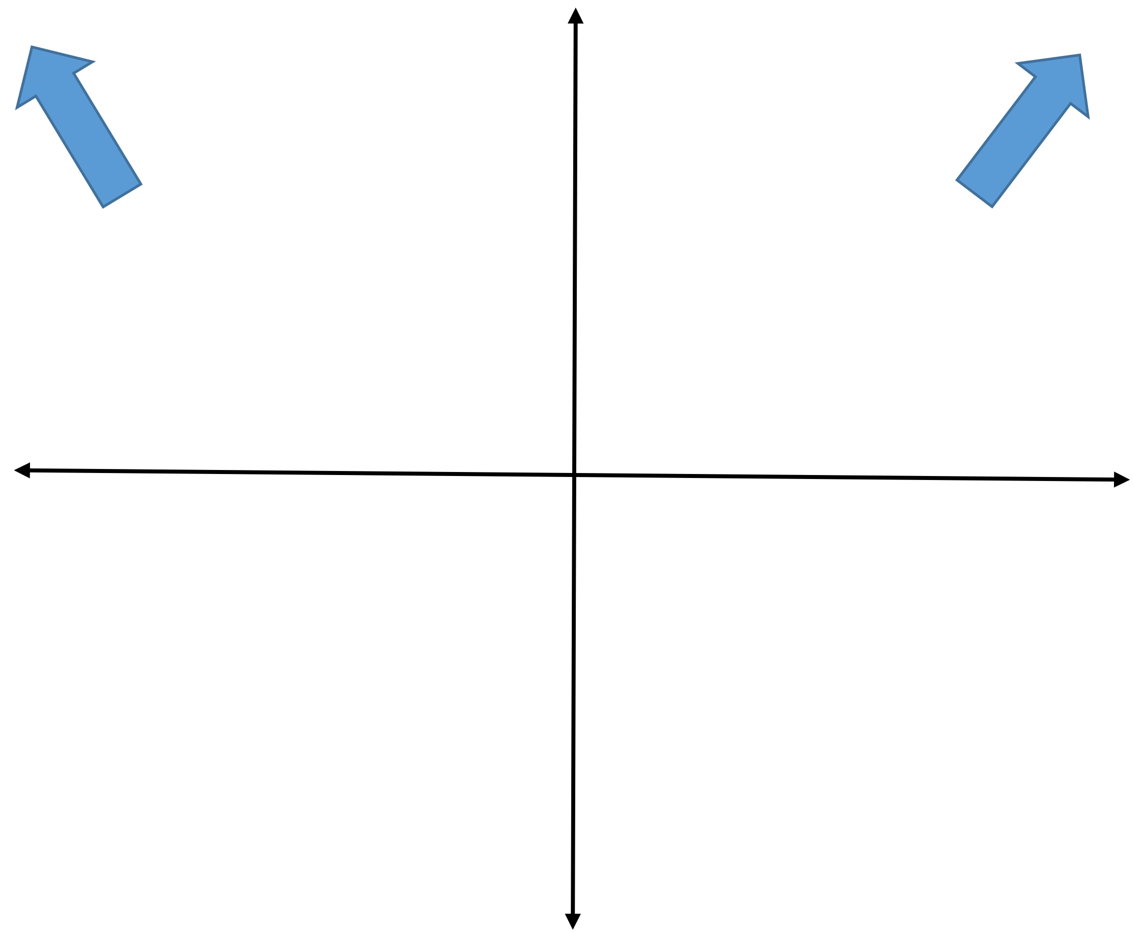
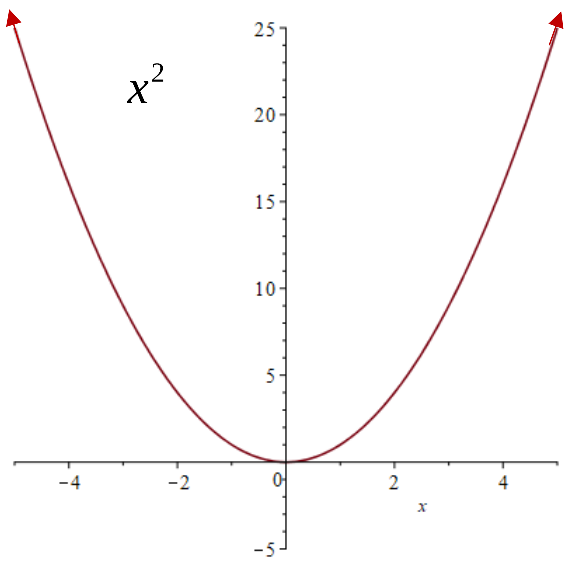
Graphing Polynomial Functions:

The Leading Coefficient Test and End Behavior:

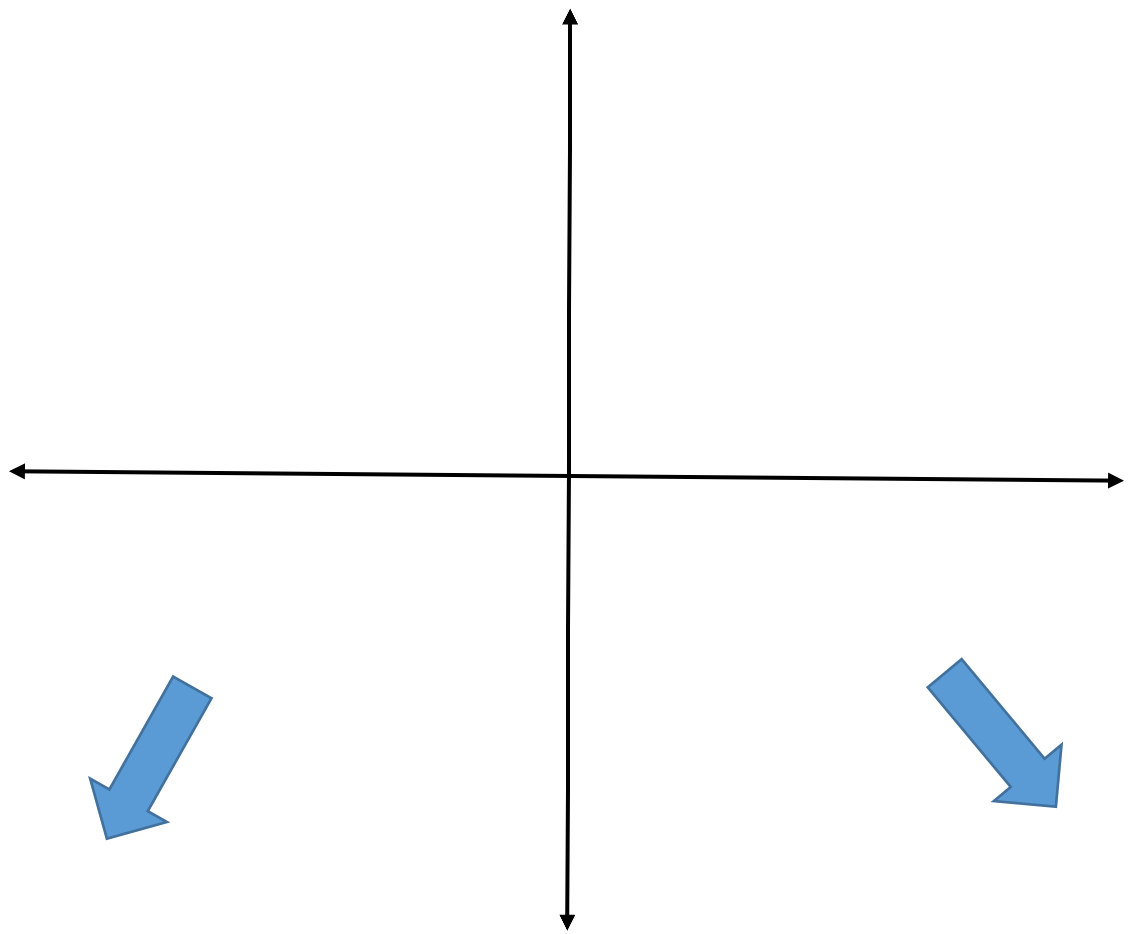
For an n^{th} – degree polynomial function $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ **with** $a_n \neq 0$**,**

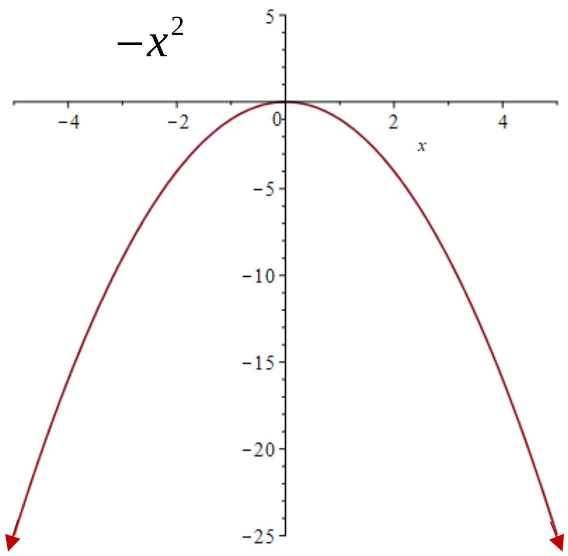
If *n* is even and $a_n > 0$**, then**



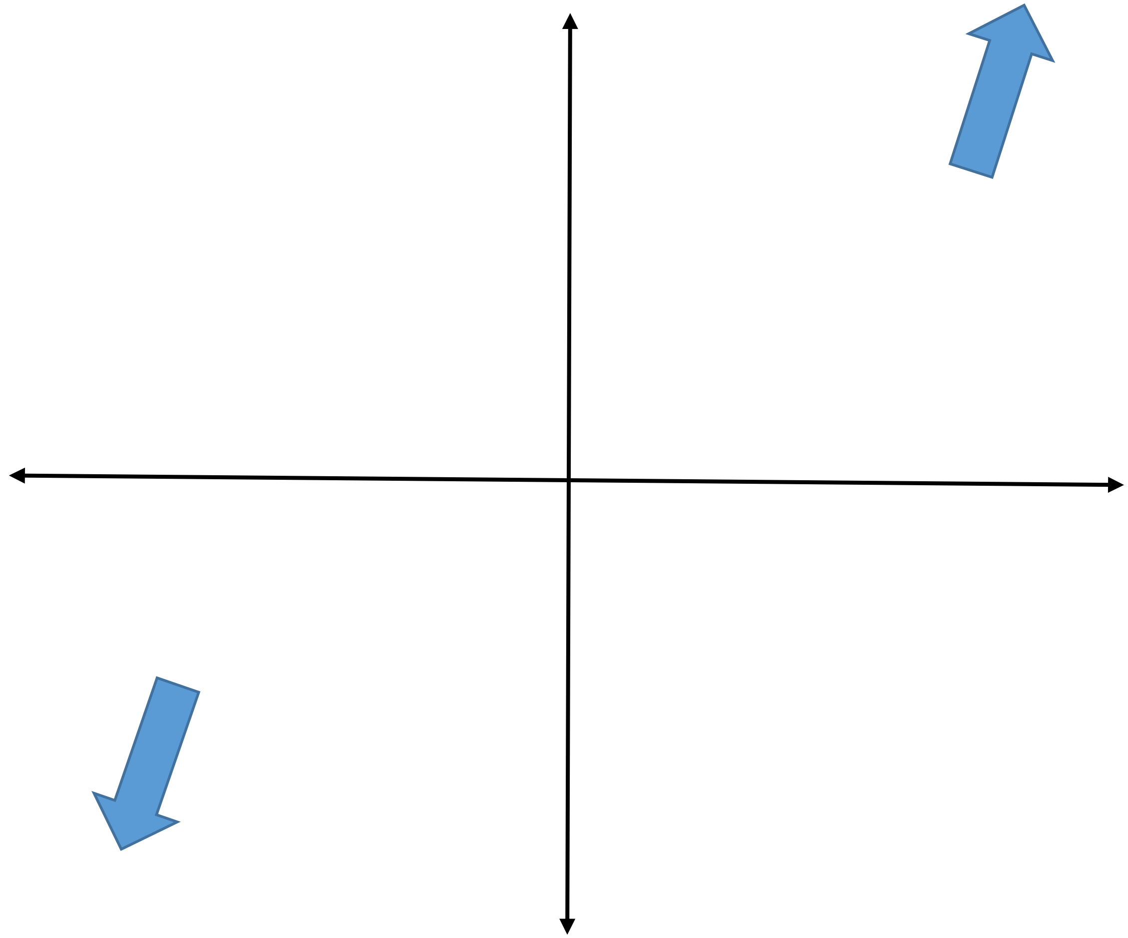


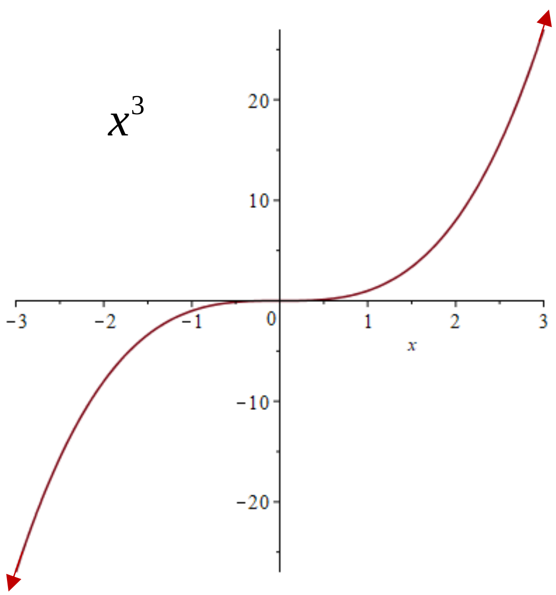
If *n* is even and $a_n < 0$, then



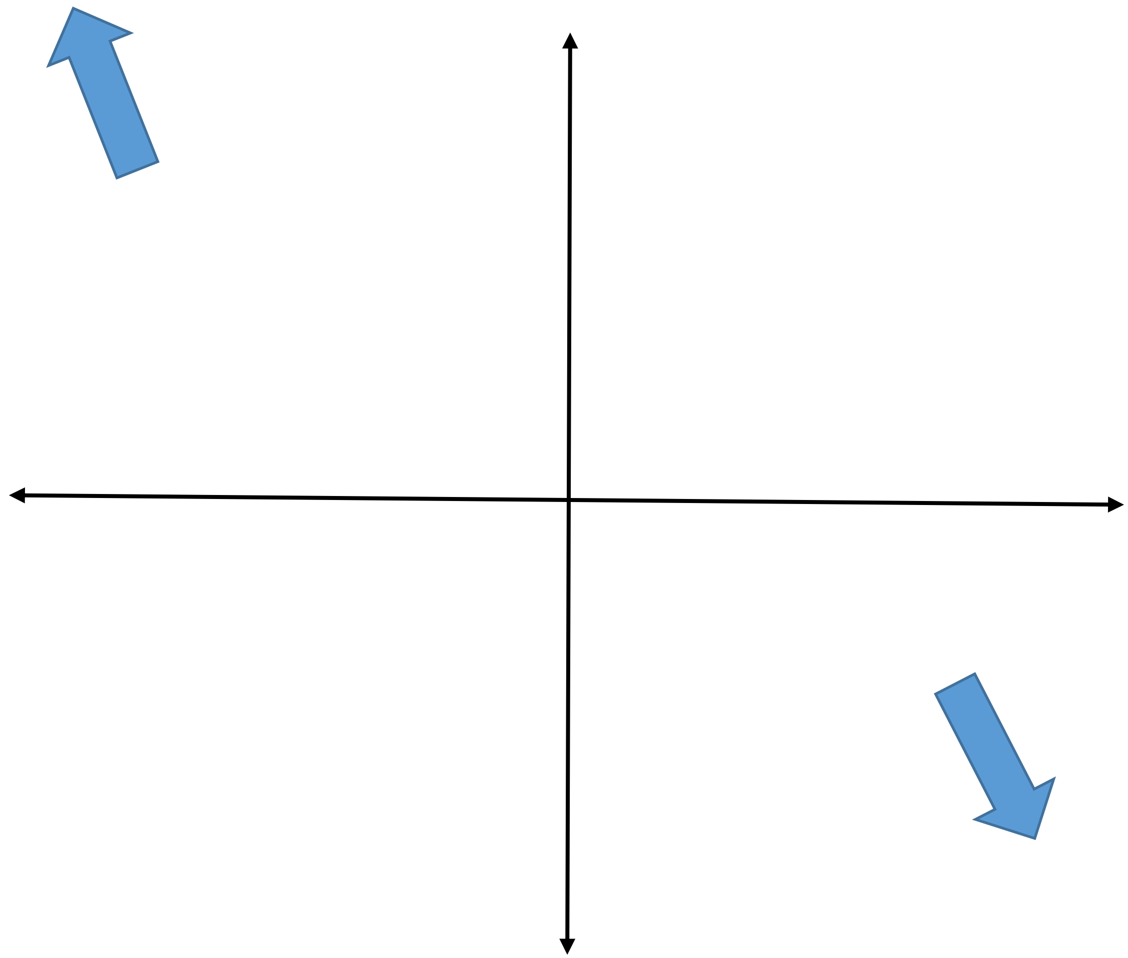


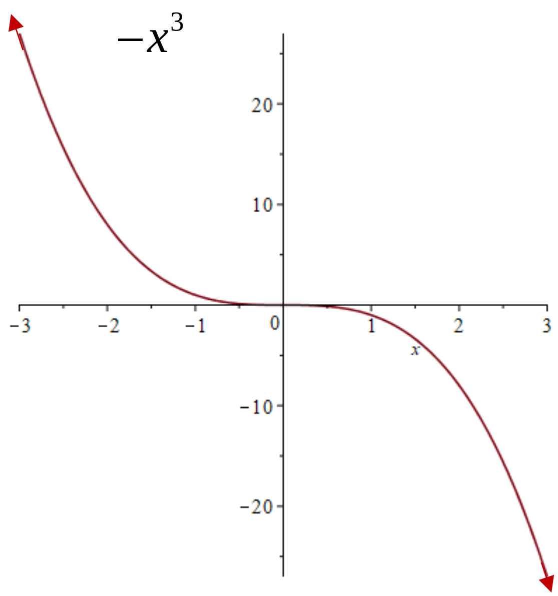
If *n* is odd and $a_n > 0$, then





If *n* is odd and $a_n < 0$, then





Determine the end behavior of the following polynomial functions.

1. $f\left(x\right)=4x-x^3$ **Left:** up **Right:** down

2. $f\left(x\right)=2x^4+12x-4$ **Left:** up **Right:** up

3. $f\left(x\right)=x^3+2x^2-8x$ **Left:** down **Right:** up

4. $f\left(x\right)=4x-x^6$ **Left:** down **Right:** down

5. $f(x) = x^2(x-3)$ **Left:** down **Right:** up

6. $f(x) = -2(x+2)(x-2)^3$ **Left:** down **Right:** down

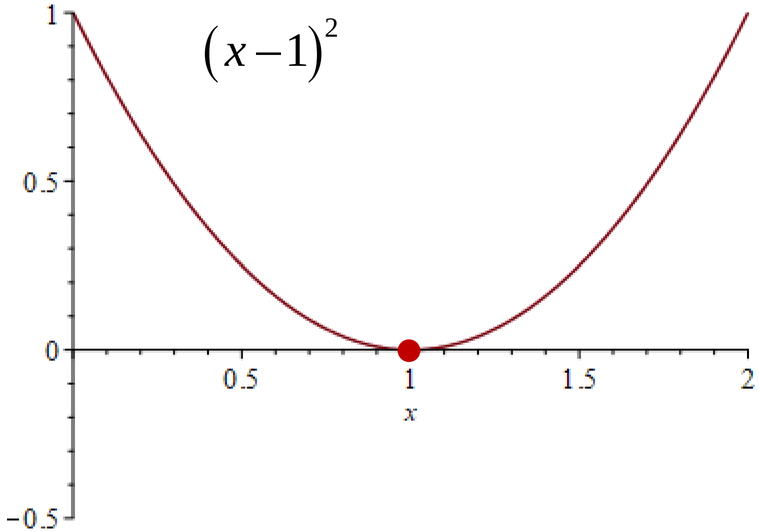
7. $f(x) = (x+1)^2(x-2)^2$ **Left:** up **Right:** up

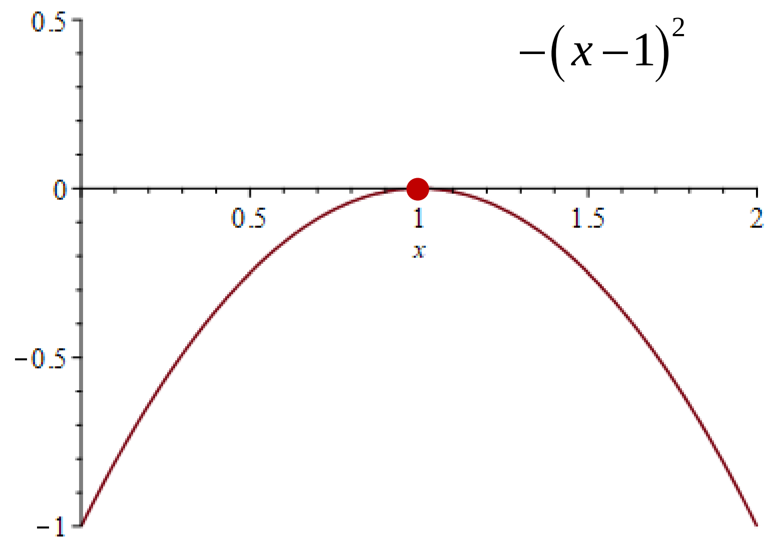
8. $f(x) = -2(x+2)^2(x-2)^3$ **Left:** up **Right:** down

Behavior at the x-intercepts:

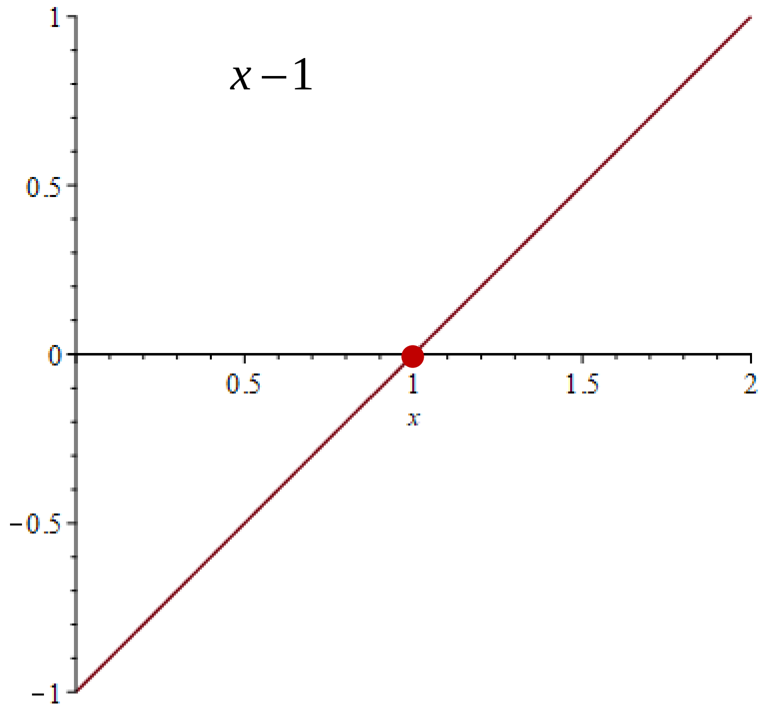
If $(x-c)^k$ is the highest power of $(x-c)$ that is a factor of $f(x)$, with *c* a real number, then

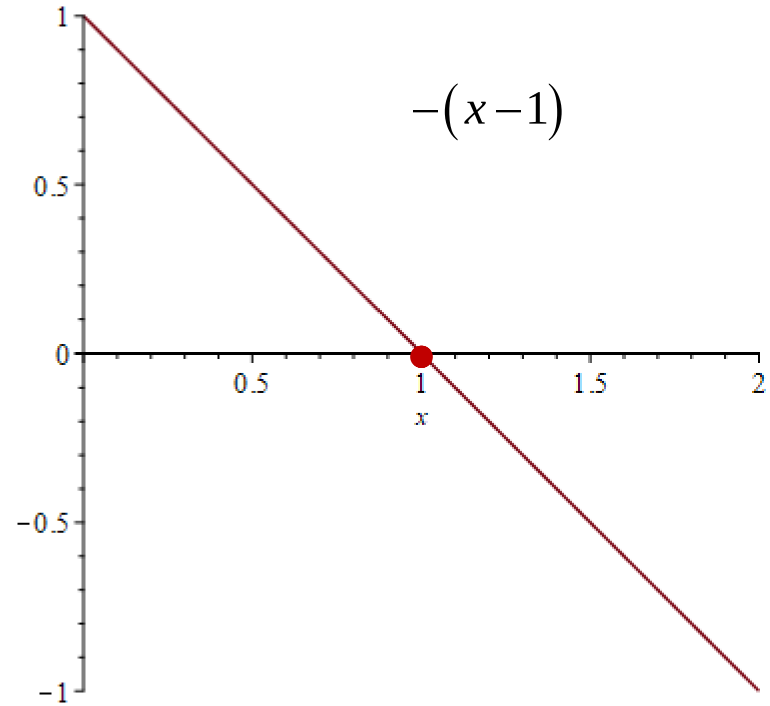
If *k* is even, then the graph touches the *x*-axis at *c* but doesn't cross the axis.



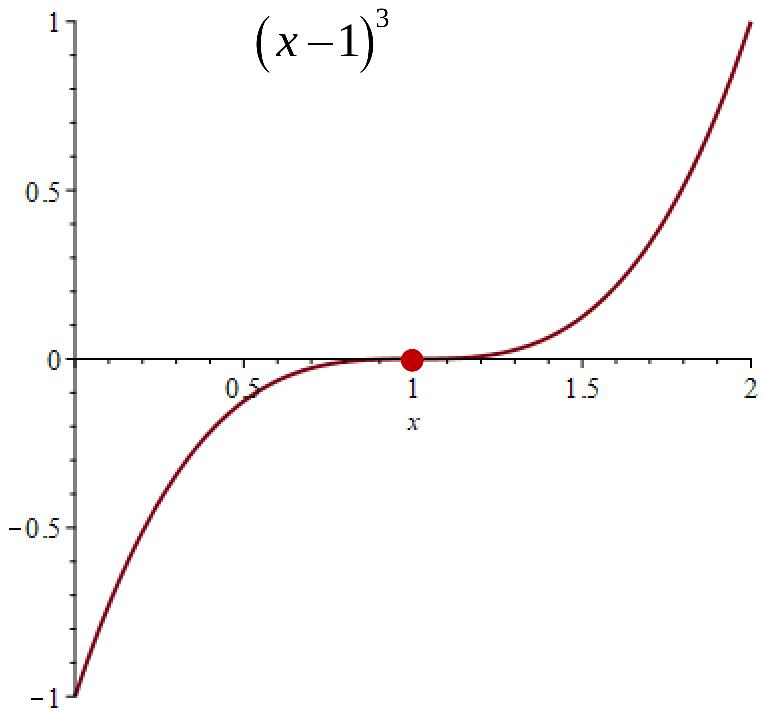


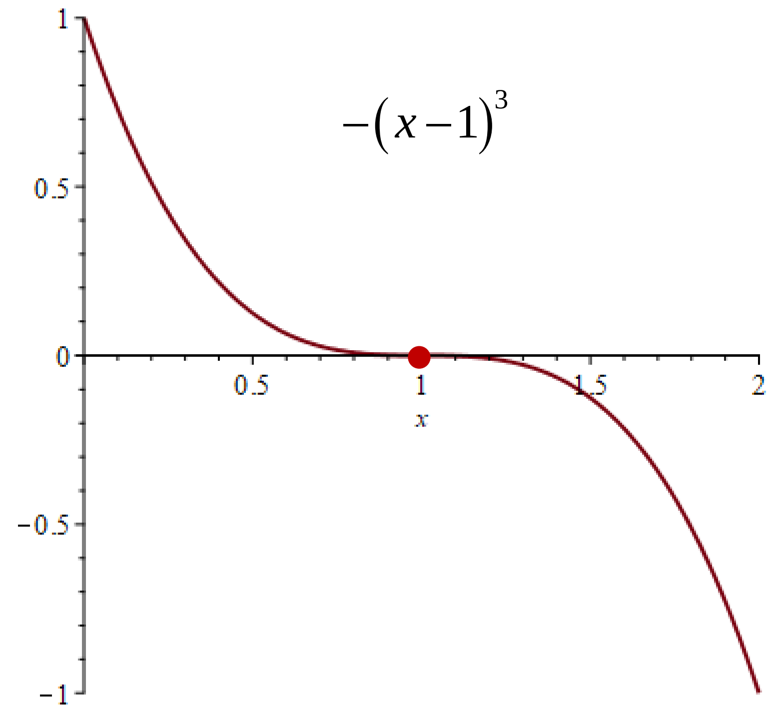
If *k* is 1, then the graph crosses the *x*-axis at *c* with a non-zero angle.





If *k* is odd and greater than 1, then the graph crosses the *x*-axis at *c* with a zero angle(flat).





Steps for sketching graphs of polynomial functions:

1. Determine the end behavior, and indicate it on the graph with arrows.

2. Find all the real zeros(*x*-intercepts) of $f(x)$, and indicate them on the graph with points.

3. Find the *y*-intercept(by setting *x* to zero), and indicate it on the graph with a point.

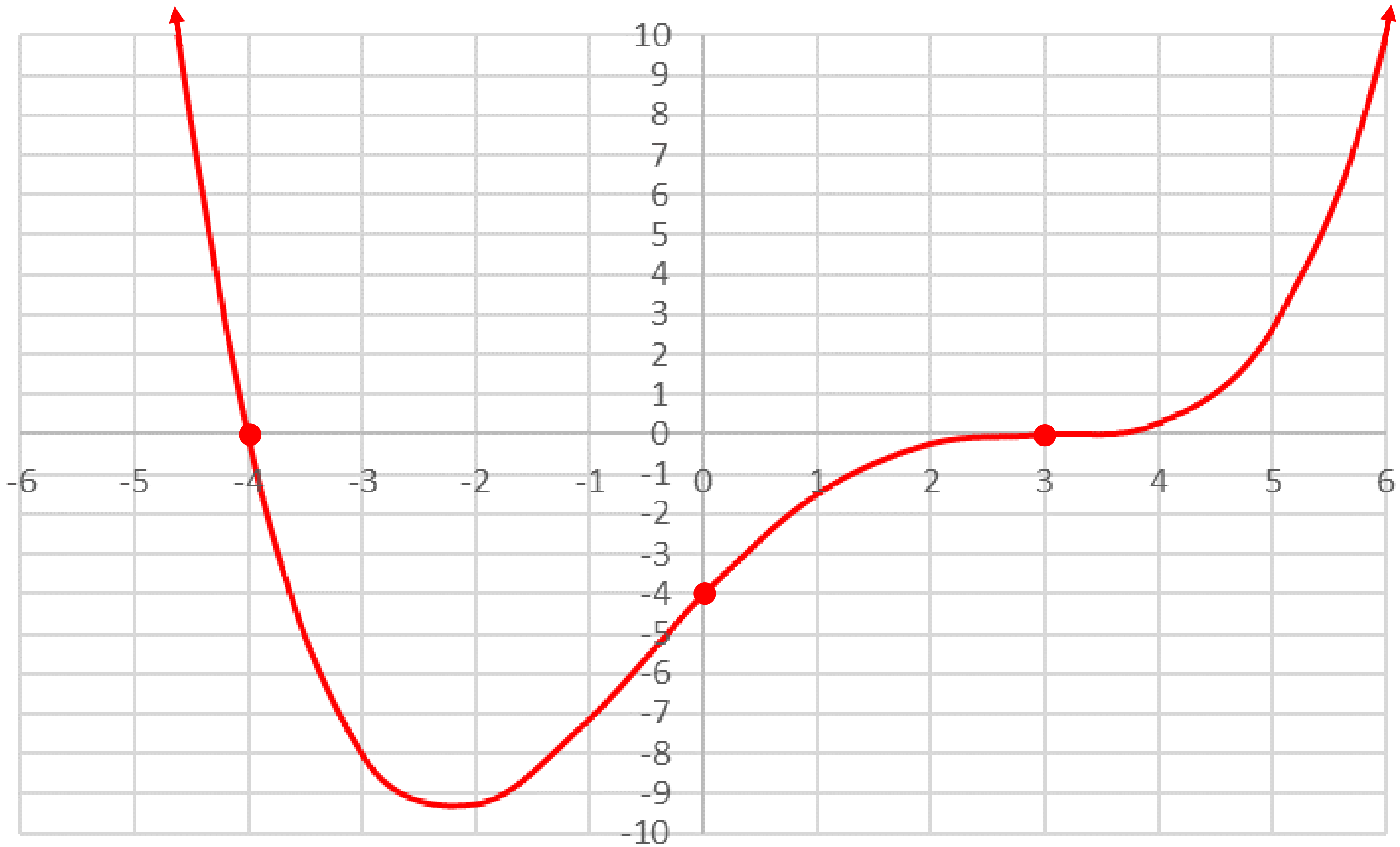
4. Use the end behavior and *x*-intercept behavior to connect the previous points and arrows into a reasonable graph.

The goal in sketching the graph of a polynomial function is to plot as few points as possible(the *x* and *y* intercepts), and use the end behavior and *x*-intercept behavior to capture the qualitative behavior of the graph. Don't worry about the vertical scaling, just produce a reasonably connected graph.

Sketch the graphs of the following polynomial functions.

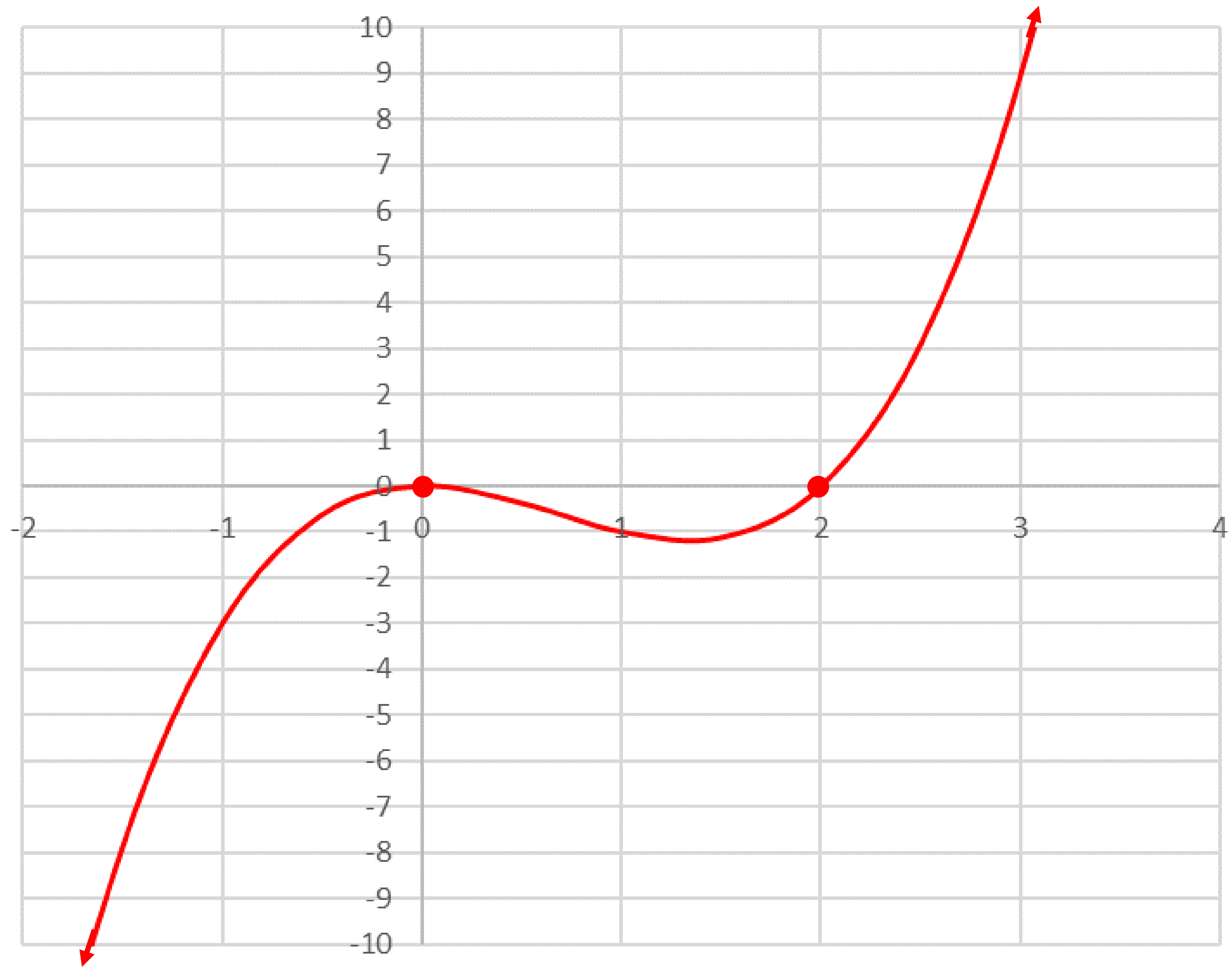
1. $f(x)=\frac{1}{27}(x+4)(x-3)^3$

$f(0)=-4$, so the y-intercept is -4.



2. $f(x) = x^2(x-2)$

$f(0) = 0$, so the y-intercept is 0.

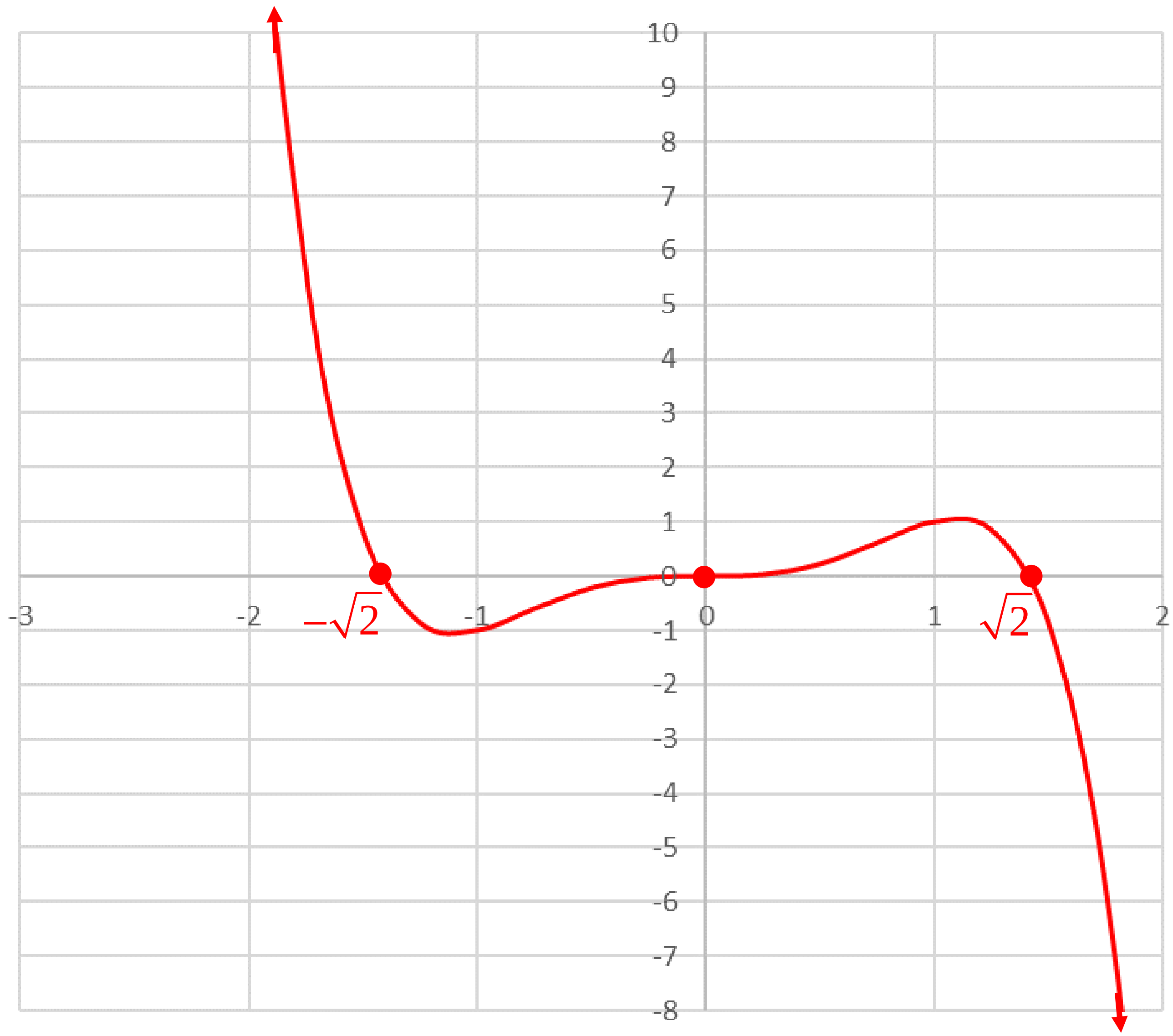


3. $f(x) = -(x+2)(x-2)^3$

$f(0) = 16$, so the y-intercept is 16.

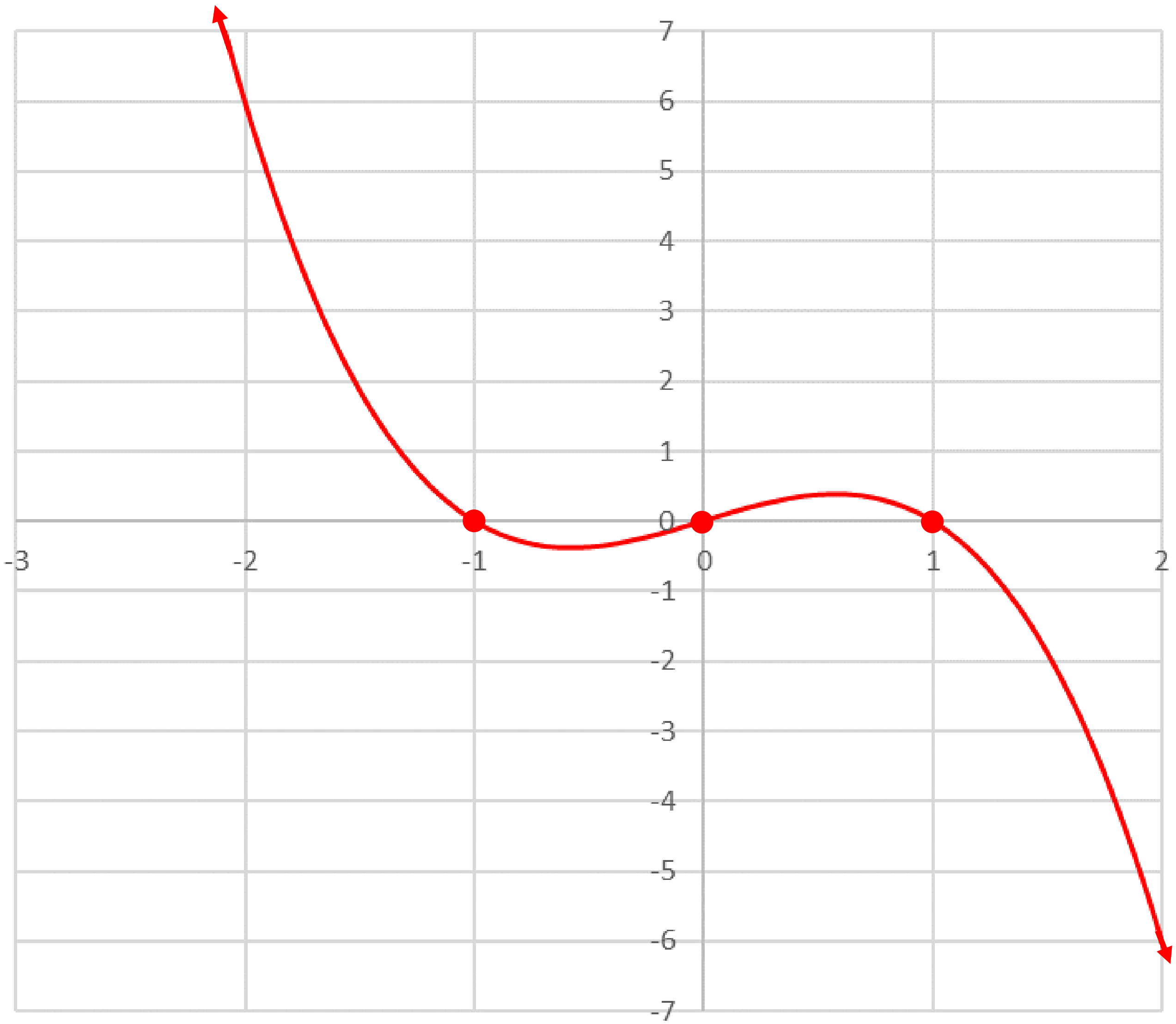
4. $f(x) = -(x^2 - 2)x^3 = -x^3(x - \sqrt{2})(x + \sqrt{2})$ $f(0) = 0$, so the y-intercept is 0.

{Factor first.}



5. $f(x) = x - x^3 = -x(x^2 - 1) = -x(x-1)(x+1)$ $f(0) = 0$, so the y-intercept is 0.

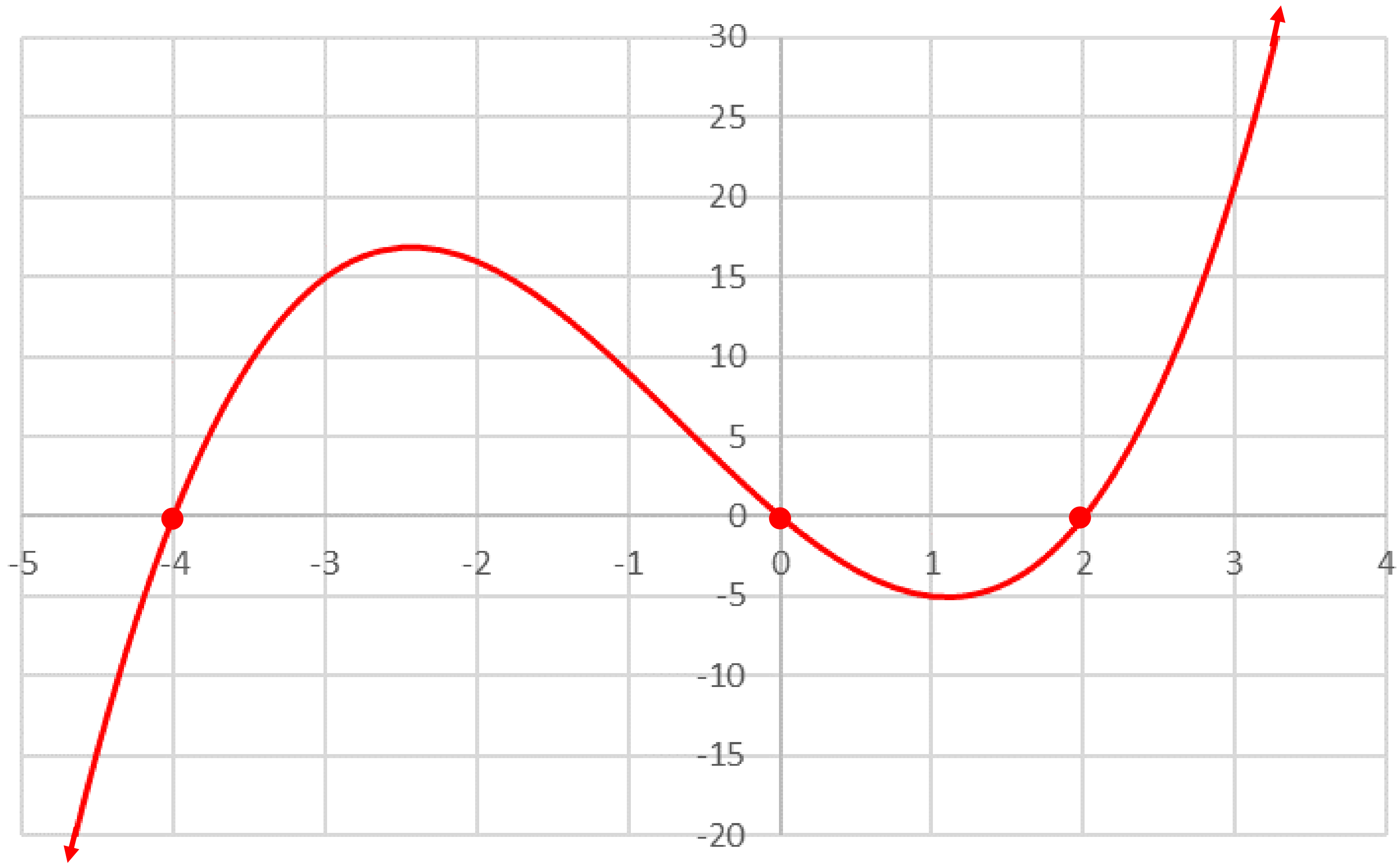
{Factor first.}



6. $f(x) = x^3 + 2x^2 - 8x = x(x^2 + 2x - 8) = x(x+4)(x-2)$

{Factor first.}

$f(0) = 0$, so the y-intercept is 0.



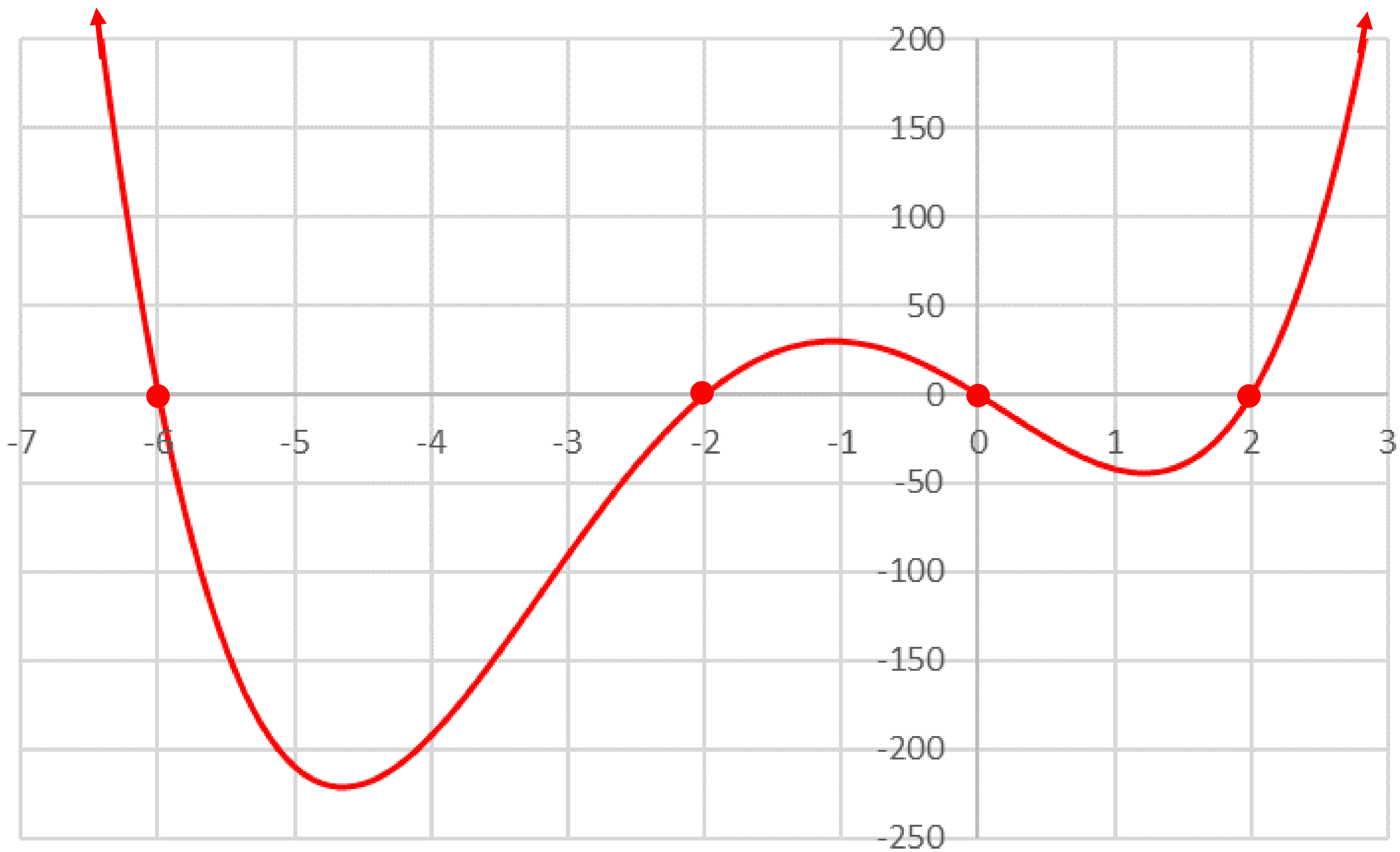
7. $f(x) = 2x^4 + 12x^3 - 8x^2 - 48x = 2x^3(x+6) - 8x(x+6)$

$= 2x(x+6)(x^2-4)$

$= 2x(x+6)(x-2)(x+2)$

$f(0) = 0$, so the *y*-intercept is 0.

{Factor first.}



8. $f(x) = x^2 - x^4 = -x^2(x^2 - 1) = -x^2(x-1)(x+1)$

{Factor first.}

$f(0) = 0$, so the *y*-intercept is 0.