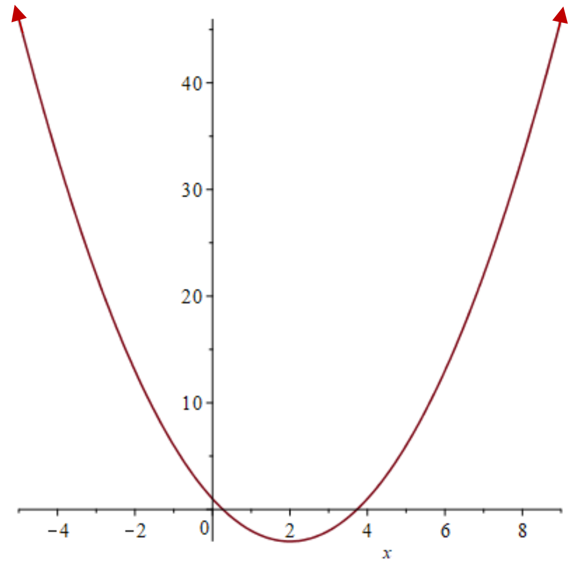
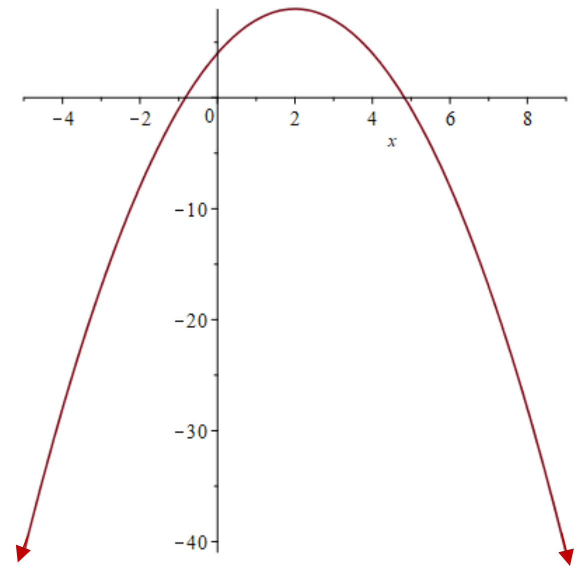
Quadratic Functions:





General Form:

$$f(x) = ax^2 + bx + c\ ;\ a \neq 0$$

Standard Form:

$$f(x) = a(x-h)^2 + k\ ;\ a \neq 0$$

Intercept Form:

$$f(x) = a(x-x_1)(x-x_2)\ ;\ a \neq 0, x_1 \text{ and } x_2 \text{ are real numbers}$$

General Form:

$f(x) = ax^2 + bx + c\ ;\ a \neq 0$

If $a > 0$, the parabola opens up; if $a < 0$, the parabola opens down. The *y*-intercept is c.

Standard Form:

$f(x) = a(x-h)^2 + k\ ;\ a \neq 0$

The vertex is at (h,k) and it's a minimum vertex if $a > 0$ and a maximum vertex if $a < 0$.

Intercept Form:

$f(x) = a(x-x_1)(x-x_2)\ ;\ a \neq 0, x_1$ and x_2 are real numbers

x_1 and x_2 **are the *x*-intercepts, and the *x*-coordinate of the vertex is $\frac{x_1 + x_2}{2}$. The vertex is a minimum if $a > 0$ and a maximum if $a < 0$.**

Graph the following quadratic functions. Indicate the vertex and all the intercepts.

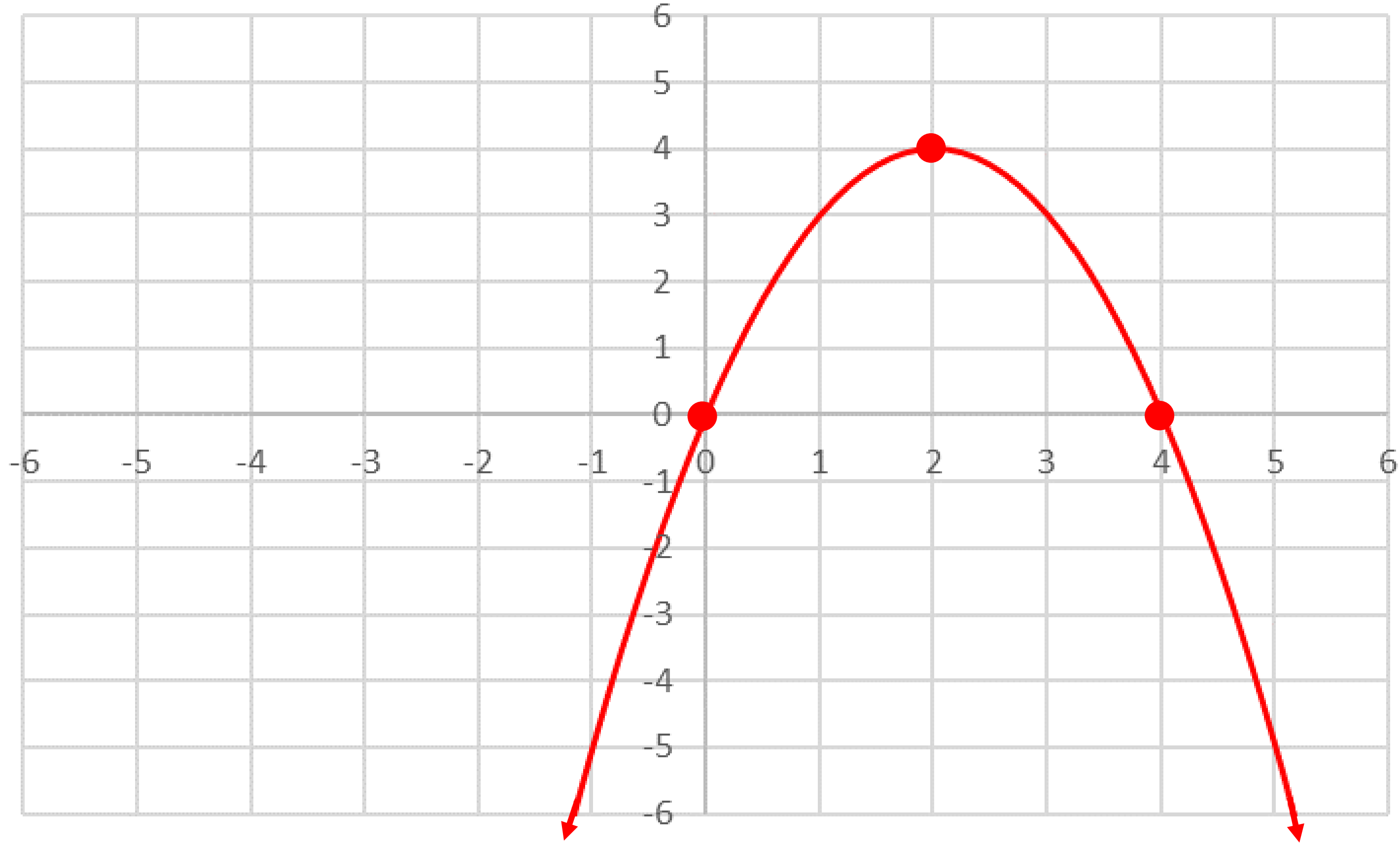
1. $f(x) = -x^2 + 4x$

$= -x(x-4)$

{Convert to intercept form by factoring out $-x$***.}***

From symmetry, the x-coordinate of the vertex is the average of the two x-intercepts, $\frac{0+4}{2} = 2$, $f(2) = 4$.

$f(0) = 0$, so the y-intercept is 0.



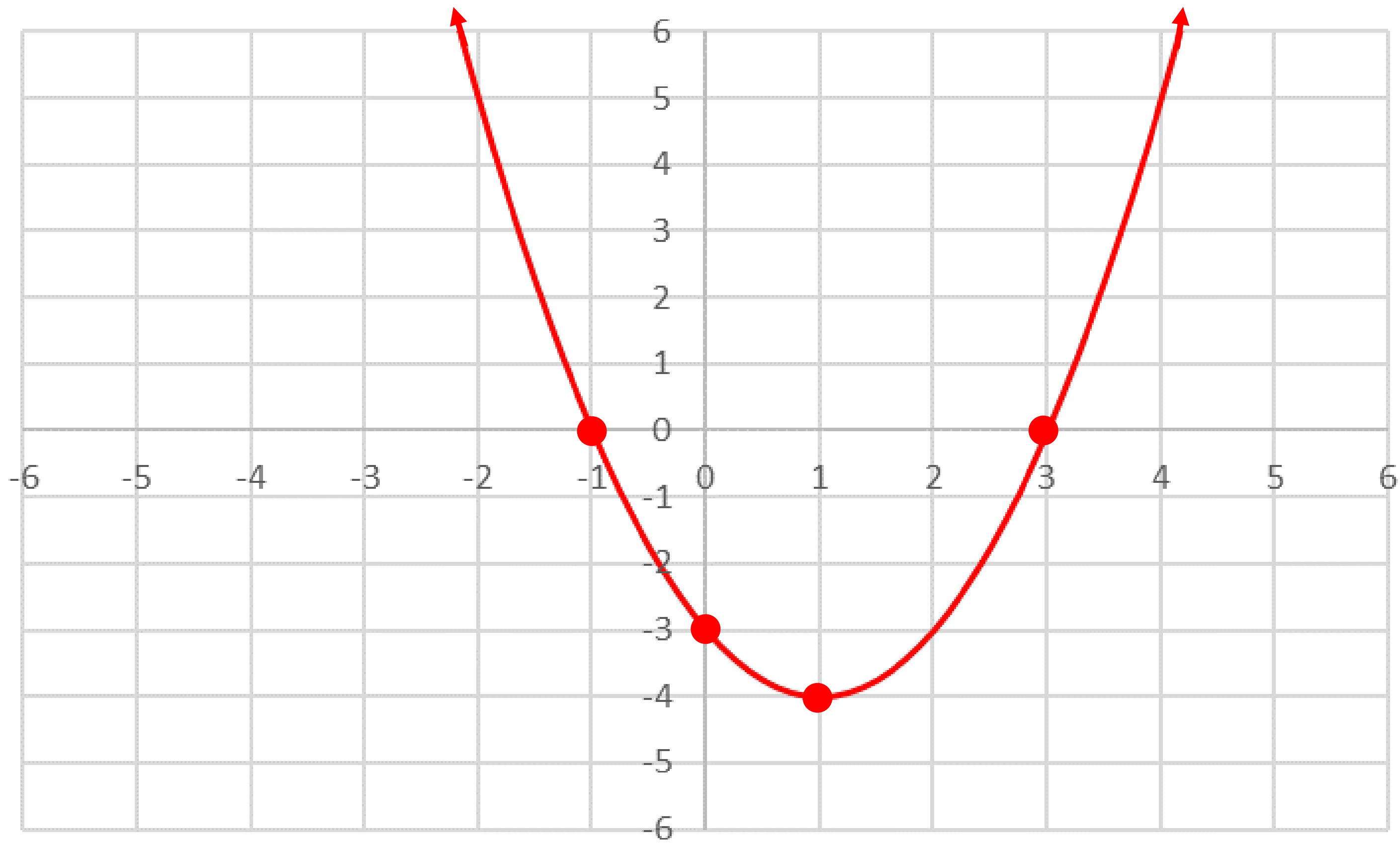
2. $f(x) = x^2 - 2x - 3$

$= (x-3)(x+1)$

{Convert to intercept form by factoring the trinomial.}

From symmetry, the x-coordinate of the vertex is the average of the two x-intercepts, $\frac{-1+3}{2} = 1$, $f(1) = -4$.

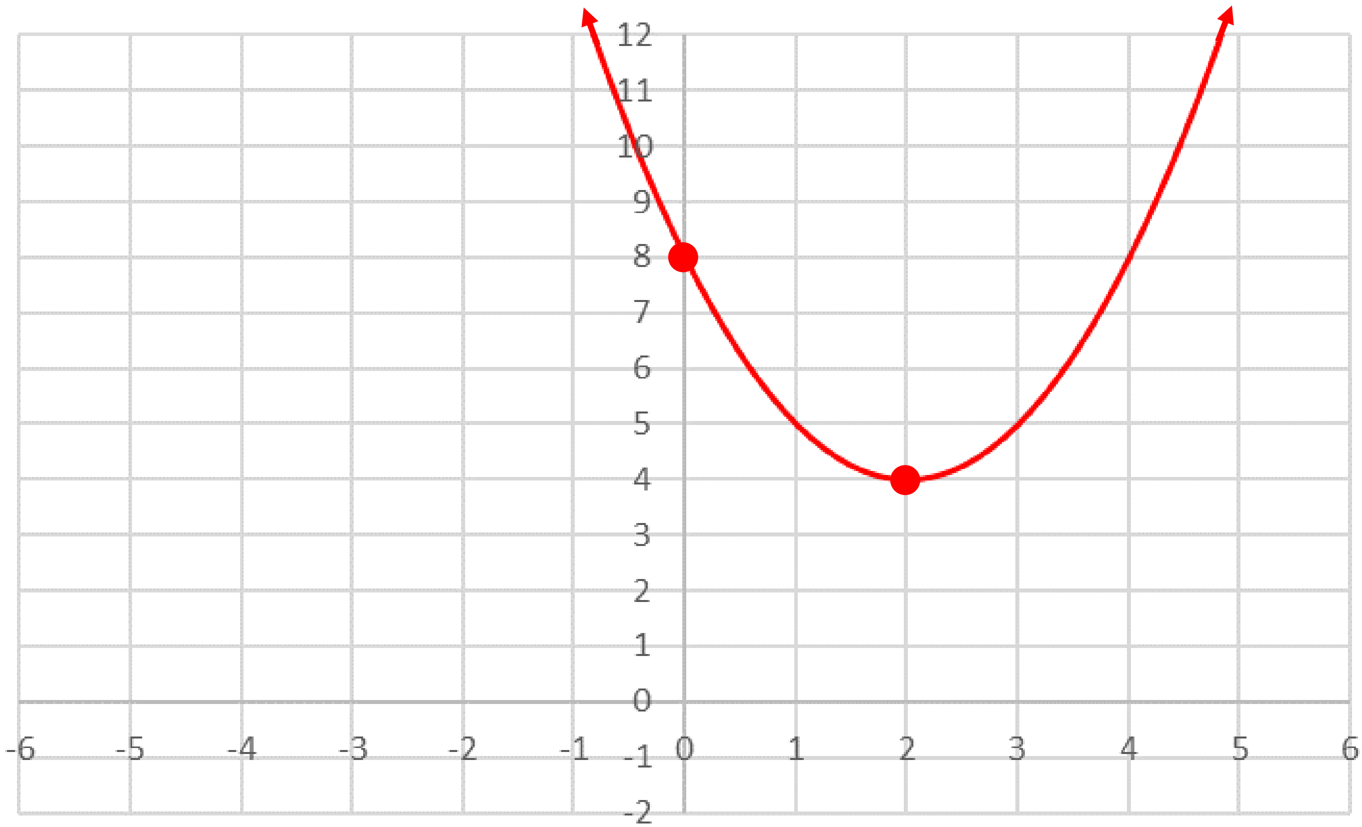
$f(0) = -3$, so the y-intercept is -3.



3. $f(x)=(x-2)^2+4$ ***{It's in standard form.}***

Vertex is at $(2,4)$.

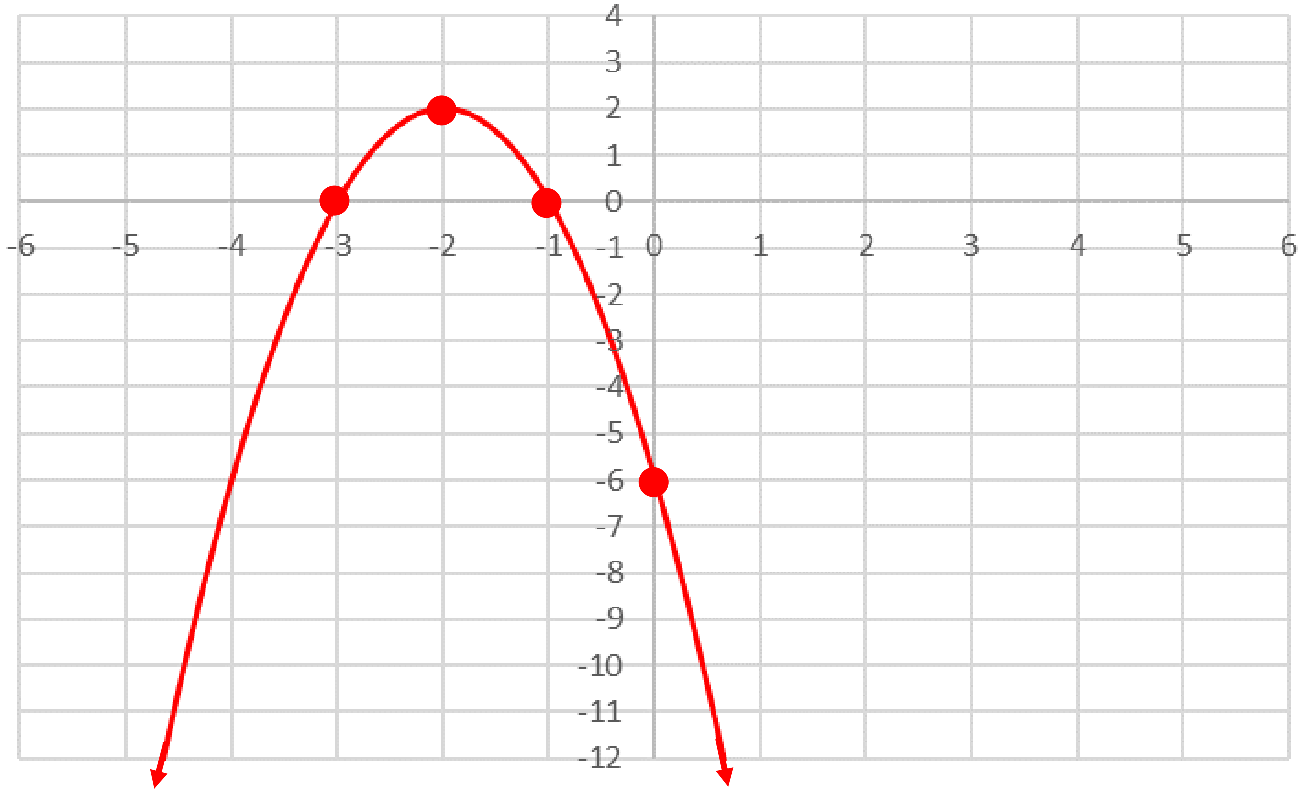
$f(0)=8$, so the y-intercept is 8. There are no x-intercepts.



4. $f(x) = -2(x+2)^2 + 2$ ***{It's in standard form.}***

Vertex is at $(-2,2)$.

$f(0) = -6$, so the *y*-intercept is -6.



$-2(x+2)^2 + 2 = 0 \Rightarrow 2(x+2)^2 = 2 \Rightarrow (x+2)^2 = 1 \Rightarrow x+2 = \pm 1 \Rightarrow x = -2 \pm 1$

so the $x-$intercepts are $-3, -1$.

5. $f(x) = 4x^2 - 2x + 1$ ***{Convert to standard form by completing the square.}***

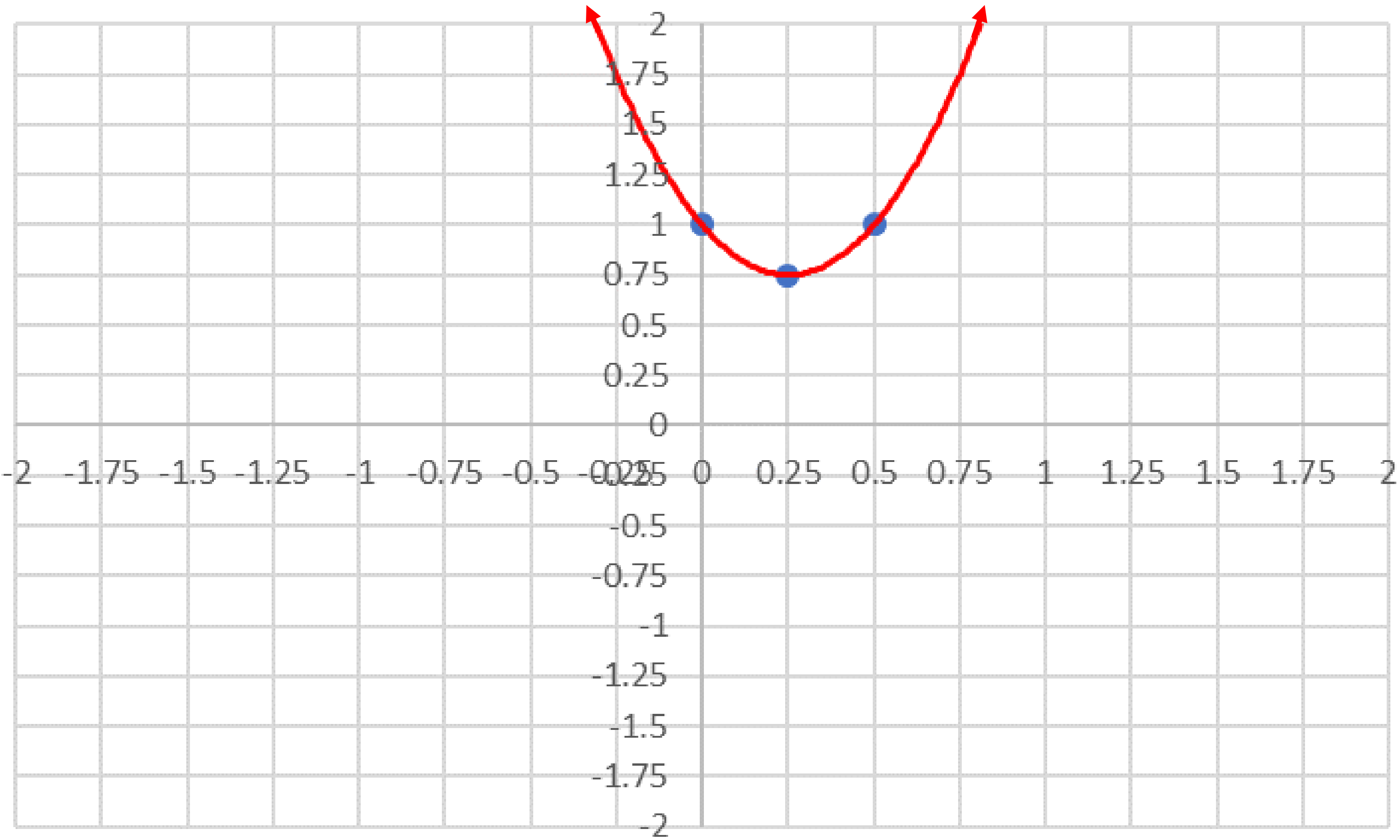
$$= 4\left(x^2 - \tfrac{1}{2}x\right) + 1$$

$$= 4\left(x^2 - \tfrac{1}{2}x + \tfrac{1}{16}\right) + 1 - \tfrac{1}{4}$$

$$= 4\left(x - \tfrac{1}{4}\right)^2 + \tfrac{3}{4}$$

Vertex is at $\left(\tfrac{1}{4}, \tfrac{3}{4}\right)$.

$f(0) = 1$, so the y-intercept is 1. There are no x-intercepts.



6. $f(x) = -2x^2 + 2x - 3$ ***{Convert to standard form by completing the square.}***

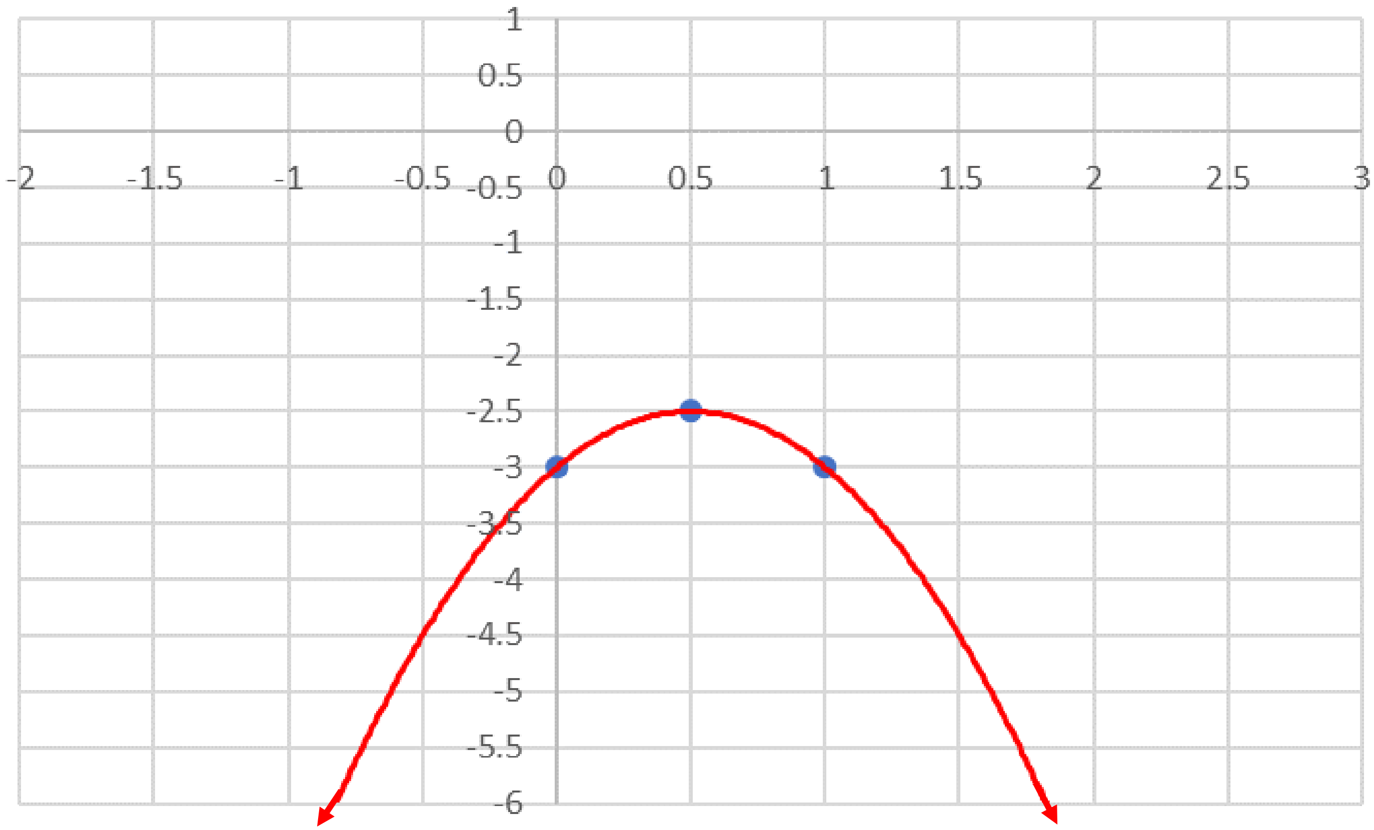
$= -2(x^2 - x) - 3$

$= -2(x^2 - x + \frac{1}{4}) - 3 + \frac{1}{2}$

Vertex is at $(\frac{1}{2}, -\frac{5}{2})$.

$= -2(x - \frac{1}{2})^2 - \frac{5}{2}$

$f(0) = -3$, so the y-intercept is -3. There are no x-intercepts.



Finding Formulas for Quadratic Functions:

1. The vertex of the parabola is $(1,2)$, and it passes through the point $(3,0)$.

$$\{f(x) = a(x-h)^2 + k\}$$

$$f(x) = a(x-1)^2 + 2$$

$$\Rightarrow 0 = a(3-1)^2 + 2$$

$$\Rightarrow 0 = 4a + 2$$

$$\Rightarrow a = -\frac{1}{2}$$

$$\Rightarrow \boxed{f(x) = -\frac{1}{2}(x-1)^2 + 2}$$

2. The *x*-intercepts are 5 and -3, and the graph passes through the point $(0,-4)$**.**

$$\left\{ f(x) = a(x - x_1)(x - x_2) \right\}$$

$$f(x) = a(x-5)(x+3)$$

$$\Rightarrow -4 = a(0-5)(0+3)$$

$$\Rightarrow -4 = -15a$$

$$\Rightarrow a = \frac{4}{15}$$

$$\Rightarrow \boxed{f(x) = \frac{4}{15}(x-5)(x+3)}$$

3. The graph passes through the points $(0,1)$, $(1,2)$, and $(-1,4)$.

$$\left\{ f(x) = ax^2 + bx + c \right\}$$

$$f(x) = ax^2 + bx + c$$

$$\Rightarrow \underline{1 = c}, 2 = a + b + c, 4 = a - b + c$$

$$\Rightarrow 1 = a + b, 3 = a - b$$

$$\Rightarrow 4 = 2a$$

$$\Rightarrow \underline{a = 2}, \underline{b = -1}$$

$$\Rightarrow \boxed{f(x) = 2x^2 - x + 1}$$

4. The graph passes through the points $(1,2)$ and $(5,2)$, and the minimum value of the function is -4.

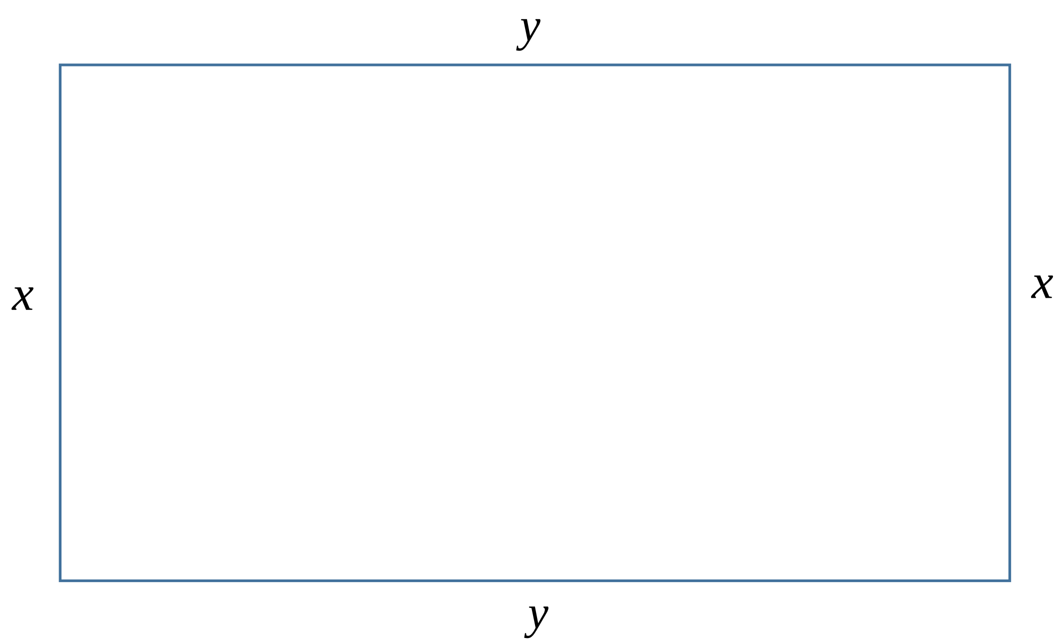
$$\left\{f(x)=a(x-h)^2+k\right\}$$

Since the two points on the graph have the same *y*-coordinate, by symmetry the average of their *x*-coordinates must be the *x*-coordinate of the vertex. The minimum value would have to be the *y*-coordinate of the vertex.

$$f(x)=a(x-3)^2-4$$
$$\Rightarrow 2=a(1-3)^2-4$$
$$\Rightarrow 2=4a-4$$
$$\Rightarrow a=\frac{3}{2}$$
$$\Rightarrow \boxed{f(x)=\frac{3}{2}(x-3)^2-4}$$

Word Problems:

1. Joe has 3,000 feet of fence available to enclose a rectangular field.



$$2x + 2y = 3000$$

$$x + y = 1500$$

$$y = 1500 - x$$

a) Express the enclosed area, *A*, as a function of *x*.

$$A = xy$$

$$\Rightarrow A = x(1500 - x)$$

$$\Rightarrow \boxed{A(x) = x(1500 - x)}$$

b) Determine the domain of the function, $A(x)$**.**

Both of the dimensions of the rectangle must be positive, so

$$x > 0, 1500 - x > 0$$

$$\Rightarrow \boxed{0 < x < 1500 \text{ or } (0,1500)}$$

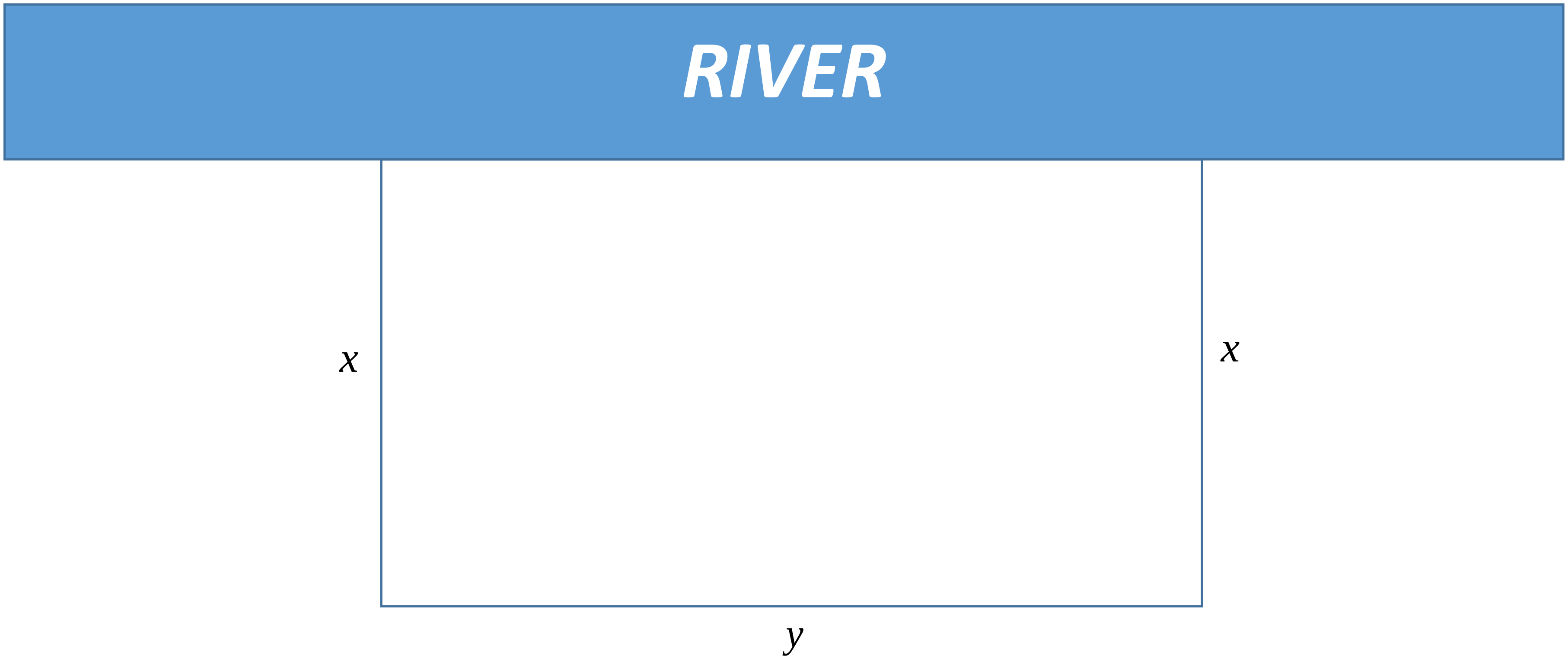
c) For what value of *x* is the enclosed area the largest?

The graph of the quadratic function that represents the enclosed area takes on the value zero at $x = 0, 1500$, so the value of x that corresponds to the maximum enclosed area is the average of these two values. So $x = \boxed{750\ ft}$.

d) What is the value of the largest enclosed area?

$$A(750) = 750 \cdot 750 = \boxed{562{,}500\ ft^2}$$

2. A farmer with 2,000 yards of fence wants to enclose a rectangular field that borders on a straight river-so he'll only need fence on three sides of the field.



$$2x + y = 2000$$

$$y = 2000 - 2x$$

a) Express the enclosed area, *A*, as a function of *x*.

$$A = xy$$

$$\Rightarrow A = x(2000 - 2x)$$

$$\Rightarrow A = 2x(1000 - x)$$

$$\Rightarrow \boxed{A(x) = 2x(1000 - x)}$$

b) Determine the domain of the function, $A(x)$**.**

Both of the dimensions of the rectangle must be positive, so

$$x > 0, 2000 - 2x > 0$$

$$\Rightarrow \boxed{0 < x < 1000 \quad \text{or} \quad (0,1000)}$$

c) For what value of *x* is the enclosed area the largest?

The graph of the quadratic function that represents the enclosed area takes on the value zero at $x = 0, 1000$, so the value of x that corresponds to the maximum enclosed area is the average of these two values. So $x = \boxed{500\,yds}$.

d) What is the value of the largest enclosed area?

$$A(500) = 1000 \cdot 500 = \boxed{500,000\,yd^2}$$