Basic Set Terminology:

A is the set of students registered for this class. *(Word Description method)*

$B = \{1, 2, 3, 4, 5\}$ *(Roster method)*

$C = \{2, 4, 6, 8\}$ *(Roster method)*

$D = \{x \mid x \text{ is a counting number less than } 6\}$ *(Set-builder notation)*

$$D = \{1, 2, 3, 4, 5\}$$

$E = \{x \mid x \text{ is an even counting number less than } 10\}$ *(Set-builder notation)*

$$E = \{2, 4, 6, 8\}$$

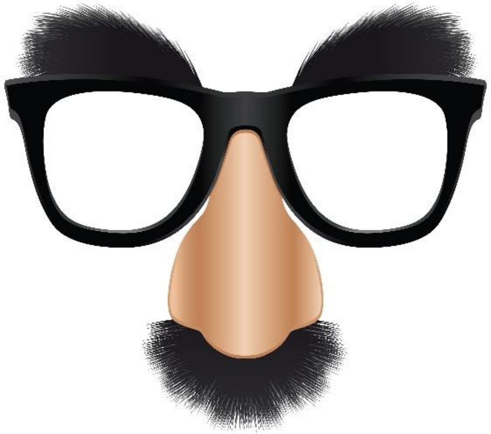
Convert $F = \{1, 2, 3, \ldots, 19\}$ into set-builder notation.

$$F = \{x \mid x \text{ is a counting number less than } 20\}$$

<u>There is a special set with no elements called the empty set.</u>

Notation: $\{\ \}$ **or** ϕ**.**

Sometimes the empty set is in disguise.



$$A = \{x \mid x \text{ is greater than 5 and less than 2}\}$$

$$A = \phi$$

There are no numbers that are greater than 5 and also less than 2!

<u>Set membership:</u>

$\in$ **means is a member or element of**

$\notin$ **means is not a member or element of**

Fill-in the blanks with either $\in$ or $\notin$.

$$3 \boxed{\in} \{3,5,7\}$$

$$6 \boxed{\notin} \{3,5,7\}$$

$$15 \boxed{\in} \{1,2,3,\ldots,20\}$$

$$3 \boxed{\notin} \{x \mid x \text{ is a counting number with } 4 \le x \le 9\}$$

$$8 \boxed{\notin} \phi$$

There's a special abbreviation for the Counting Numbers or Natural numbers:

$$\mathrm{N} = \{1,2,3,4,5,6,\ldots\}$$

Cardinal Number and Cardinality:

The cardinal number or cardinality of a set, *A*, is the number of elements in the set *A*.

Notation: $n(A)$

Determine the following cardinal numbers:

$$n(\{2,4,6,8\}) = 4$$

$$n(\{x \mid x \in N \text{ with } 4 \le x \le 12\}) = 9$$

$$n(\{x \mid x \in N \text{ with } x \le 4 \text{ and } x > 7\}) = 0$$

$$n(\{2,2,4,6,8\}) = 4$$

Duplicated items in the list of elements in a set are not counted as additional elements!

Equivalent Sets:

Sets are equivalent if they have the same cardinality*(number of elements)*.

Determine if the following pairs of sets are equivalent sets:

$$\{1,2,3,4,5\} \text{ and } \{a,b,c,d,e\}$$

equivalent

$$\{x \mid x \in N \text{ with } 3 \le x \le 8\} \text{ and } \{1,2,3,4,5\}$$

not equivalent

$$\{x \mid x \in N \text{ with } x \le 2 \text{ and } x \ge 11\} \text{ and } \phi$$

equivalent

$$\{1,2,3,4\} \text{ and } \{1,2,3,4\}$$

equivalent

Equality of Sets:

Sets are equal if they have the same elements.

Notation: =

Determine if the following pairs of sets are equal:

$$\{1,2,3,4,5\} \text{ and } \{a,b,c,d,e\}$$

not equal

$$\{x \mid x \in N \text{ with } 3 \le x \le 8\} \text{ and } \{3,4,5,6,7,8\}$$

equal

$$\{x \mid x \in N \text{ with } x \le 2 \text{ and } x \ge 11\} \text{ and } \phi$$

equal

$$\{1,2,3,4,5,6,7,8\} \text{ and } \{1,2,3,\ldots,8\}$$

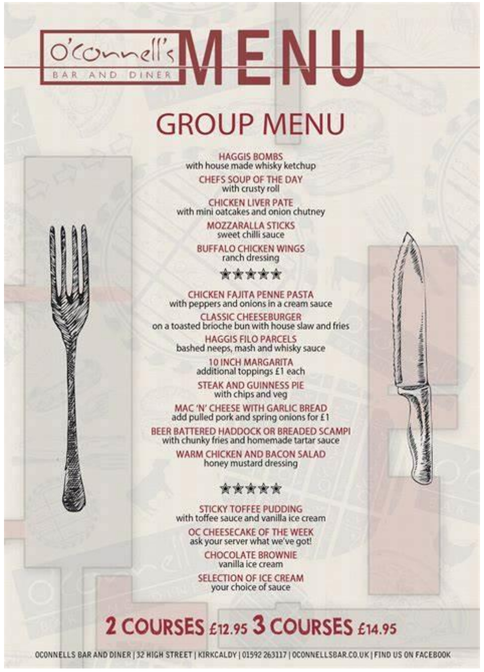
equal

Subsets:

A set *A* is a subset of the set *B* if each element of *A* is also an element of *B*.

Notation: $A \subseteq B$ *{Think of B as a menu and a subset A as an order from the menu.}*

Fill-in the blanks with either $\subseteq$ or $\not\subseteq$.



$$\{3,7\} \boxed{\subseteq} \{3,5,7\}$$

$$\{3,6\} \boxed{\not\subseteq} \{3,5,7\}$$

$$\{4,5,6,\ldots,13\} \boxed{\subseteq} \{1,2,3,\ldots,20\}$$

$$\{1,2,3,4,5\} \boxed{\not\subseteq} \phi$$

$$\phi \boxed{\subseteq} \{a,b,c\}$$

$$\{1,2,3\} \boxed{\subseteq} \{1,2,3\}$$

$$\phi \boxed{\subseteq} \phi$$

If you're not hungry and you're not thirsty, then you don't order anything from the menu-i.e. the empty set is considered to be a subset of every set.

Proper Subsets:

A set *A* is a proper subset of the set *B* if each element of *A* is also an element of *B*, but $A \neq B$.

Notation: $A \subset B$

Fill-in the blanks with either $\subset$ or $\not\subset$.

$$\{3,7\} \boxed{\subset} \{3,5,7\}$$

$$\{3,5,7\} \boxed{\not\subset} \{3,5,7\}$$

$$\{4,5,6,\ldots,13\} \boxed{\subset} \{1,2,3,\ldots,20\}$$

$$\{1,2,3,4,5,6,7\} \boxed{\not\subset} \{1,2,3,\ldots,7\}$$

$$\phi \boxed{\subset} \{a,b,c\}$$

$$\phi \boxed{\not\subset} \phi$$

The empty set is a proper subset of every set, except for itself!

How many subsets or proper subsets does a set have?

Set A	$n(A)$	Subsets of A	Proper Subsets of A	# of subsets	# of proper subsets
ϕ	0	ϕ	none	1	0
$\{a\}$	1	$\phi, \{a\}$	ϕ	2	1
$\{a,b\}$	2	$\phi, \{a\}, \{b\}, \{a,b\}$	$\phi, \{a\}, \{b\}$	4	3
$\{a,b,c\}$	3	$\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}$	$\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}$	8	7

Notice that each time a new element is added, you still have the previous subsets along with the new subsets generated by adding the new element to each of the previous subsets. This is why the number of subsets doubles.

Use inductive reasoning to complete the following:

If a set has *n* elements, then it has _____2^n_______ subsets.

If a set has *n* elements, then it has _____$2^n - 1$_______ proper subsets.

How many subsets are there of the set $\{a, b, c, d, e\}$?

$$2^5 = \boxed{32}$$

How many proper subsets are there of the set $\{a, b, c, d, e\}$?

$$2^5 - 1 = \boxed{31}$$

A pizza can be ordered with some, none, or all of the following toppings:

$\{pepperoni, sausage, mushroom, onion, peppers, black\ olives, green\ olives, hamburger\}$.

How many different pizzas are possible?

$$2^8 = \boxed{256}$$

In this example, what would correspond to the empty set?

a cheese pizza!