any democratic voting procedure to always satisfy the four fairness criteria.