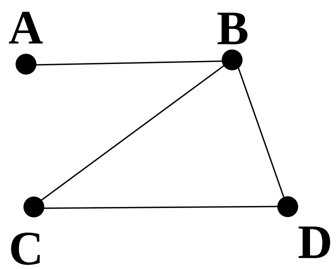
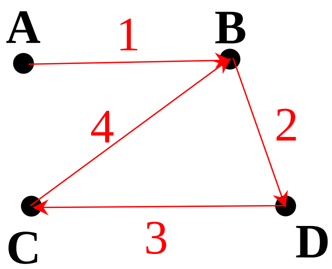
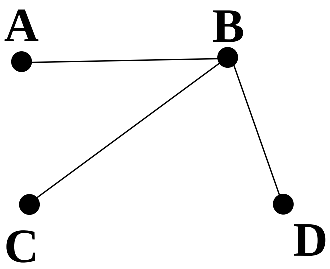
Euler Path:

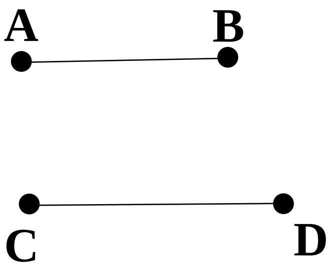
It's a path that uses every edge of a graph exactly once.





A,B,D,C,B is an example of an Euler path for the graph above.



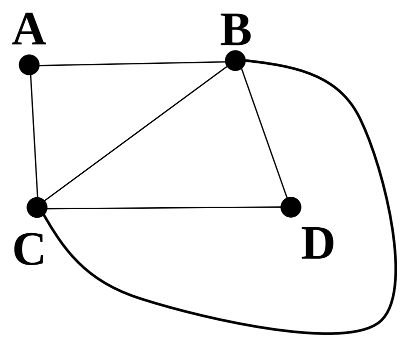


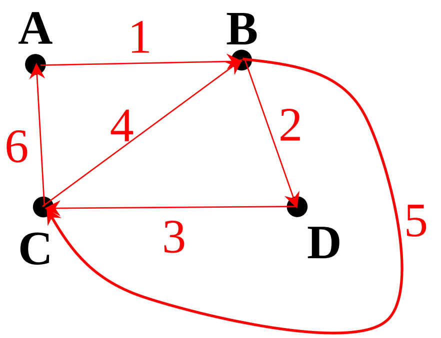
There is no Euler path for this graph.

There is no Euler path for this graph.

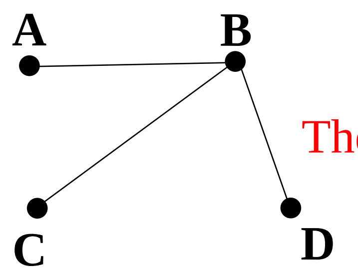
Euler Circuit:

It's a circuit that uses every edge of a graph exactly once. Every Euler circuit is also an Euler path.





A,B,D,C,B,C,A is an example of an Euler circuit for the graph above.



There is no Euler circuit for this graph.

<u>Euler's Theorem:</u>

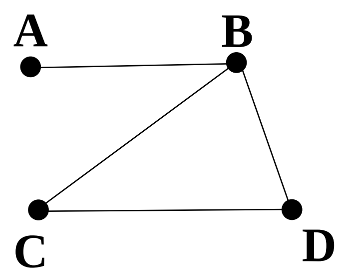
For a connected graph:

1. **If a graph has exactly two odd vertices, then it has at least one Euler path, but no Euler circuit. Each Euler path must start at one of the odd vertices and end at the other odd vertex.**

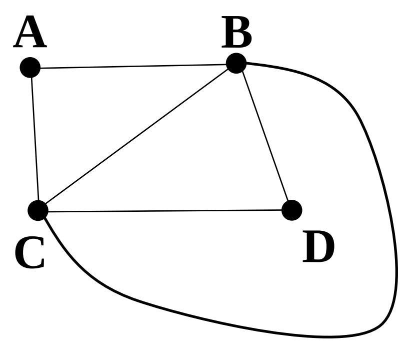
2. **If a graph has no odd vertices, then it has at least one Euler circuit(and therefore an Euler path). An Euler circuit can start and end at any vertex.**

3. **If a graph has more than two odd vertices, then it has no Euler paths and no Euler circuits.**

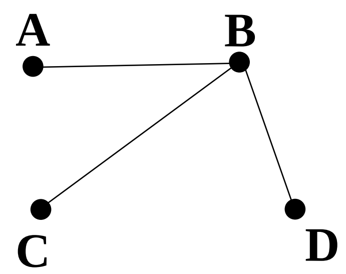
Determine if the following graphs have an Euler circuit, Euler path, or neither, using Euler's Theorem.



This graph has exactly two odd vertices, so it will have an Euler path, but no Euler circuit.



This graph has no odd vertices, so it will have an Euler circuit, (and therefore an Euler path, as well.)



This graph has more than two odd vertices, so it will have neither an Euler path nor an Euler circuit.

Vertex	Degree
A	21
B	23
C	12
D	14

This graph has exactly two odd vertices, so it will have an Euler path, but no Euler circuit.

Vertex	Degree
A	21
B	23
C	11
D	14
E	15

This graph has more than two odd vertices, so it will have neither an Euler path nor an Euler circuit.

Vertex	Degree
A	22
B	24
C	18
D	14
E	16

This graph has no odd vertices, so it will have an Euler circuit, (and therefore an Euler path, as well.)

How do you find an Euler path or Euler circuit?

<u>Fleury's Algorithm:</u>

If Euler's Theorem indicates an Euler path or circuit, then

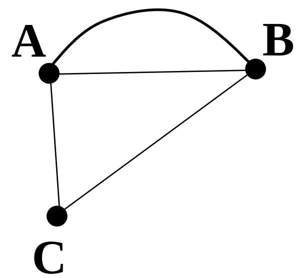
1. **If the graph has exactly two odd vertices(and therefore an Euler path), choose one of them as a starting vertex. If the graph has no odd vertices(and therefore an Euler circuit), start at any vertex.**

2. **Number the edges as you use them by obeying the following rules:**

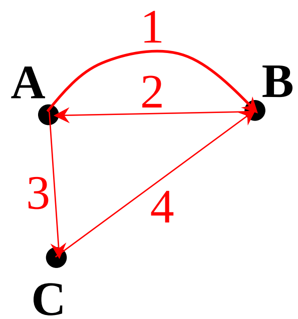
   a. **Dash out edges that you have already used.**

   b. **When you have a choice of edges, avoid choosing a bridge, if possible.**

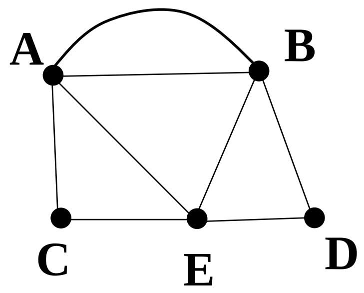
Examples:



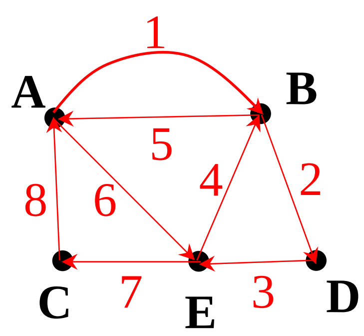
This graph has exactly two odd vertices, so it will have an Euler path but no Euler circuit. Any Euler path must start at one of the odd vertices and end at the other odd vertex. Let's start at A and end at B.



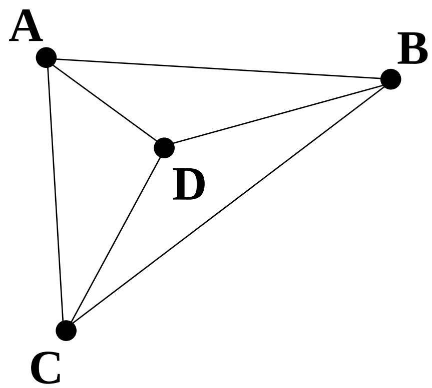
A,B,A,C,B



This graph has no odd vertices, so it will have an Euler circuit. An Euler circuit can start and end at any of the vertices, so let's start and end at A.

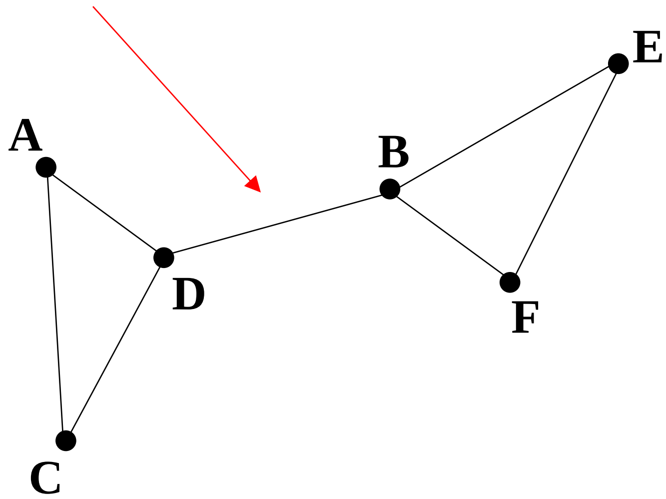


A,B,D,E,B,A,E,C,A

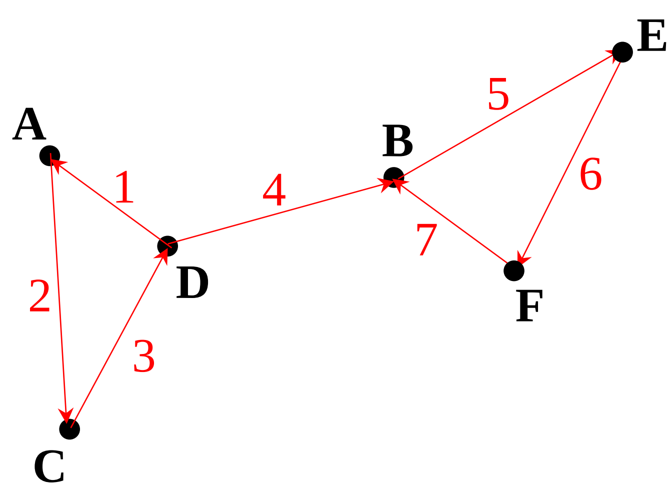


This graph has more than two odd vertices, so it will have neither an Euler path nor Euler circuit.

Watch out for the bridge!

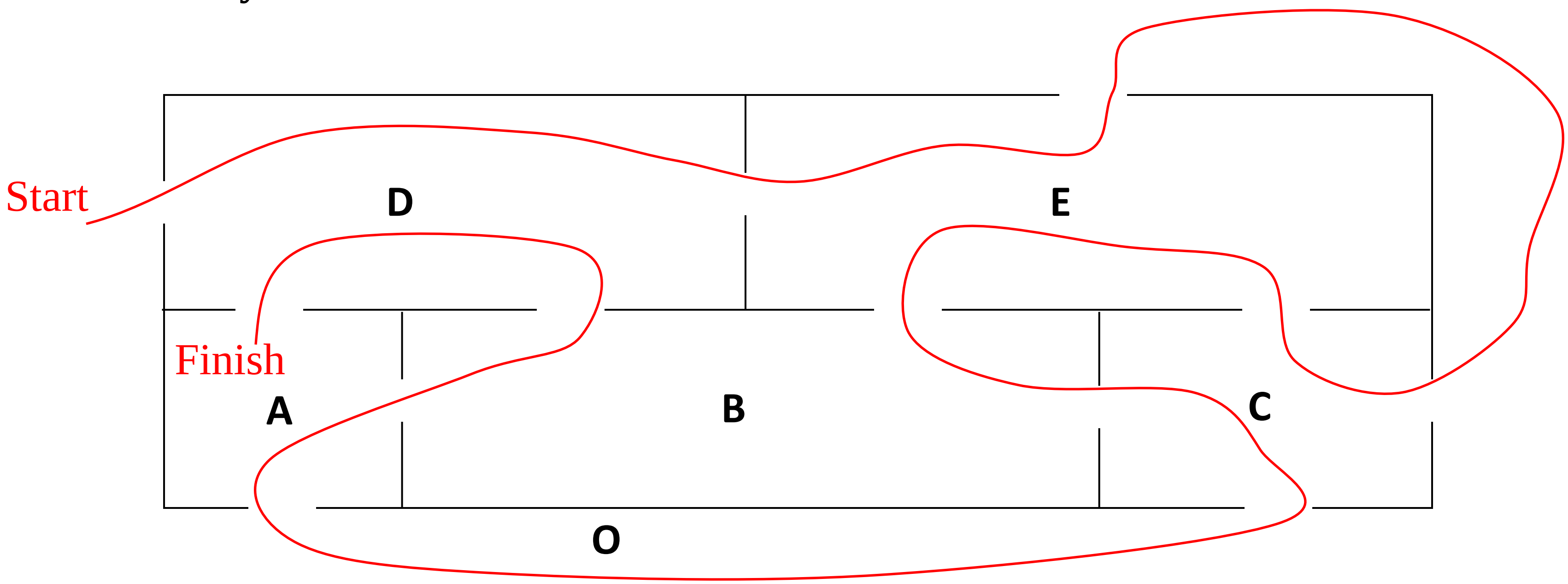


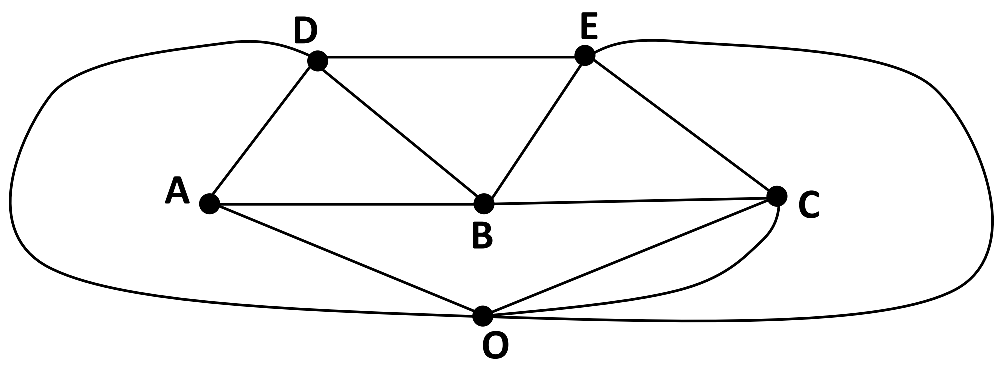
This graph has exactly two odd vertices, so it will have an Euler path but no Euler circuit.
Any Euler path must start at one of the odd vertices and end at the other odd vertex.
Let's start at D and end at B.



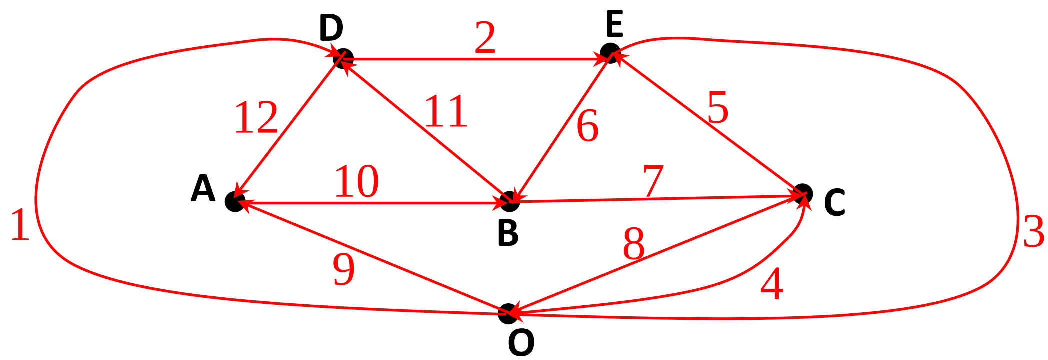
D,A,C,D,B,E,F,B

Indicate a tour of the building that starts on the outside, passes through each door exactly once and ends in room A.





Let’s start with the graph, and then transfer back to the floorplan. There are exactly two odd vertices-A and O, so we can find an Euler path that starts at O and ends at A



O,D,E,O,C,E,B,C,O,A,B,D,A