Exponential Functions:

A function of the form $f\left(x\right)=b^{x}$ with $b>0$ and $b\neq 1$ is a called an exponential function with base *b*.

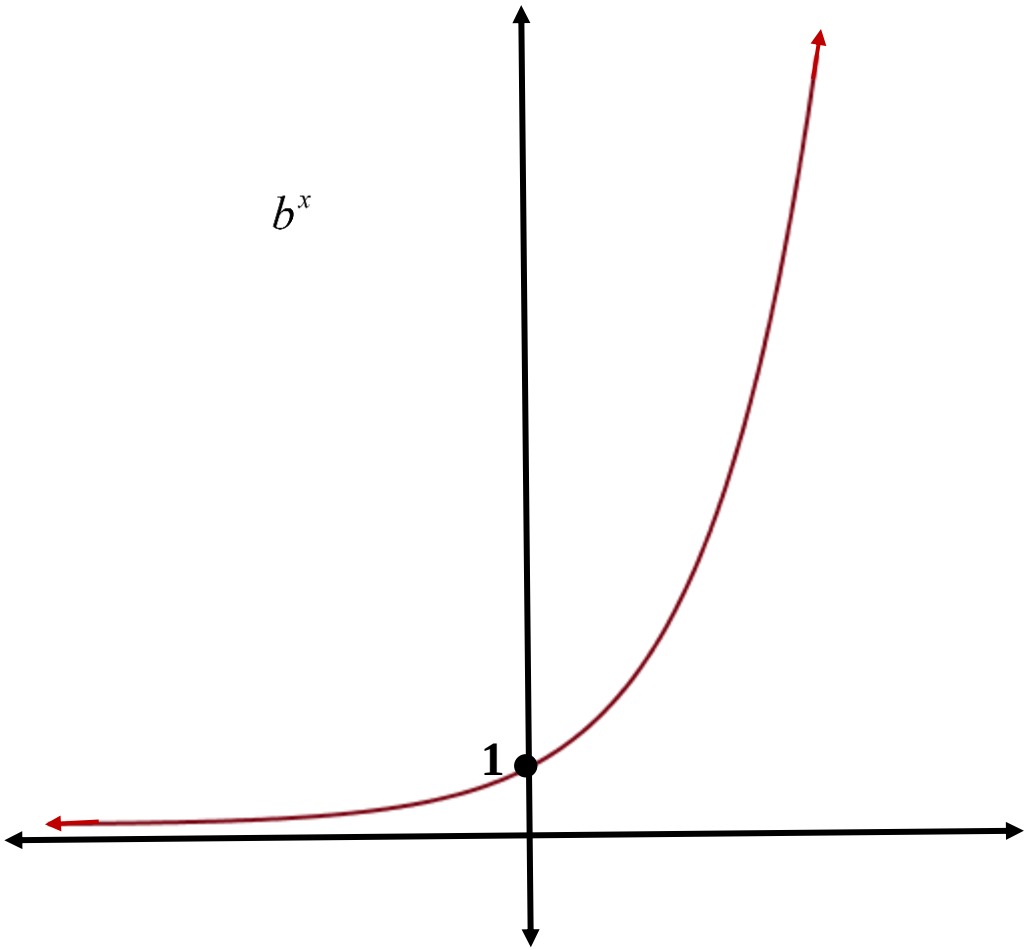
The bases naturally divide into two categories:

$$b>1$$

And

$$0<b<1$$

For $b > 1$,



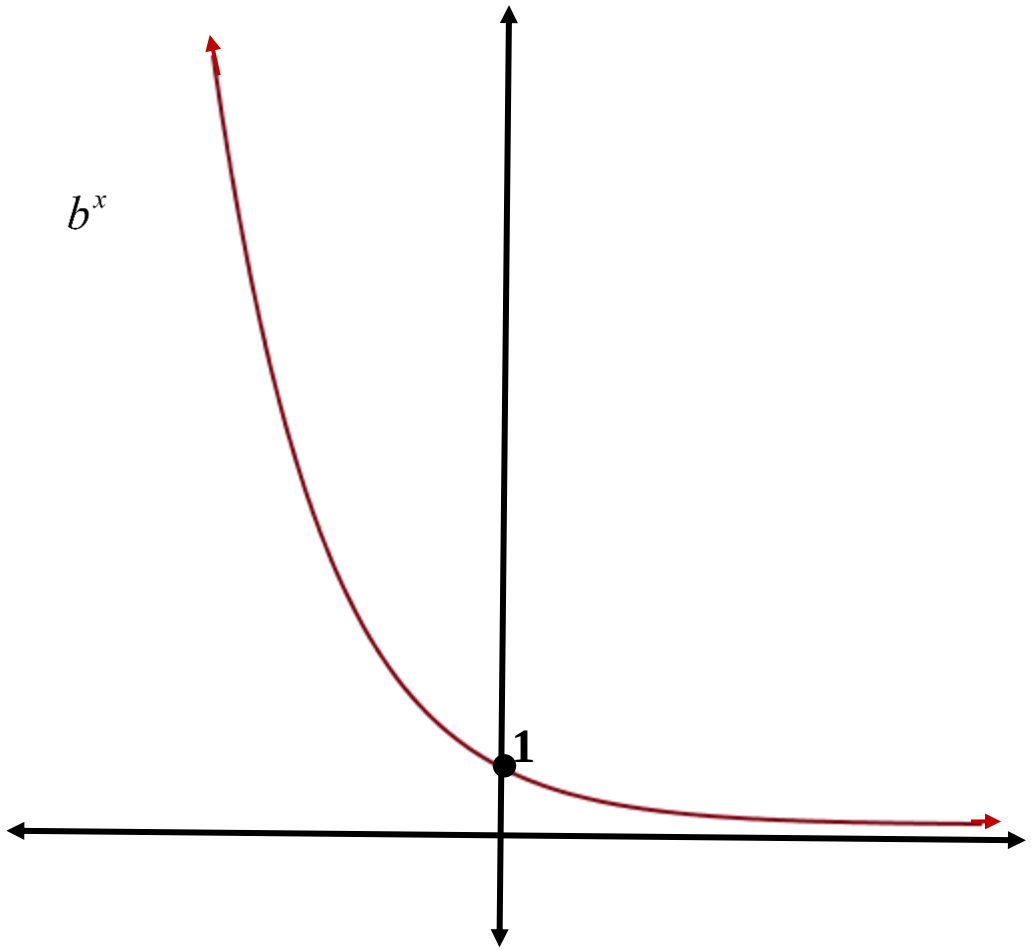
Domain: $(-\infty,\infty)$

Horizontal Asymptote: $y = 0$ **to the left**

Range: $(0,\infty)$

Increasing: $(-\infty,\infty)$

For $0 < b < 1$**,**



Domain: $(-\infty,\infty)$

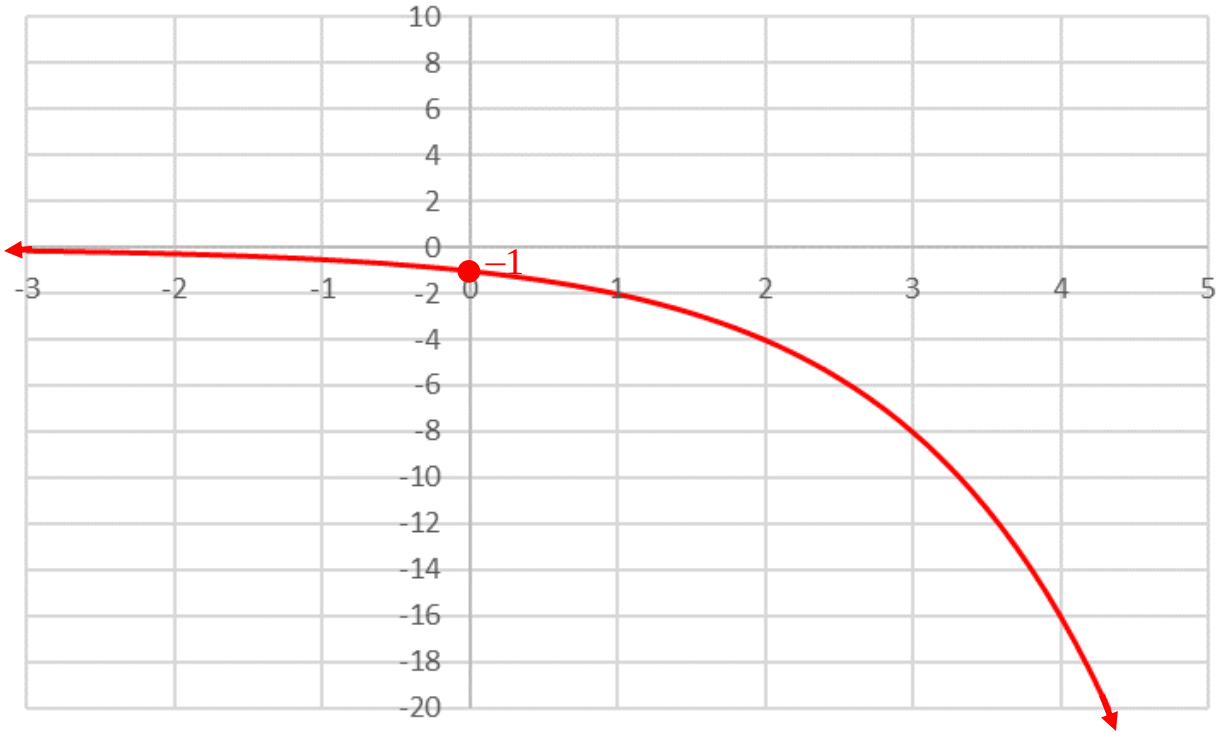
Horizontal Asymptote: $y = 0$ **to the right**

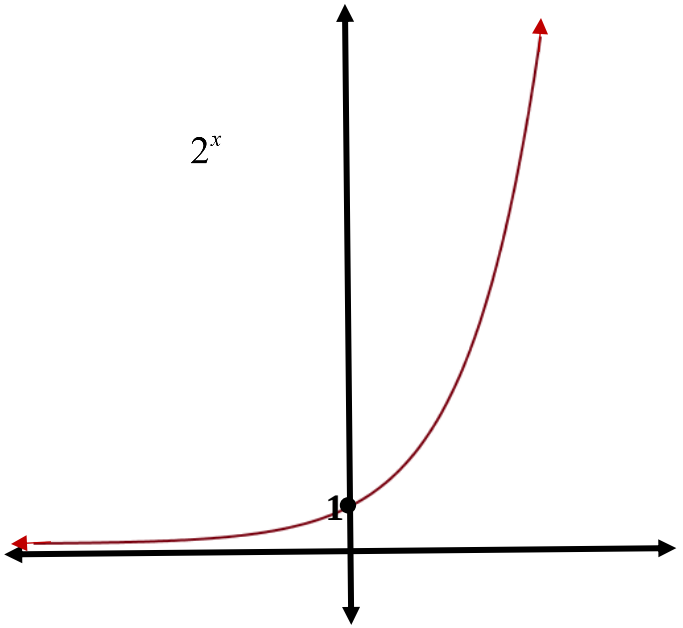
Range: $(0,\infty)$

Decreasing: $(-\infty,\infty)$

Transformations of Exponential Functions:

1. $f(x) = -2^x$

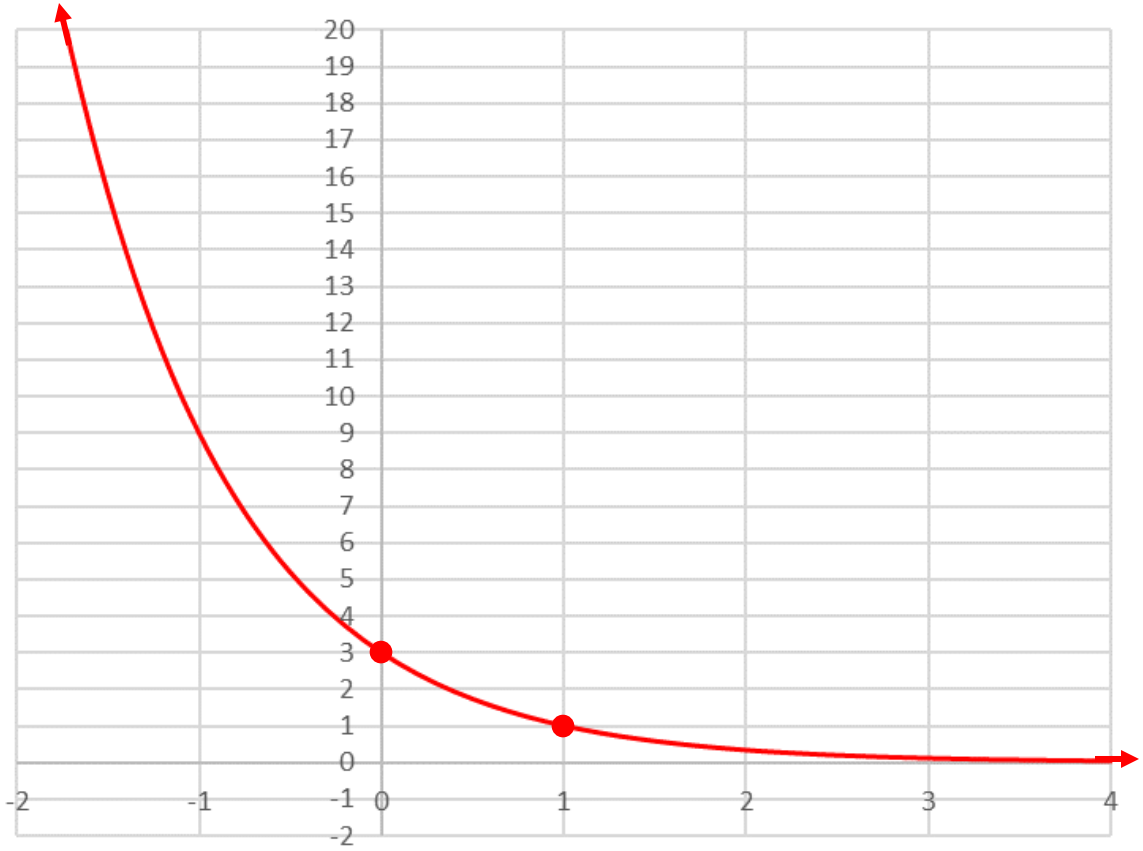


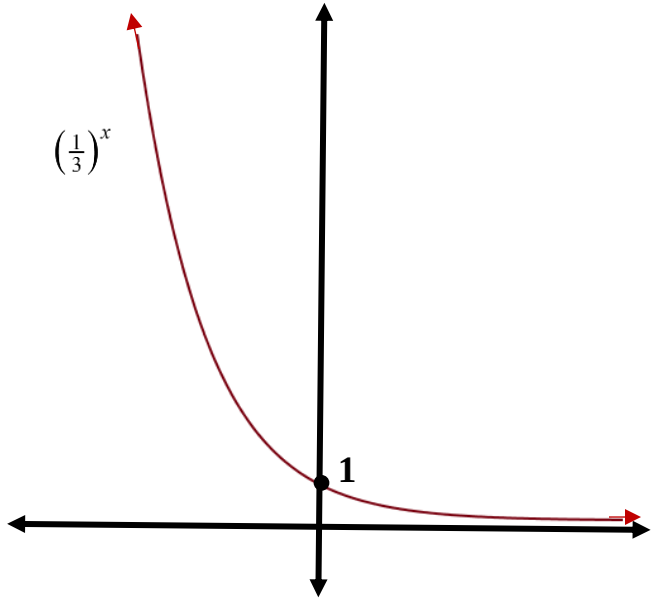


Domain: $(-\infty, \infty)$

Range: $(-\infty, 0)$

2. $f(x)=\left(\frac{1}{3}\right)^{x-1}$

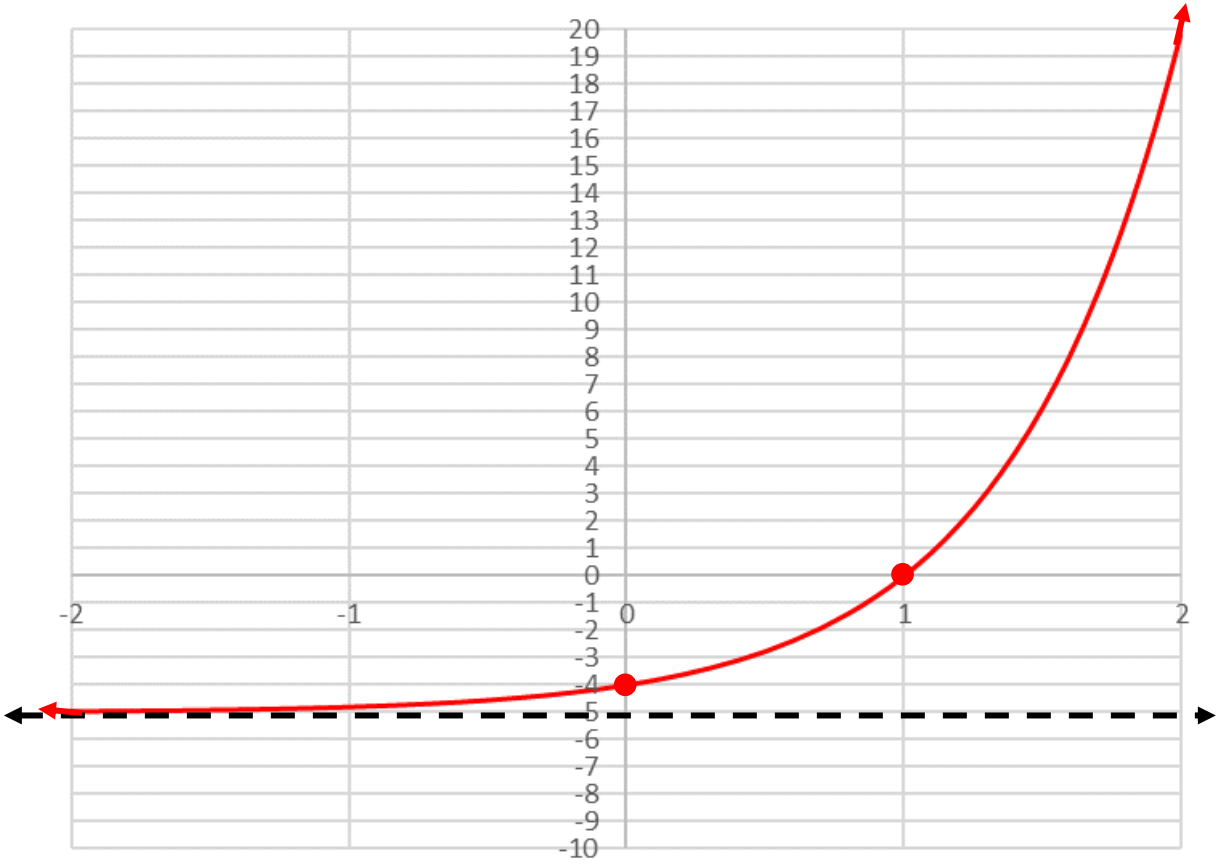


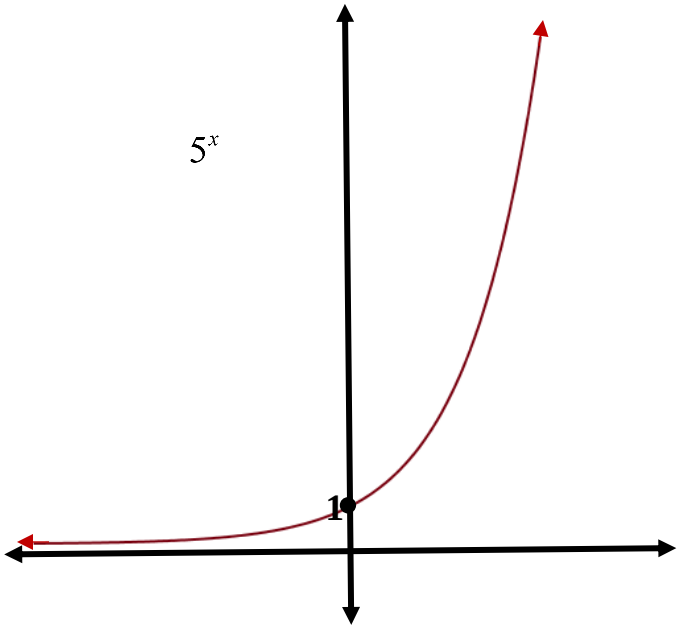


Domain: $(-\infty,\infty)$

Range: $(0,\infty)$

3. $f(x) = 5^x - 5$

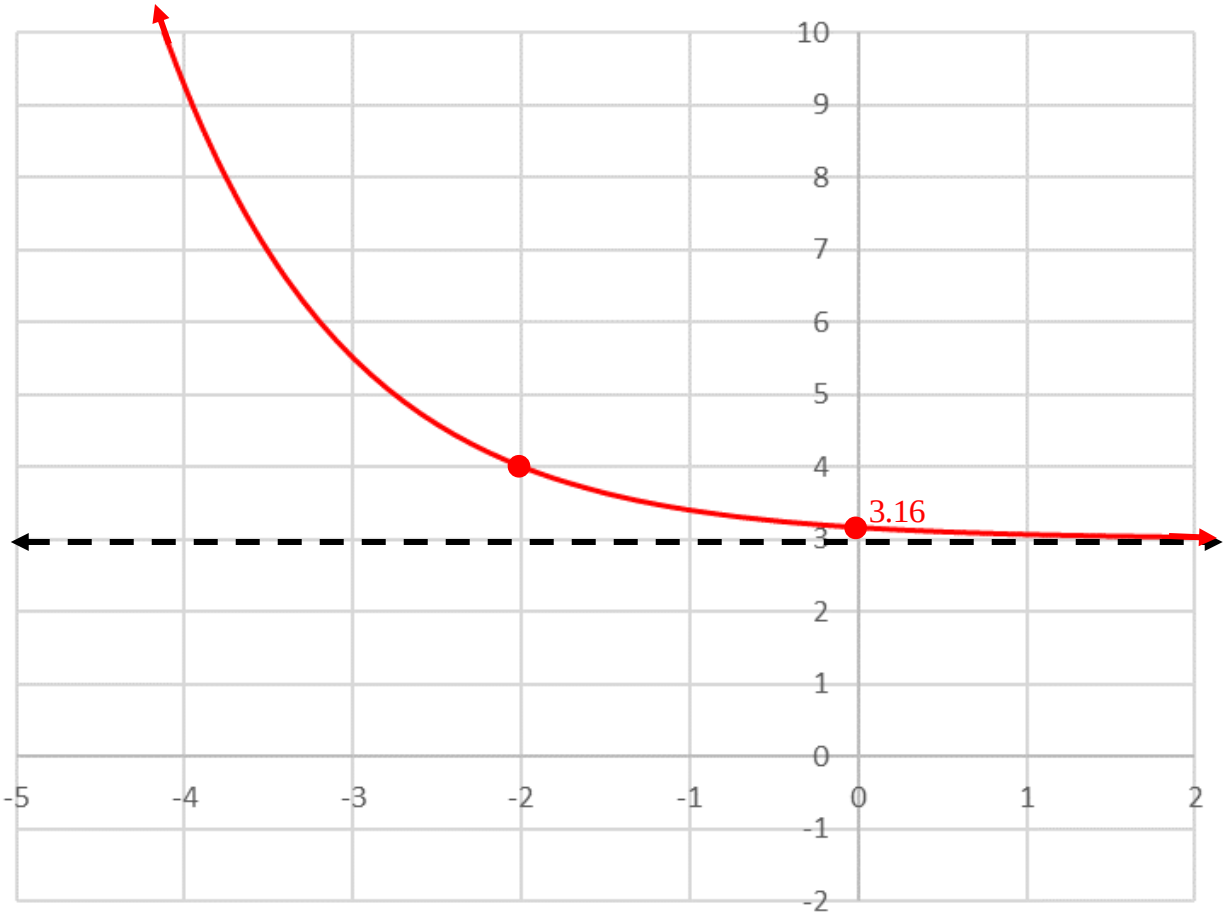


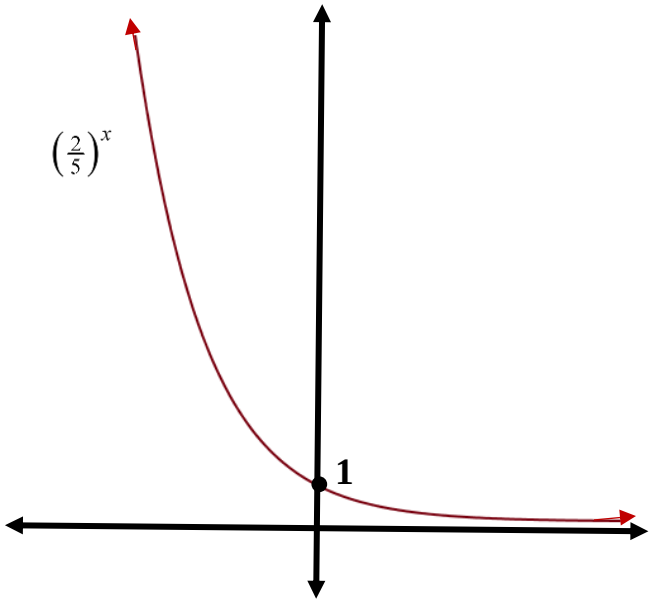


Domain: $(-\infty, \infty)$

Range: $(-5, \infty)$

4. $f(x)=\left(\frac{2}{5}\right)^{x+2}+3$





Domain: $(-\infty,\infty)$

Range: $(3,\infty)$

An Important Exponential Property:

If $b^x = b^y$**, then** $x = y$**.**

Solve the following basic exponential equations:

1. $5^x = 5^{-6}$

$$x = \boxed{-6}$$

2. $3^{-x} = 81$

$$3^{-x} = 3^4 \Rightarrow -x = 4 \Rightarrow x = \boxed{-4}$$

3. $4^{x^2} = 2^x$

$$\left(2^2\right)^{x^2} = 2^x \Rightarrow 2^{2x^2} = 2^x \Rightarrow 2x^2 = x \Rightarrow 2x^2 - x = 0$$

$$\Rightarrow x(2x-1) = 0 \Rightarrow x = \boxed{0, \frac{1}{2}}$$

4. $9^{-x+15} = 27^x$

$$\left(3^2\right)^{-x+15} = \left(3^3\right)^x \Rightarrow 3^{-2x+30} = 3^{3x} \Rightarrow -2x + 30 = 3x$$

$$\Rightarrow 5x = 30 \Rightarrow x = \boxed{6}$$

5. $5^{x^2+8} = 125^{2x}$

$$5^{x^2+8} = \left(5^3\right)^{2x} \Rightarrow 5^{x^2+8} = 5^{6x} \Rightarrow x^2 + 8 = 6x$$

$$\Rightarrow x^2 - 6x + 8 = 0 \Rightarrow (x-2)(x-4) = 0 \Rightarrow x = \boxed{2,4}$$