Polynomial Division:

Long Division:

$$\left(6x^2+13x+9\right)\div\left(2x+1\right)$$

$$\begin{array}{r}
3x+5 \\
2x+1\overline{)\,6x^2+13x+9} \\
\underline{-\left(6x^2+3x\right)} \\
10x+9 \\
\underline{-\left(10x+5\right)} \\
4
\end{array}$$

So $\left(6x^2+13x+9\right)\div\left(2x+1\right)=3x+5\quad r4$ **or** $\underbrace{6x^2+13x+9}_{\text{dividend}}=\underbrace{\left(2x+1\right)}_{\text{divisor}}\underbrace{\left(3x+5\right)}_{\text{quotient}}+\underbrace{4}_{\textit{remainder}}$.

$$\left(2x^4-3x^3-3x^2+9x+2\right)\div\left(x^2-2x+1\right)$$

$$\begin{array}{r|l} & 2x^2+x-3 \\ \hline x^2-2x+1 & 2x^4-3x^3-3x^2+9x+2 \\ & \underline{-\left(2x^4-4x^3+2x^2\right)} \\ & x^3-5x^2+9x+2 \\ & \underline{-\left(x^3-2x^2+x\right)} \\ & -3x^2+8x+2 \\ & \underline{-\left(-3x^2+6x-3\right)} \\ & \qquad\qquad 2x+5 \end{array}$$

So $\left(2x^4-3x^3-3x^2+9x+2\right)\div\left(x^2-2x+1\right)=2x^2+x-3 \quad r\left(2x+5\right)$ or

$$\underbrace{2x^4-3x^3-3x^2+9x+2}_{\text{dividend}}=\underbrace{\left(x^2-2x+1\right)}_{\text{divisor}}\underbrace{\left(2x^2+x-3\right)}_{\text{quotient}}+\underbrace{\left(2x+5\right)}_{\text{remainder}}.$$

$$\left(8x^2-2x-15\right)\div\left(4x+5\right)$$

$$\begin{array}{r|l} & 2x-3 \\ \hline 4x+5 & 8x^2-2x-15 \\ & -\left(8x^2+10x\right) \\ \hline & -12x-15 \\ & -\left(-12x-15\right) \\ \hline & 0 \end{array}$$

So $\left(8x^2-2x-15\right)\div\left(4x+5\right)=2x-3$ or $\underbrace{8x^2-2x-15}_{\text{dividend}}=\underbrace{\left(4x+5\right)}_{\text{divisor}}\underbrace{\left(2x-3\right)}_{\text{quotient}}$.

Synthetic Division:

Only works if you're dividing by $(x \pm c)$.

Examples:

$(2x^2 + x - 16) \div (x - 2)$

2	2	1	-16
		4	10
	2	5	-6

So $(2x^2 + x - 16) \div (x - 2) = 2x + 5 \quad r(-6)$ or $\underbrace{2x^2 + x - 16}_{\text{dividend}} = \underbrace{(x - 2)}_{\text{divisor}}\underbrace{(2x + 5)}_{\text{quotient}} + \underbrace{(-6)}_{\text{remainder}}$.

$$\left(3x^2-2x+5\right)\div\left(x-3\right)$$

3	3	-2	5
		9	21
	3	7	26

$$3x^2-2x+5=\left(x-3\right)\left(3x+7\right)+26$$

$$\left(x^2-8x-12\right)\div\left(x+4\right)$$

-4	1	-8	-12
		-4	48
	1	-12	36

$$x^2-8x-12=\left(x+4\right)\left(x-12\right)+36$$

$$\left(2x^2+8\right)\div\left(x+3\right)$$

-3	2	0	8
		-6	18
	2	-6	26

$$2x^2+8=\left(x+3\right)\left(2x-6\right)+26$$

$$\left(x^3-2x^2+5x-1\right)\div\left(x-5\right)$$

5	1	-2	5	-1
		5	15	100
	1	3	20	99

$$x^3-2x^2+5x-1=\left(x-5\right)\left(x^2+3x+20\right)+99$$

Remainder Theorem:

When the polynomial function $f(x)$ is divided by $(x-c)$, the remainder is equal to $f(c)$.

Here's why: $f(x)=(x-c)q(x)+r$**, so**

$$\begin{aligned} f(c) &= (c-c)q(c)+r \\ &= 0\cdot q(c)+r \\ &= r \end{aligned}$$

Examples:

1. Use synthetic division along with the Remainder Theorem to find the value of $f(2)$ for the polynomial function $f(x)=3x^3-2x^2+x-3$.

2	3	-2	1	-3
		6	8	18
	3	4	9	15

So $f(2)=\boxed{15}$.

2. Use synthetic division along with the Remainder Theorem to find the value of $f(-3)$ for the polynomial function $f(x) = 5x^2 - 2x^3 + x - 4$.

-3	-2	5	1	-4
		6	-33	96
	-2	11	-32	92

So $f(-3) = \boxed{92}$.

Factor Theorem:

For the polynomial function $f(x)$, if $f(c) = 0$, then $(x - c)$ is a factor of $f(x)$.

Here's why:

$f(x) = (x - c)q(x) + r$**, so if** $f(c) = 0$**, then** $r = 0$**, and so** $(x - c)$ **is a factor of** $f(x)$**.**

Examples:

1. Use synthetic division and the Factor Theorem to show that $x-3$ is a factor of the polynomial function $f(x)=x^3-3x^2+x-3$, and then find all of the zeros of the polynomial function.

3	1	-3	1	-3
		3	0	3
	1	0	1	0

$f(3)=0$, so the Factor Theorem implies that $x-3$ is a factor of $f(x)=x^3-3x^2+x-3$, and from the synthetic division, $x^3-3x^2+x-3=(x-3)(x^2+1)$, which means that the zeros are $\boxed{3, i, -i}$.

2. Use synthetic division and the Factor Theorem to show that $x+2$ is a factor of the polynomial function $f(x)=2x^3+3x^2-5x-6$, and then find all of the zeros of the polynomial function.

-2	2	3	-5	-6
		-4	2	6
	2	-1	-3	0

$f(-2)=0$, so the Factor Theorem implies that $x+2$ is a factor of $f(x)=2x^3+3x^2-5x-6$, and from the synthetic division, $2x^3+3x^2-5x-6=(x+2)(2x^2-x-3)=(x+2)(2x-3)(x+1)$, which means that the zeros are $\boxed{-2,\frac{3}{2},-1}$.

3. Find the value of *k* so that $x-1$ is a factor of $x^5-4x^3+2x^2-3x+k$.

If $x-1$ is a factor of $x^5-4x^3+2x^2-3x+k$, then from the Factor Theorem, when you plug in 1for x in the polynomial function, you get zero.

$1^5-4\cdot1^3+2\cdot1^2-3\cdot1+k=0\Rightarrow-4+k=0\Rightarrow k=\boxed{4}$