Logarithmic Functions:

A function of the form $f(x) = log_b x$ **with** $b > 0$ **and** $b \neq 1$ **is a called a logarithmic function with base *b*. It is the inverse of the exponential function** b^x**, so we can get the graph of a logarithmic function by reflecting the graph of the corresponding exponential function about the line** $y = x$**.**

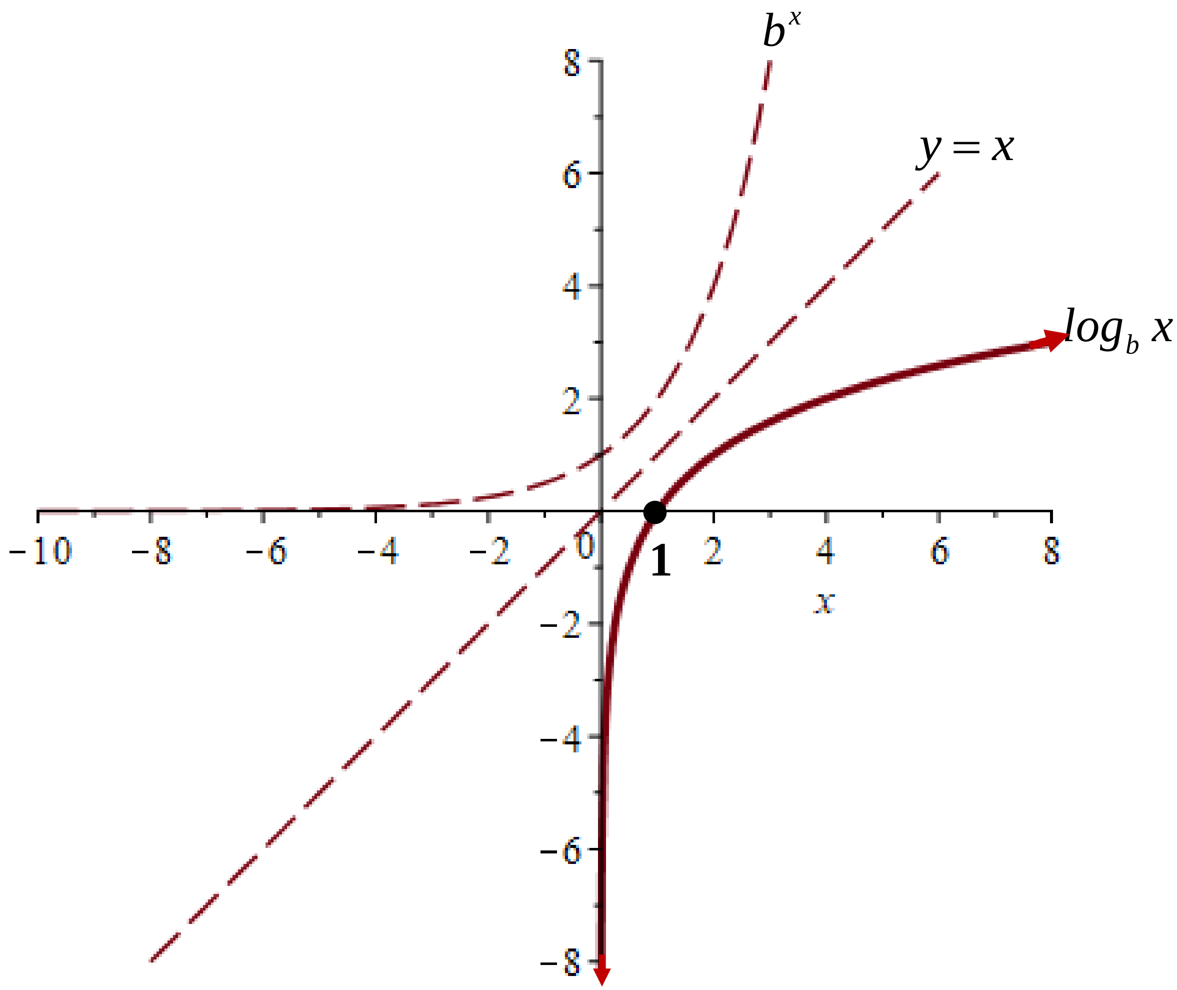
Just like the exponential functions, the bases separate into two categories:

$$b > 1$$

And

$$0 < b < 1$$

For $b > 1$,



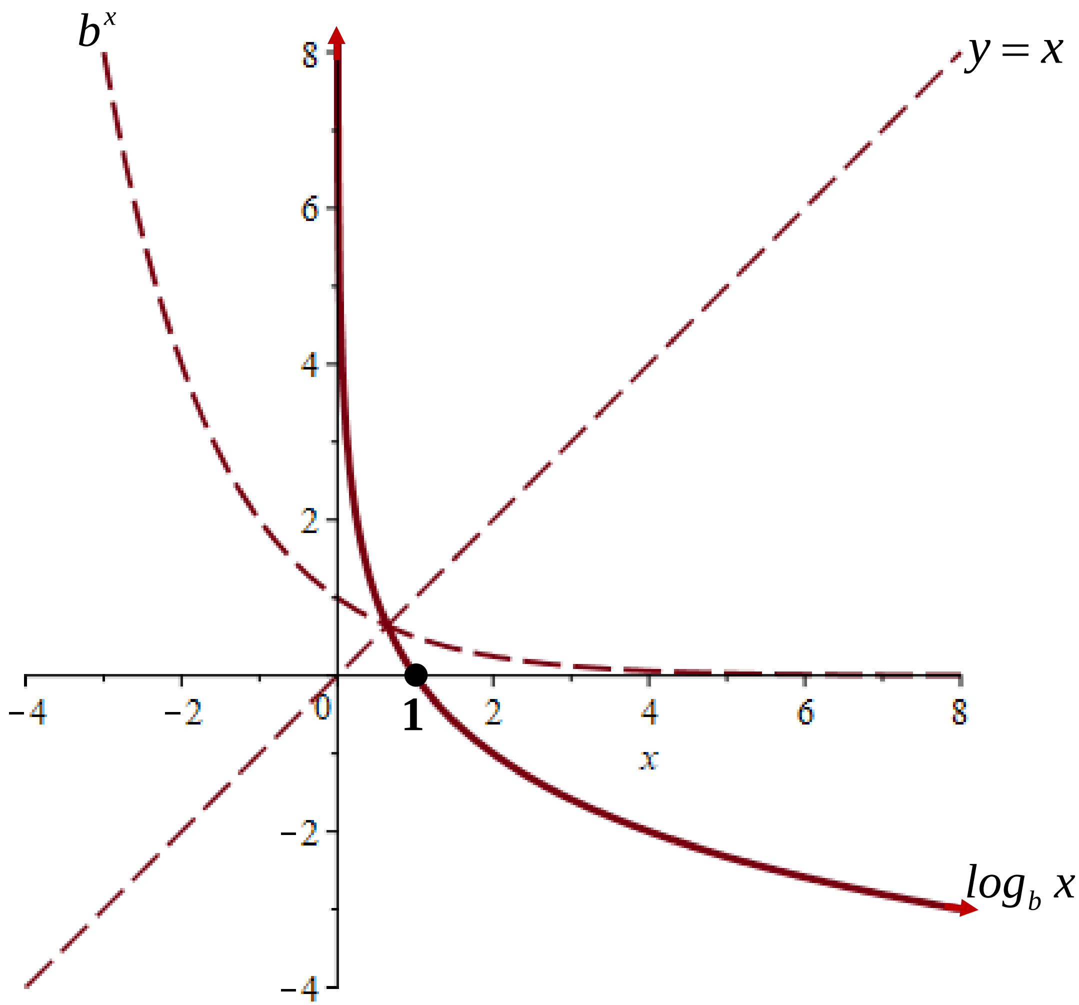
Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Vertical Asymptote: $x = 0$ **from the right**

Increasing: $(0, \infty)$

For $0 < b < 1$**,**



Domain: $(0, \infty)$

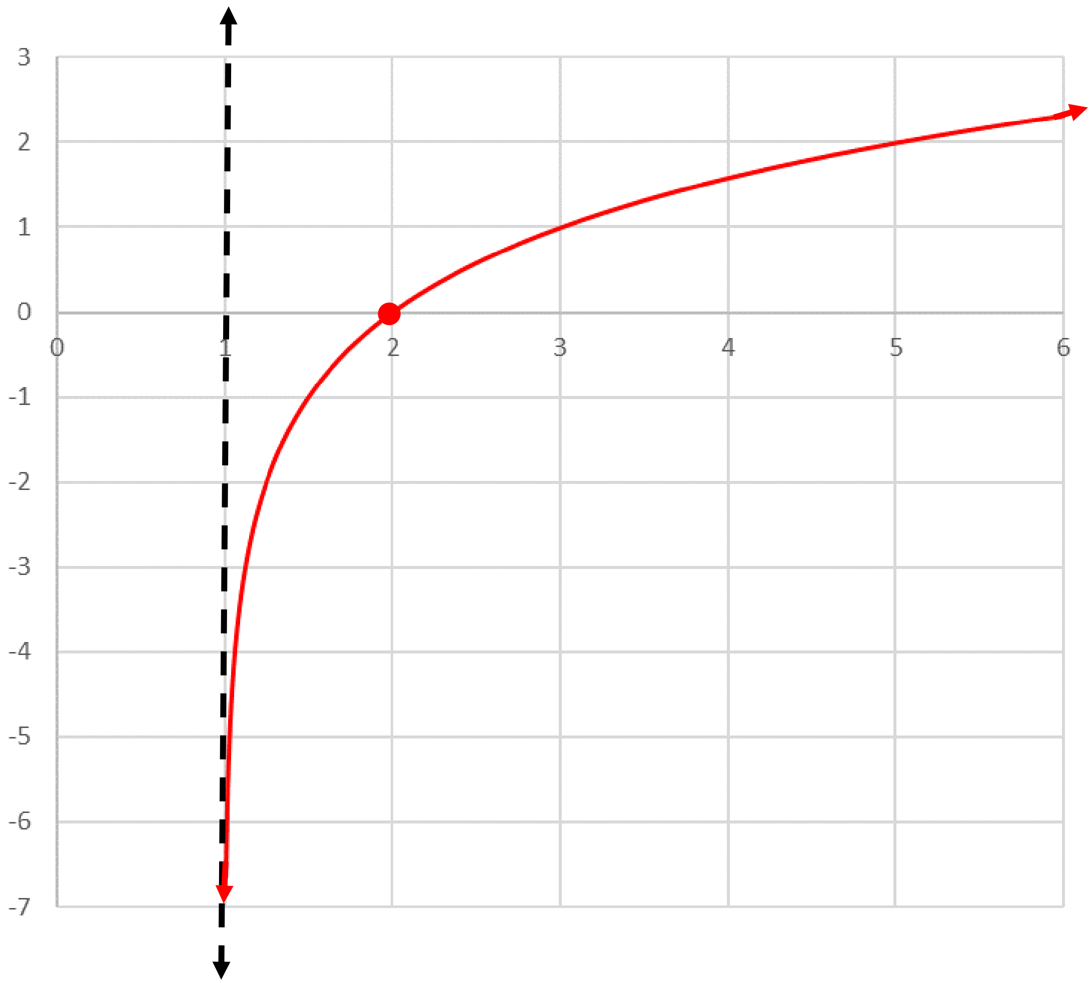
Range: $(-\infty, \infty)$

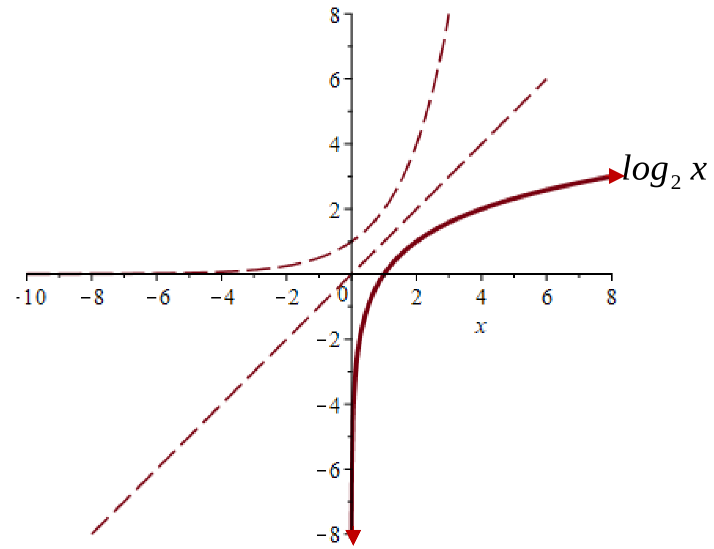
Vertical Asymptote: $x = 0$ **from the right**

Decreasing: $(0, \infty)$

Transformations of Logarithmic Functions:

1. $f(x) = log_2(x-1)$

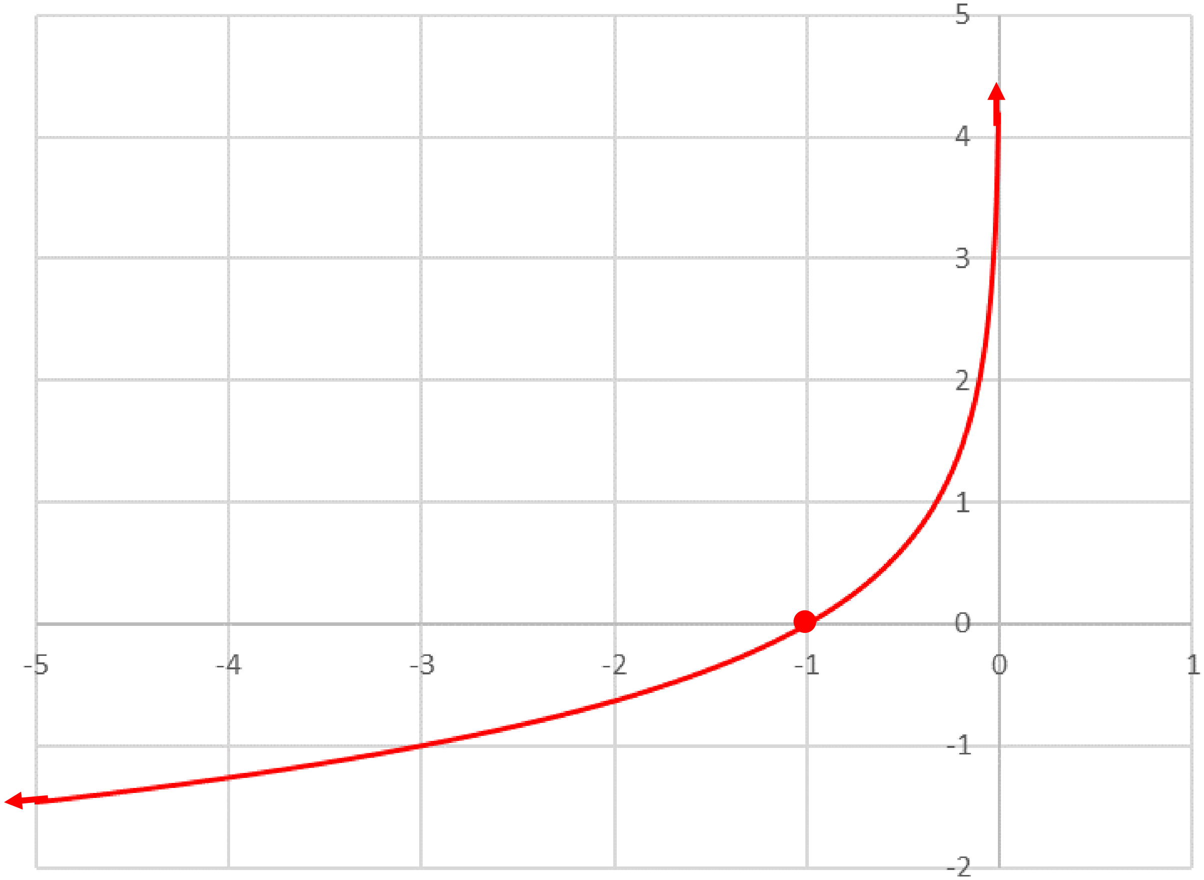


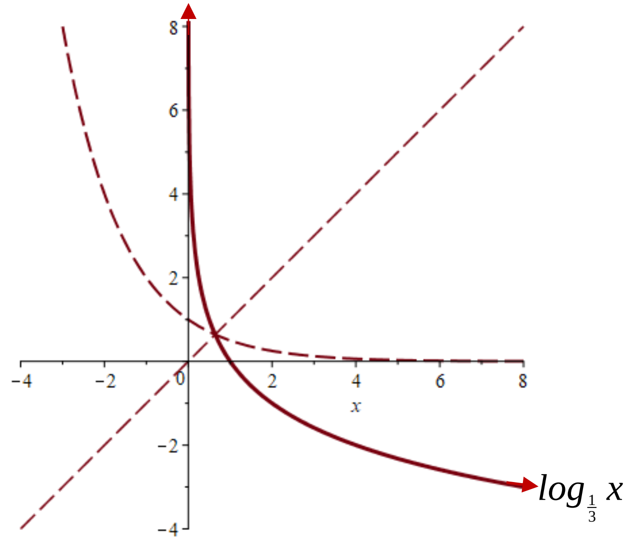


Domain: $(1, \infty)$

Range: $(-\infty, \infty)$

2. $f(x) = log_{\frac{1}{3}}(-x)$

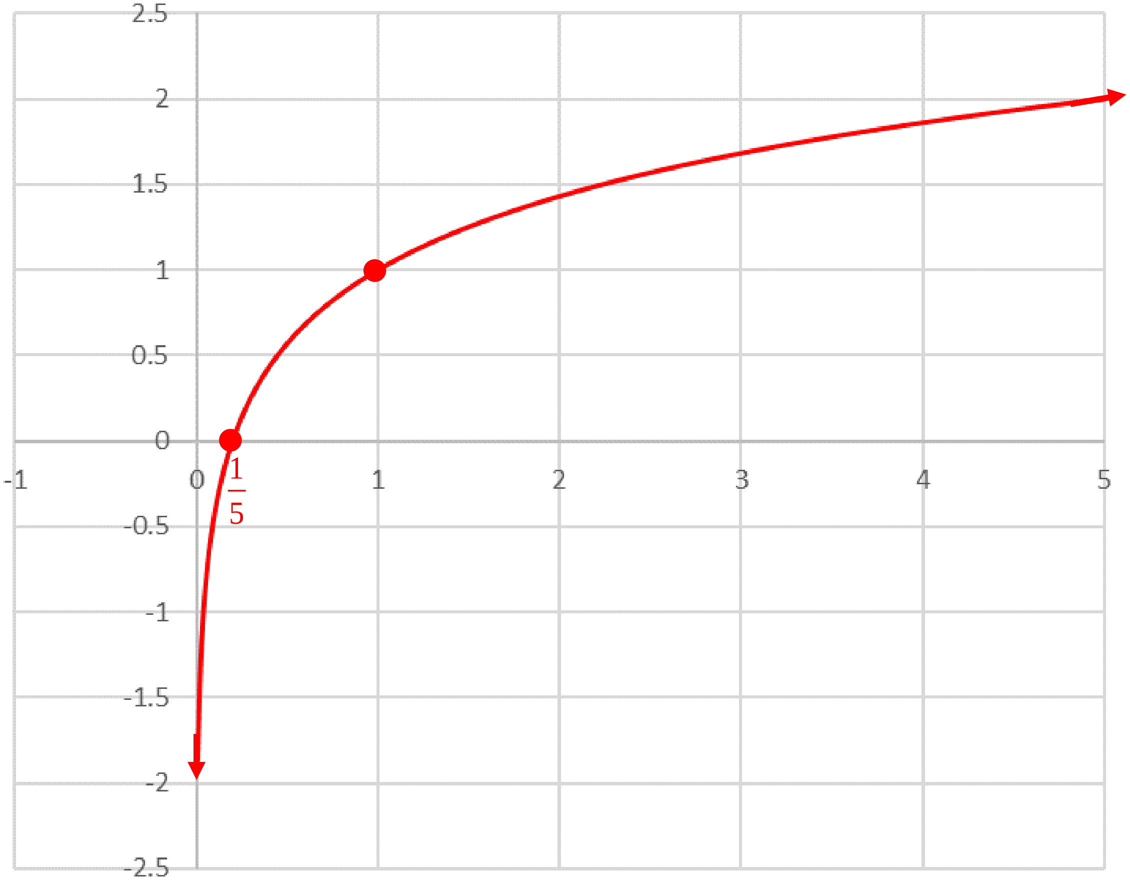


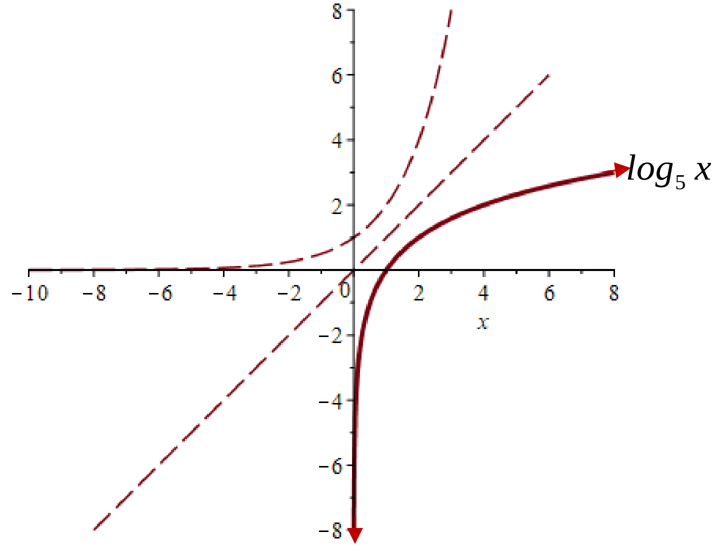


Domain: $(-\infty, 0)$

Range: $(-\infty, \infty)$

3. $f(x) = log_5 x + 1$

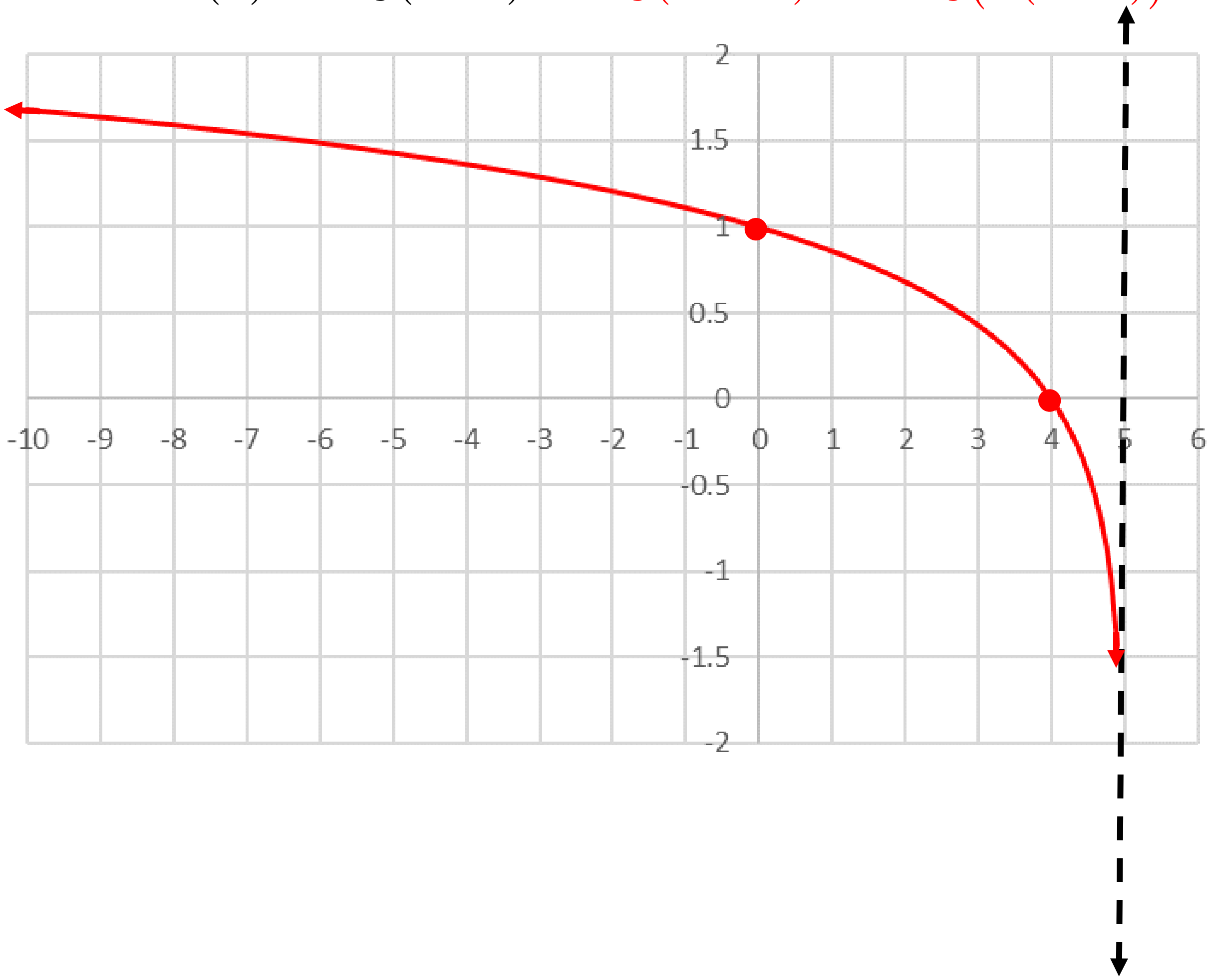


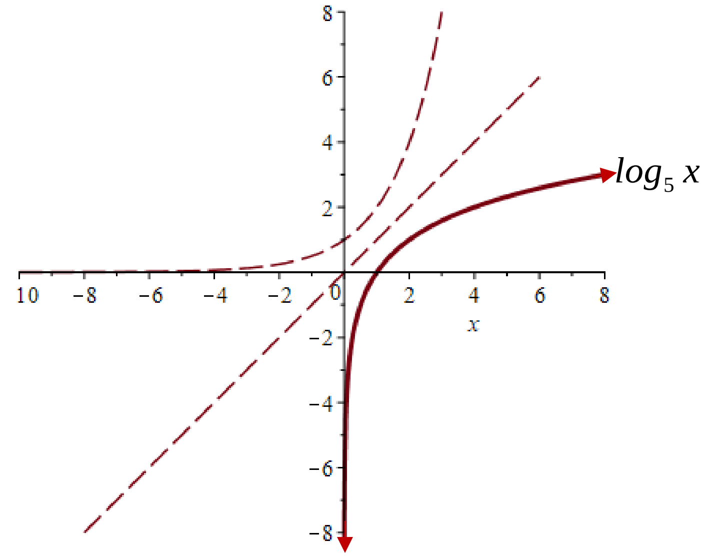


Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

4. $f(x) = log_5(5 - x) = log_5(-x + 5)$ or $log_5(-(x - 5))$





Domain: $(-\infty, 5)$

Range: $(-\infty, \infty)$

Logarithms are actually exponents. $log_b x$ **is the power or exponent that you raise *b* to in order to get *x*.**

1. $log_8 8$

1

2. $log_3 9$

2

3. $log_2 \frac{1}{4}$

-2

4. $log_{\frac{1}{2}} 8$

-3

5. $log_6 \sqrt{6}$

$\sqrt{6} = 6^{\frac{1}{2}} \Rightarrow \boxed{\frac{1}{2}}$

6. $log_5 \sqrt[3]{25}$

$\sqrt[3]{25} = 5^{\frac{2}{3}} \Rightarrow \boxed{\frac{2}{3}}$

7. $log_{\sqrt{3}} 9$

4

Logarithmic and Exponential Form of Equations:

Logarithmic Form:

$$3 = log_{10} 1{,}000$$

Exponentiate to get the exponential form.

$$10^3 = 1{,}000$$

Exponential Form:

$$3^4 = 81$$

Apply a logarithmic function to get the logarithmic form.

$$4 = \log_3 81$$

An Important Logarithmic Property:

$$log_b\left(b^x\right) = x; \text{for all } x$$

$$b^{log_b x} = x; \text{for } x > 0$$

Solve the following basic logarithmic equations:

1. $log_5 x = 2$

$$\Rightarrow 5^{log_5 x} = 5^2 \Rightarrow x = \boxed{25}$$

2. $log_3\left(3x-2\right) = 3$

$$\Rightarrow 3^{log_3\left(3x-2\right)} = 3^3 \Rightarrow 3x-2 = 27$$

$$\Rightarrow 3x = 29 \Rightarrow x = \boxed{\frac{29}{3}}$$

3. $log_x 4 = 2$

$\Rightarrow x^{log_x 4} = x^2 \Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2 \Rightarrow x = \boxed{2}$

The base must be positive.

4. $log_4 64 = x$

$log_4 64 = x \Rightarrow x = \boxed{3}$

5. $log_3\left(x^2+1\right) = 2$

$\Rightarrow 3^{log_3\left(x^2+1\right)} = 3^2 \Rightarrow x^2 + 1 = 9$

$\Rightarrow x^2 = 8 \Rightarrow x = \boxed{\pm\sqrt{8}}$

6. $log_5\left(x^2+4x+4\right) = 2$

$\Rightarrow 5^{log_5\left(x^2+4x+4\right)} = 5^2 \Rightarrow x^2 + 4x + 4 = 25$

$\Rightarrow x^2 + 4x - 21 = 0 \Rightarrow (x+7)(x-3) = 0$

$\Rightarrow x = \boxed{-7,3}$