Exponential and Logarithmic Equations:

The goal in solving exponential and logarithmic equations is to remove the exponential and logarithmic parts, eventually.

$$b^x = b^y \Rightarrow x = y$$

$$log_b\, x = log_b\, y \Rightarrow x = y$$

$$log_b\left(b^x\right) = x; \text{for all } x$$

$$b^{log_b\, x} = x; \text{for } x > 0$$

Examples:

1. $log\left(x+6\right)=1$

$\Rightarrow 10^{log(x+6)}=10^{1}\Rightarrow x+6=10\Rightarrow x=\boxed{4}$

2. $log_4\left(x+2\right)=log_4 8$

$\Rightarrow x+2=8\Rightarrow x=\boxed{6}$

3. $log_4\left(x+2\right)=log_4\left(2x+7\right)$ **{Be careful!}**

$\Rightarrow x+2=2x+7\Rightarrow x=-5$

$\Rightarrow$ $\boxed{\text{no solution}}$

-5 doesn't satisfy the original equation.

4. $2\log_5 x = 3\log_5 4$

$\Rightarrow \log_5 x^2 = \log_5 4^3 \Rightarrow x^2 = 64$

$\Rightarrow x = \pm 8 \Rightarrow x = \boxed{8}, -8$ doesn't satisfy the original equation.

5. $\log_6 (x+4) + \log_6 (x+3) = 1$

$\Rightarrow \log_6 \left[(x+4)(x+3) \right] = 1 \Rightarrow (x+4)(x+3) = 6$

$\Rightarrow x^2 + 7x + 12 = 6 \Rightarrow x^2 + 7x + 6 = 0$

$\Rightarrow (x+1)(x+6) = 0 \Rightarrow x = -1, -6 \Rightarrow x = \boxed{-1}, -6$ doesn't satisfy the original equation.

6. $\log_3 x - 2\log_3 5 = \log_3 (x+1) - 2\log_3 10$

$$\Rightarrow \log_3 \left(\frac{x}{25} \right) = \log_3 \left[\frac{(x+1)}{100} \right] \Rightarrow \frac{x}{25} = \frac{(x+1)}{100}$$

$$\Rightarrow 100x = 25x + 25 \Rightarrow 75x = 25 \Rightarrow x = \boxed{\frac{1}{3}}$$

7. $3^{2x}+3^{x}-2=0$

$\Rightarrow \left(3^{x}+2\right)\left(3^{x}-1\right)=0 \Rightarrow 3^{x}=-2, 3^{x}=1$

$\Rightarrow x=\boxed{0}$

8. $2^{2x}+2^{x+2}-12=0$

$\Rightarrow 2^{2x}+4\cdot 2^{x}-12=0 \Rightarrow \left(2^{x}+6\right)\left(2^{x}-2\right)=0$

$\Rightarrow 2^{x}=-6, 2^{x}=2 \Rightarrow x=\boxed{1}$

9. $3^{1-2x}=4^{x}$

$\Rightarrow ln\left(3^{1-2x}\right)=ln\left(4^{x}\right) \Rightarrow (1-2x)ln3=x\,ln4 \Rightarrow ln3=(2ln3+ln4)x$

$\Rightarrow x=\boxed{\dfrac{ln3}{2ln3+ln4}}$

10. $5^{2x}-8\cdot 5^{x}=-16$

$$\Rightarrow 5^{2x}-8\cdot 5^{x}+16=0\Rightarrow \left(5^{x}-4\right)^{2}=0$$

$$\Rightarrow 5^{x}=4\Rightarrow x=\boxed{\frac{log\,4}{log\,5}}$$

11. $3^{x}-14\cdot 3^{-x}=5$

$$\Rightarrow 3^{x}\left(3^{x}-14\cdot 3^{-x}\right)=5\cdot 3^{x}\Rightarrow 3^{2x}-14=5\cdot 3^{x}\Rightarrow 3^{2x}-5\cdot 3^{x}-14=0$$

$$\Rightarrow \left(3^{x}-7\right)\left(3^{x}+2\right)=0\Rightarrow 3^{x}=7,3^{x}=-2\Rightarrow x=\boxed{\frac{ln\,7}{ln\,3}}$$