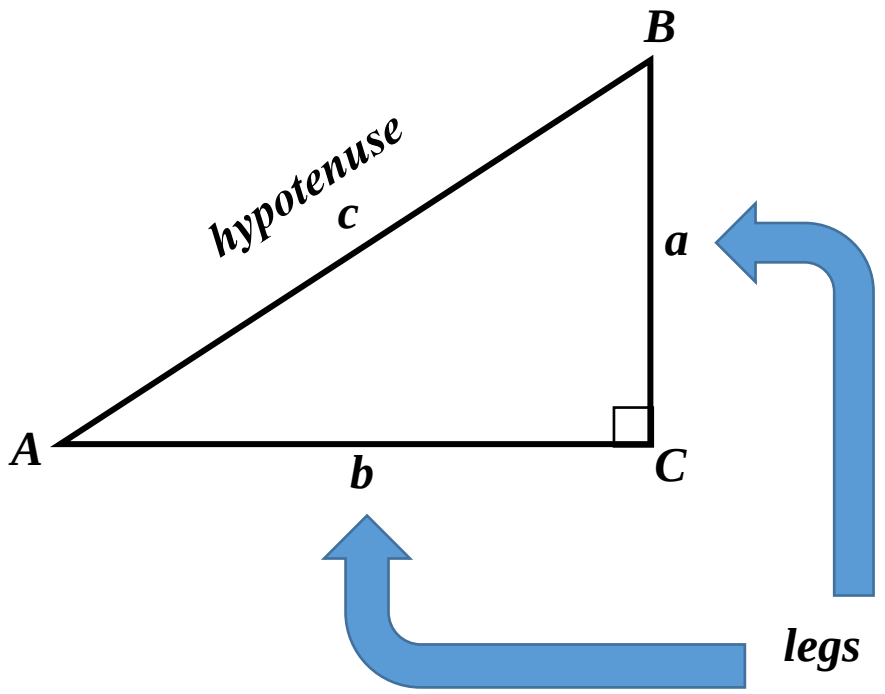
Right Triangle Ratios:

For *A*, one of the acute angles in a right triangle, the trigonometric ratios-sine, cosine, and tangent of *A* are defined as follows.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$

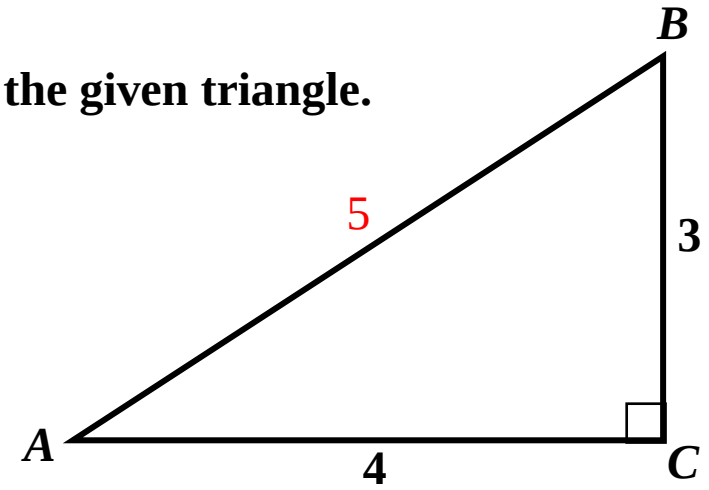


Example:

Find the following trigonometric ratio values from the given triangle.

First, use the Pythagorean Theorem $(a^2 + b^2 = c^2)$

to find the length of the hypotenuse.



The length of the hypotenuse must be $\sqrt{3^2 + 4^2} = \sqrt{25} = 5.$

$$\sin A = \frac{3}{5} \qquad \sin B = \frac{4}{5}$$

$$\cos A = \frac{4}{5} \qquad \cos B = \frac{3}{5}$$

$$\tan A = \frac{3}{4} \qquad \tan B = \frac{4}{3}$$

The values of trigonometric ratios for specific angles can be determined using a scientific calculator. Just make sure that the angle measure on the calculator is set to *degrees*.

Examples: Find the values of the following trigonometric ratios to four decimal places.

$\sin 10° = .1736$ $\sin 80° = .9848$

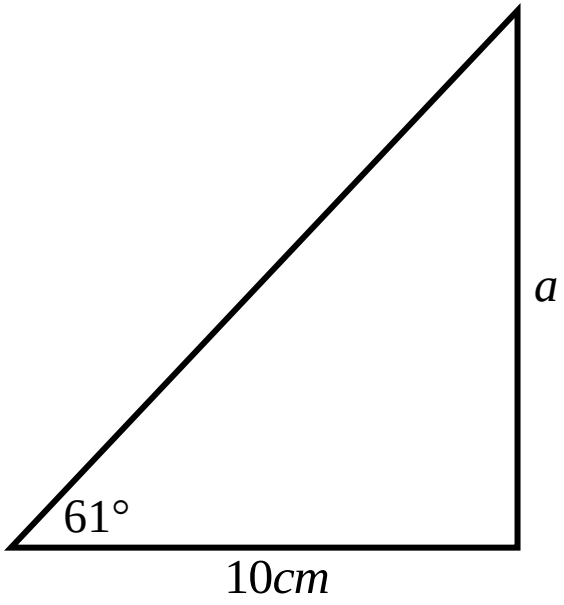
$\cos 15° = .9659$ $\cos 75° = .2588$

$\tan 25° = .4663$ $\tan 65° = 2.1445$

Finding the length of a leg of a right triangle:

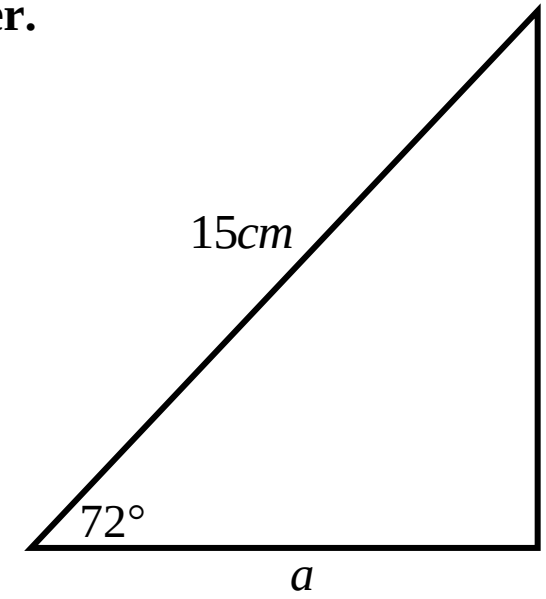
1. Find the value of *a* to the nearest whole centimeter.

$$\tan 61° = \frac{a}{10} \Rightarrow a = 10\tan 61° \Rightarrow a = 18.0404... = \boxed{18cm}$$

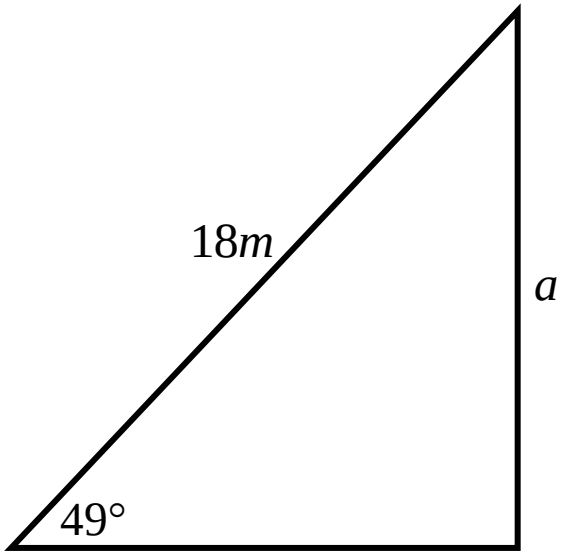


2. Find the value of *a* to the nearest tenth of a centimeter.

$$\cos 72° = \frac{a}{15} \Rightarrow a = 15\cos 72° = 4.6352... = \boxed{4.6\,cm}$$



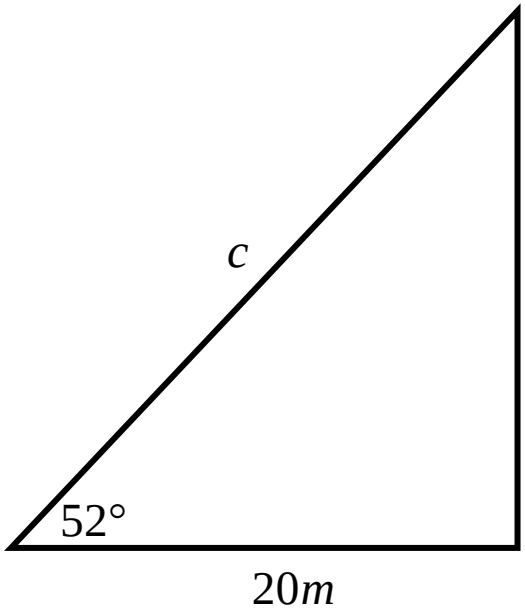
3. Find the value of *a* to the nearest tenth of a meter.



$$\sin 49° = \frac{a}{18} \Rightarrow a = 18 \sin 49° = 13.584... = \boxed{13.6m}$$

Finding the length of the hypotenuse of a right triangle:

Find the value of *c* to the nearest tenth of a meter.



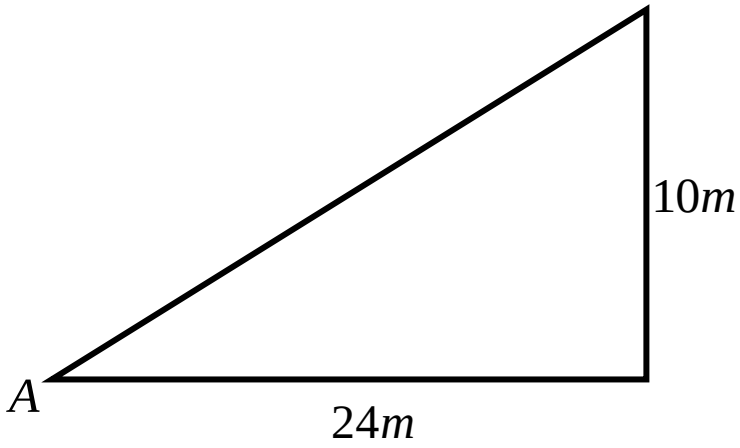
$$\cos 52° = \frac{20}{c} \Rightarrow c \cdot \cos 52° = 20$$

$$\Rightarrow c = \frac{20}{\cos 52°} = 32.48538... = \boxed{32.5m}$$

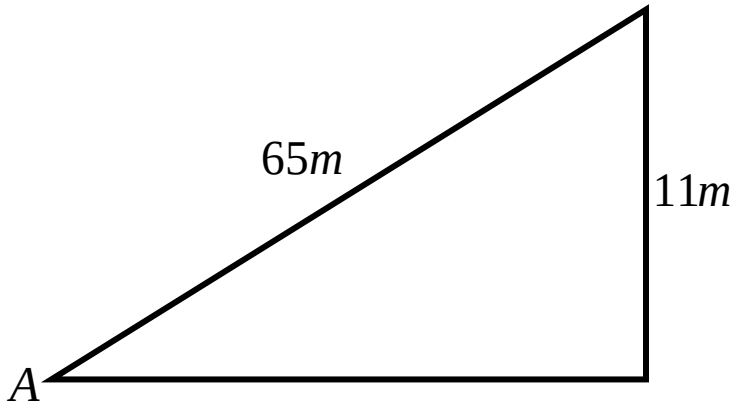
Scientific calculators can also read a given trigonometric ratio value back to a specific angle value. These reversals are done by using one of the three inverse trigonometric keys: $sin^{-1}, cos^{-1}, tan^{-1}$**.**

1. Find the measure of angle *A* to the nearest tenth of a degree.

$$\angle A = tan^{-1}\frac{\text{opposite}}{\text{adjacent}} = tan^{-1}\frac{10}{24} = 22.619... = \boxed{22.6°}$$

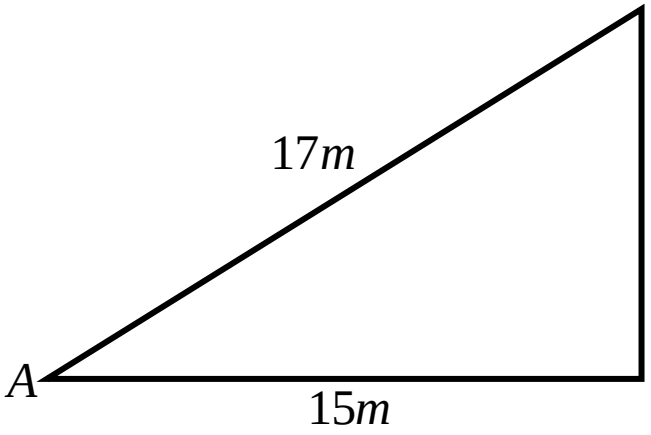


2. Find the measure of angle *A* to the nearest tenth of a degree.



$$\angle A = sin^{-1}\frac{\text{opposite}}{\text{hypotenuse}} = sin^{-1}\frac{11}{65} = 9.743... = \boxed{9.7°}$$

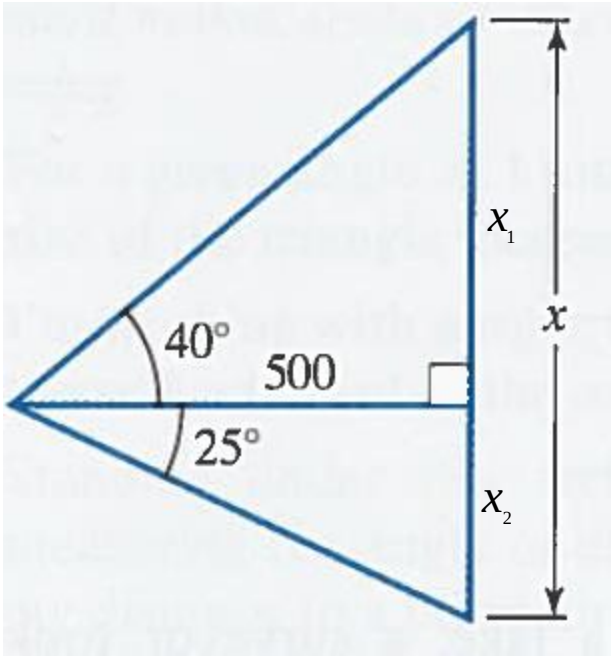
3. Find the measure of angle *A* to the nearest tenth of a degree.



$$\angle A = cos^{-1}\frac{\text{adjacent}}{\text{hypotenuse}} = cos^{-1}\frac{15}{17} = 28.0724... = \boxed{28.1°}$$

Finding missing measurements using more than one trigonometric ratio.

1. Find the value of *x* to the nearest whole number.



Notice that $x = x_1 + x_2$**, and that** $tan\,40° = \dfrac{x_1}{500}$

So $x_1 = 500(tan\,40°)$

and $tan\,25° = \dfrac{x_2}{500}$**.**

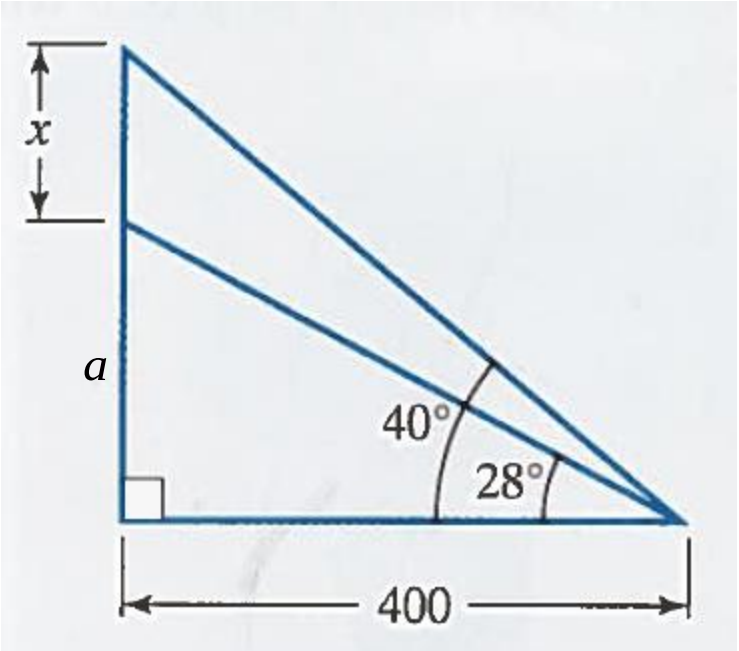
So $x_2 = 500(tan\,25°)$

So $x = 500(tan\,40°) + 500(tan\,25°)$

$= 652.703... = \boxed{653}$

2. Find the value of *x* to the nearest whole number.

From $\tan 40° = \frac{x+a}{400}$ and $\tan 28° = \frac{a}{400}$



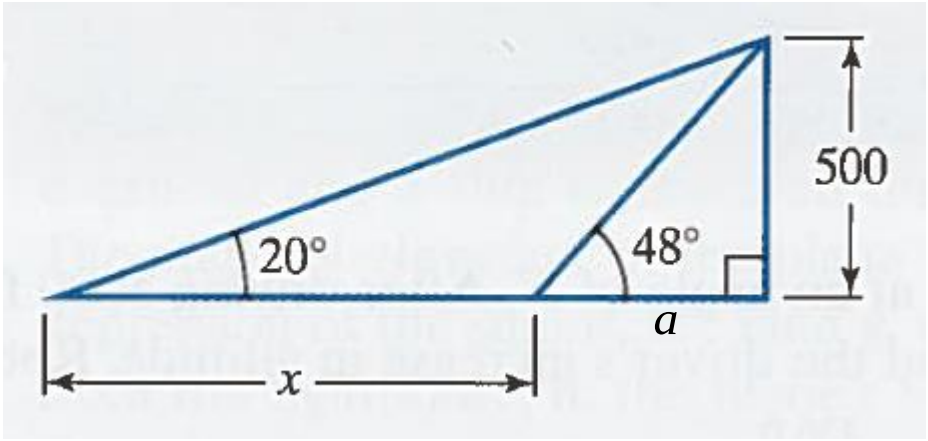
Notice that $x + a = 400(\tan 40°)$ **and**

$a = 400(\tan 28°)$**.**

So $x = 400(\tan 40°) - 400(\tan 28°)$

$= 122.956... = \boxed{123}$

3. Find the value of x to the nearest whole number.



Notice that $\frac{500}{x+a} = tan\,20°$ **and**

$\frac{500}{a} = tan\,48°$**.**

These lead to $x + a = \frac{500}{tan\,20°}$ **and** $a = \frac{500}{tan\,48°}$**.**

So $x = \frac{500}{tan\,20°} - \frac{500}{tan\,48°}$

$= 923.5366... = \boxed{924}$