Transformations of the Graphs of Functions:

Vertical Shift

Horizontal Shift

Reflection about the *x*-axis

Reflection about the *y*-axis

Vertical Stretch/Compress

Horizontal Stretch/Compress

<u>Vertical Shift:</u>

For $c > 0$**,**

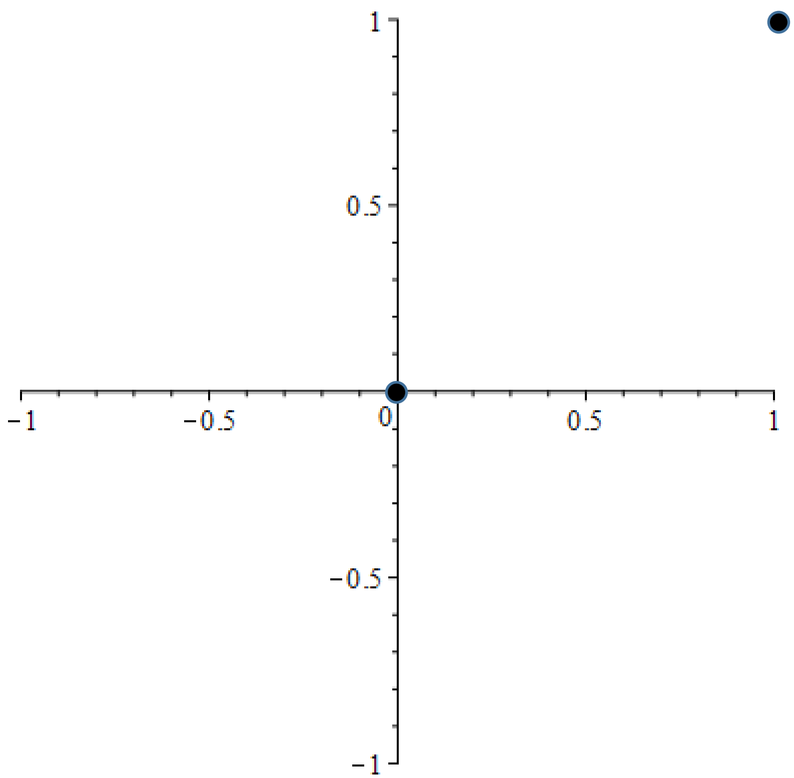
The graph of $g(x) = f(x) + c$**, is the graph of** $f(x)$ **shifted *c* units up.**

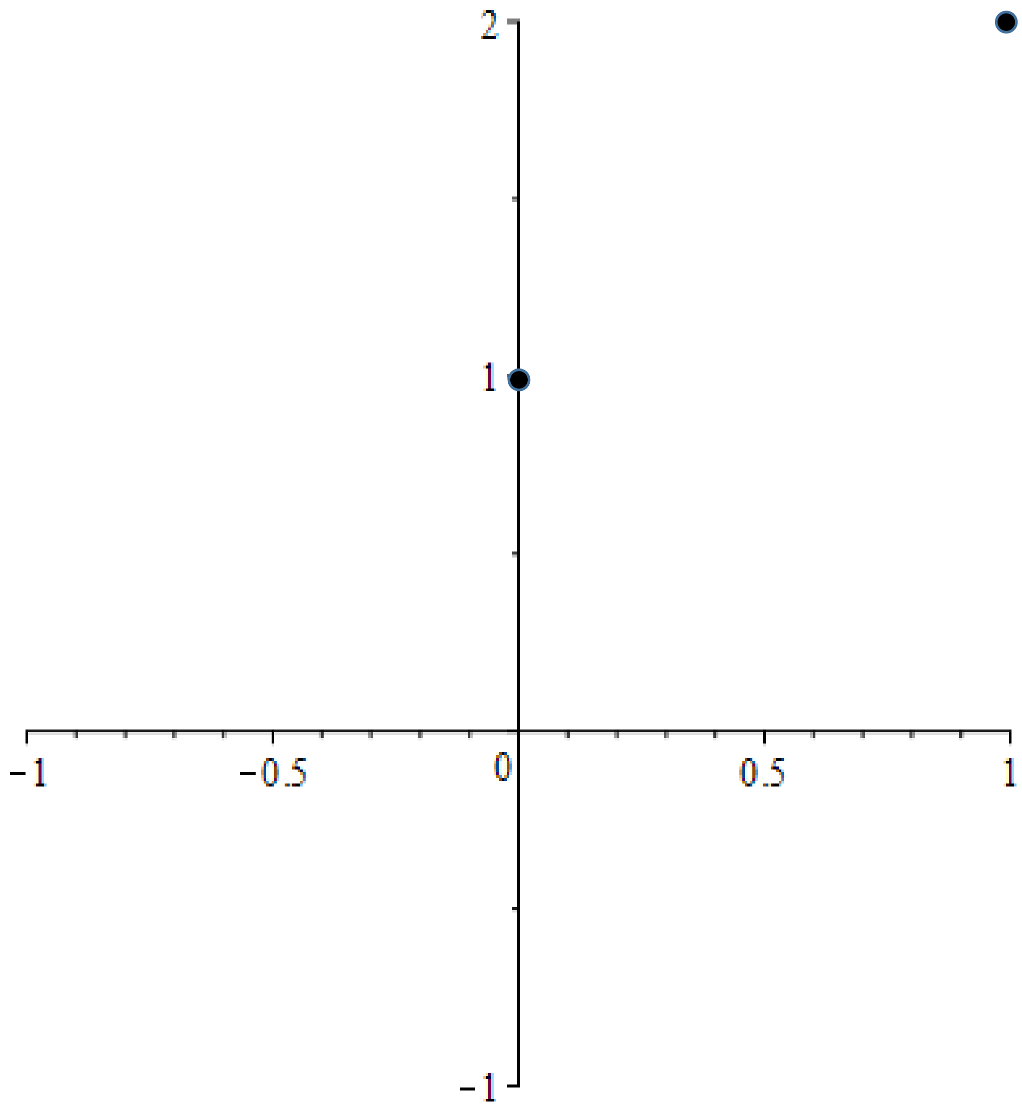
The graph of $g(x) = f(x) - c$**, is the graph of** $f(x)$ **shifted *c* units down.**

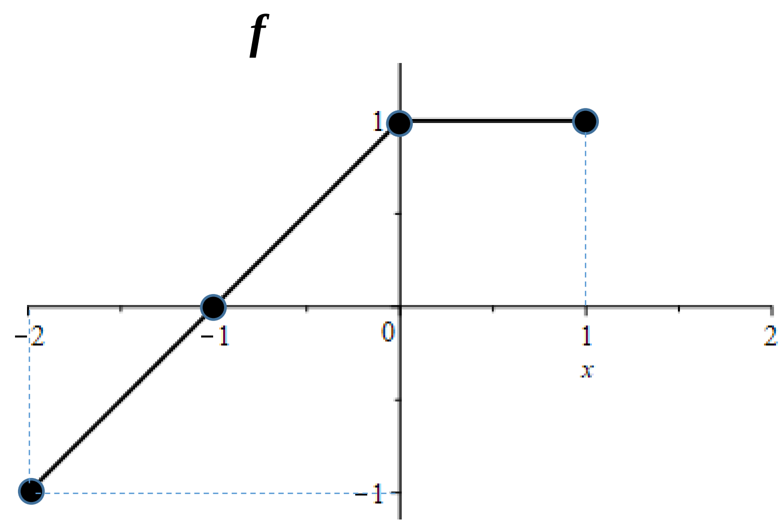
For a vertical shift, the *y*-coordinates change, but the *x*-coordinates remain the same.

$f = \{(0,0),(1,1)\}$

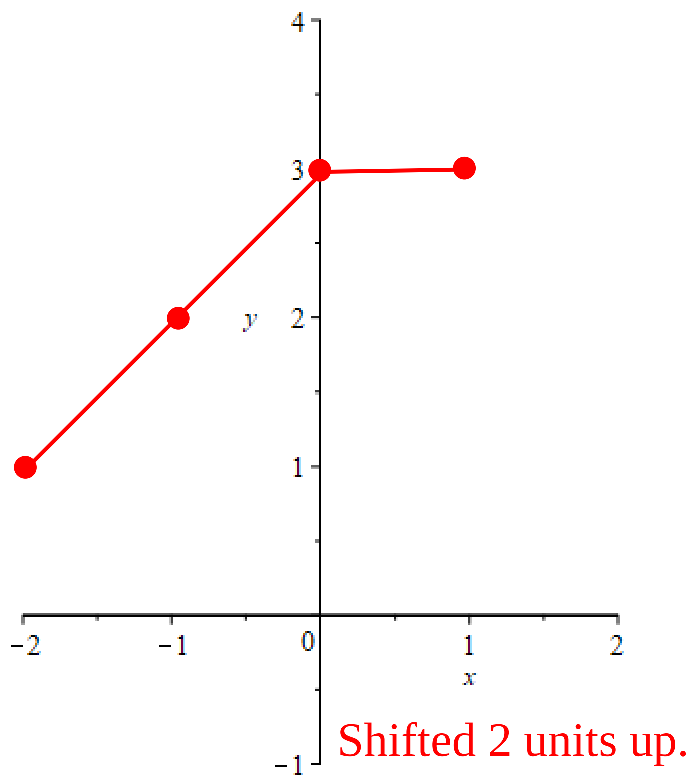
$g(x) = f(x) + 1,\ g = \{(0, \boxed{1}), (1, \boxed{2})\}$



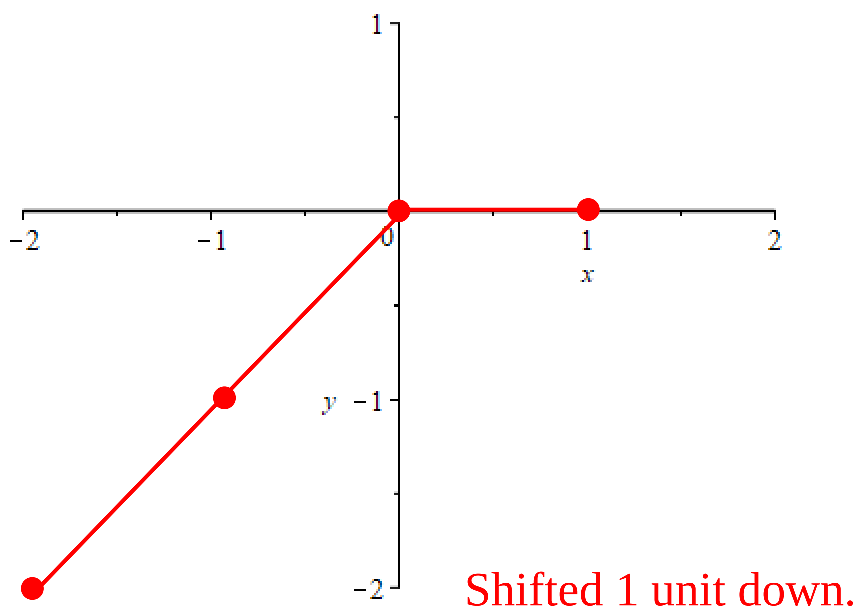



f
1
-1
-2
-1
0
1
2
x

$g(x) = f(x) + 2$


4
3
2
1
-1
y
-2
-1
0
1
2
x
Shifted 2 units up.

$h(x) = f(x) - 1$


1
-1
-2
y
-2
-1
0
1
2
x
Shifted 1 unit down.

<u>Horizontal Shift:</u>

For $c > 0$**,**

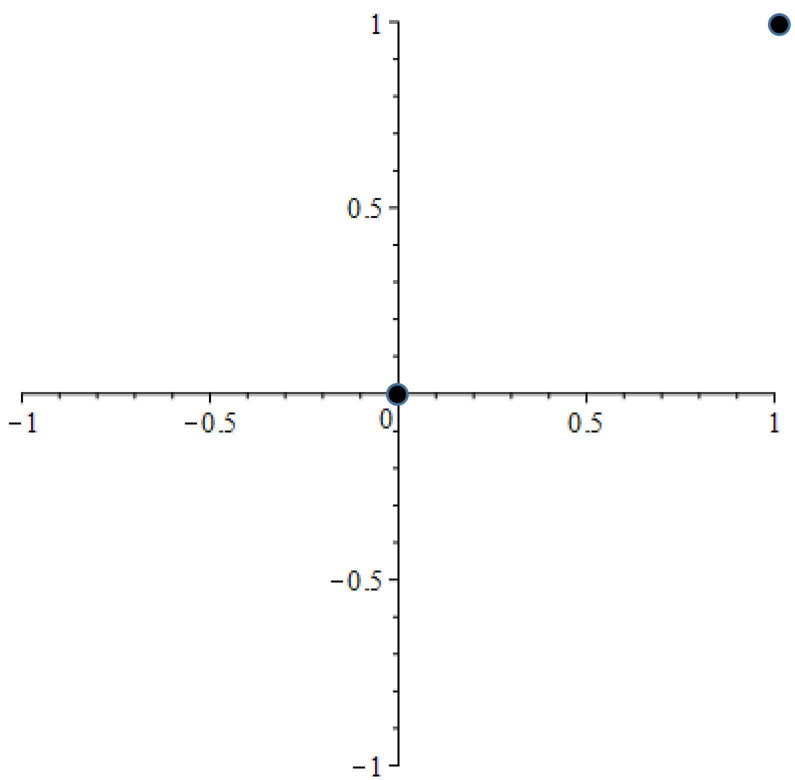
The graph of $g(x) = f(x - c)$**, is the graph of** $f(x)$ **shifted *c* units to the right.**

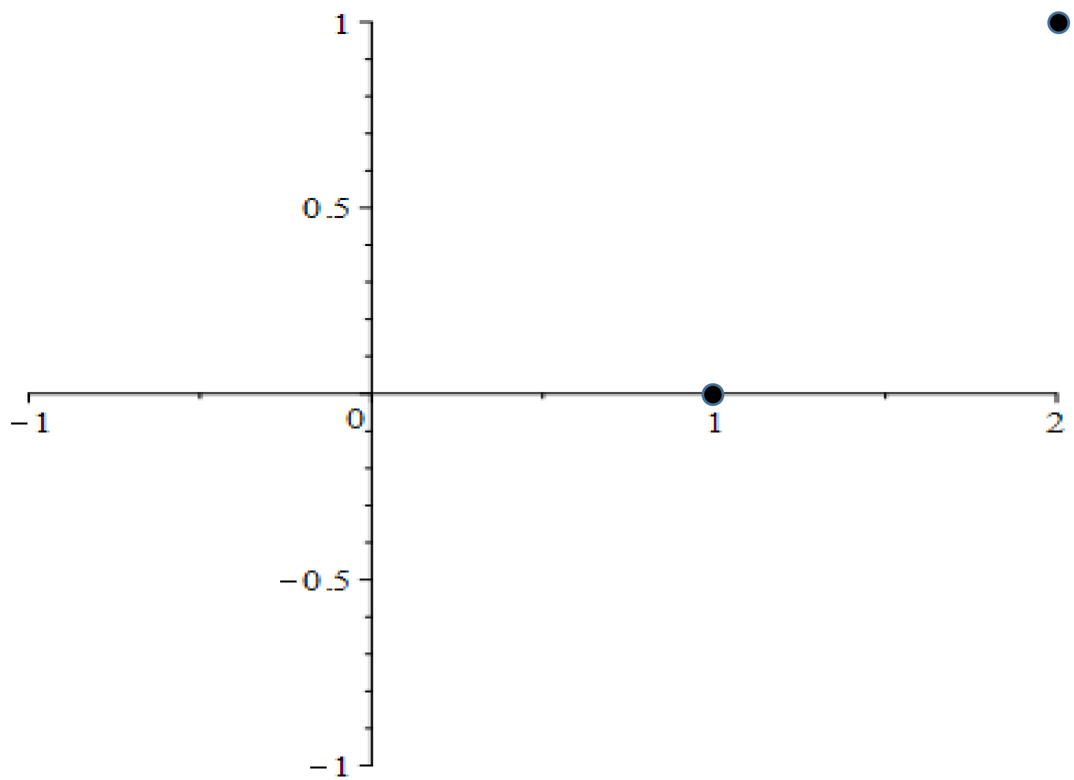
The graph of $g(x) = f(x + c)$**, is the graph of** $f(x)$ **shifted *c* units to the left.**

For a horizontal shift, the *x*-coordinates change, but the *y*-coordinates remain the same.

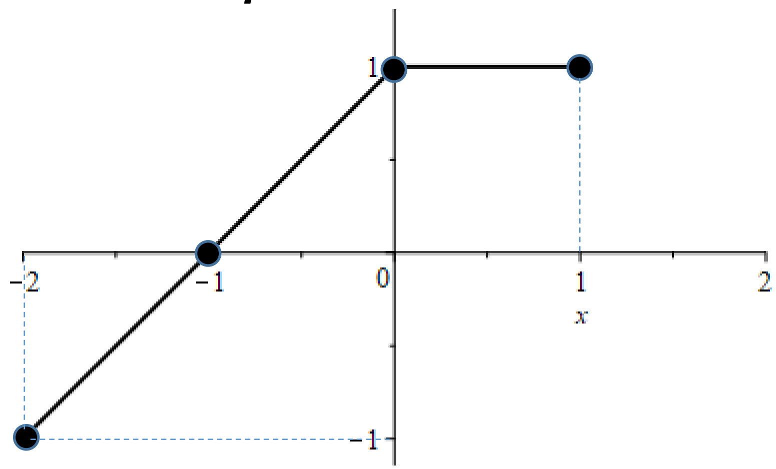
$f = \{(0,0),(1,1)\}$

$g(x) = f(x-1),\ g = \{(\boxed{1},0),(\boxed{2},1)\}$

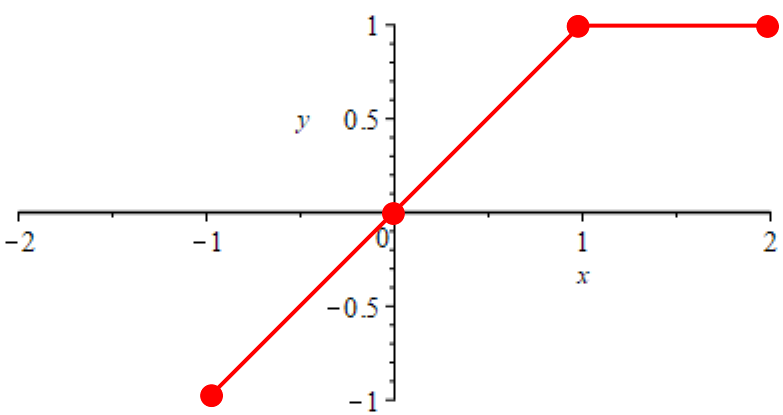




f

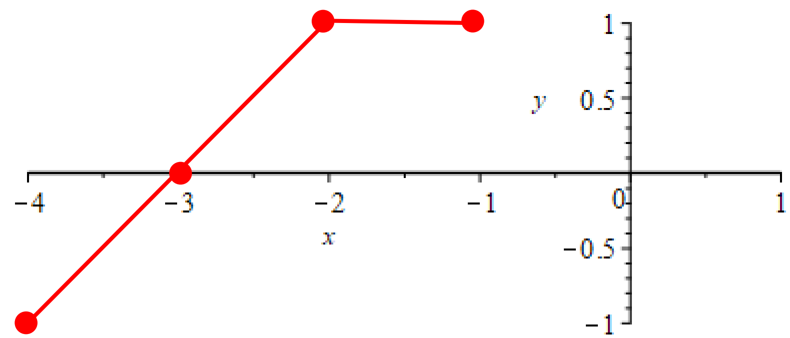


$g(x) = f(x-1)$



Shifted 1 unit to the right.

$h(x) = f(x+2)$



Shifted 2 units to the left.

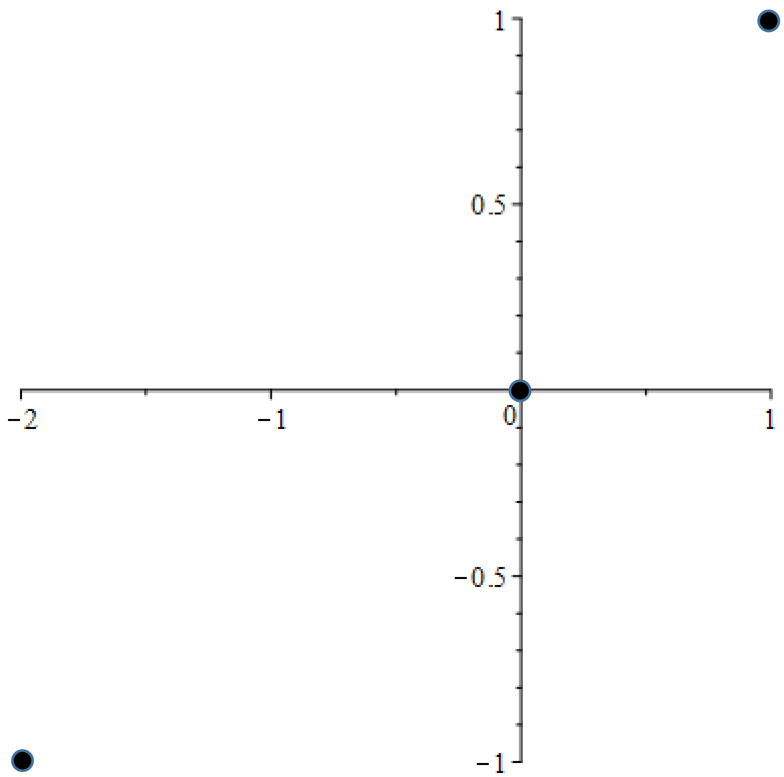
Reflection about the x-axis:

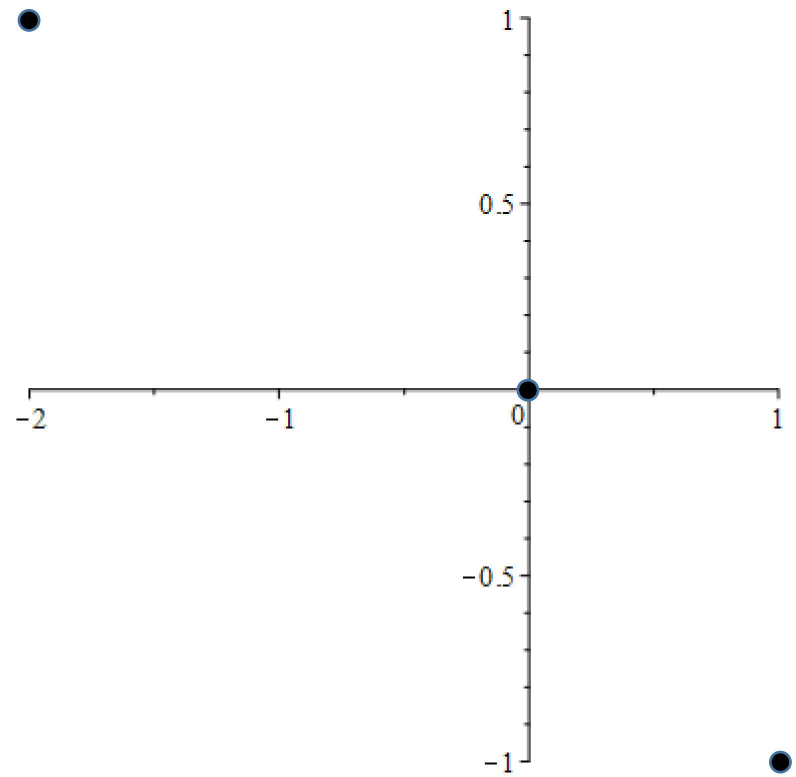
The graph of $g(x) = -f(x)$ **is the graph of** $f(x)$ **reflected about the *x*-axis.**

For reflection about the *x*-axis, the non-zero *y*-coordinates change, but the *x*-coordinates remain the same.

$f = \{(0,0),(1,1),(-2,-1)\}$

$g(x) = -f(x),\ g = \{(0,\boxed{0}),(1,\boxed{-1}),(-2,\boxed{2})\}$





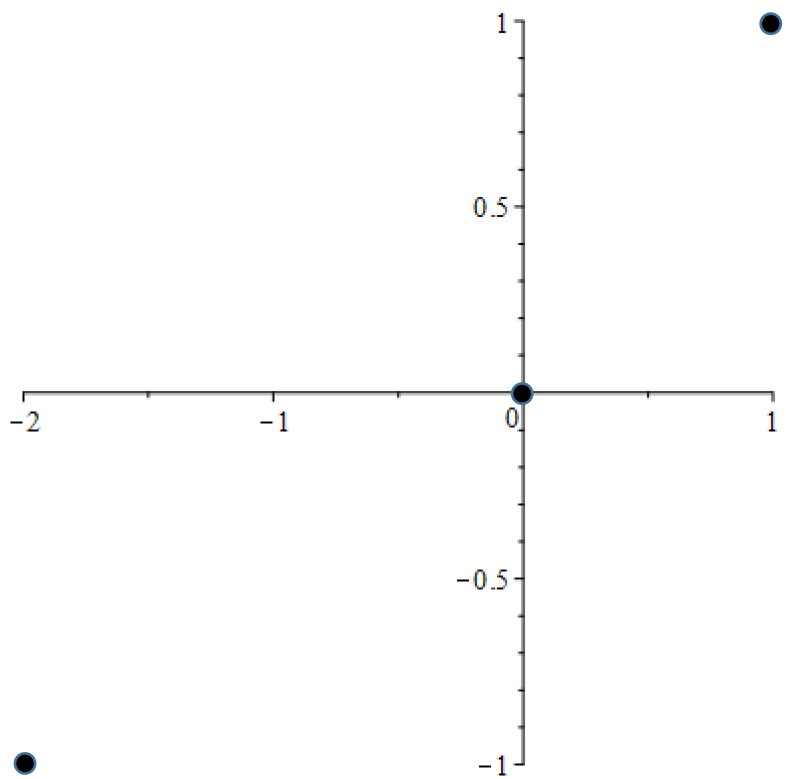
Reflection about the y-axis:

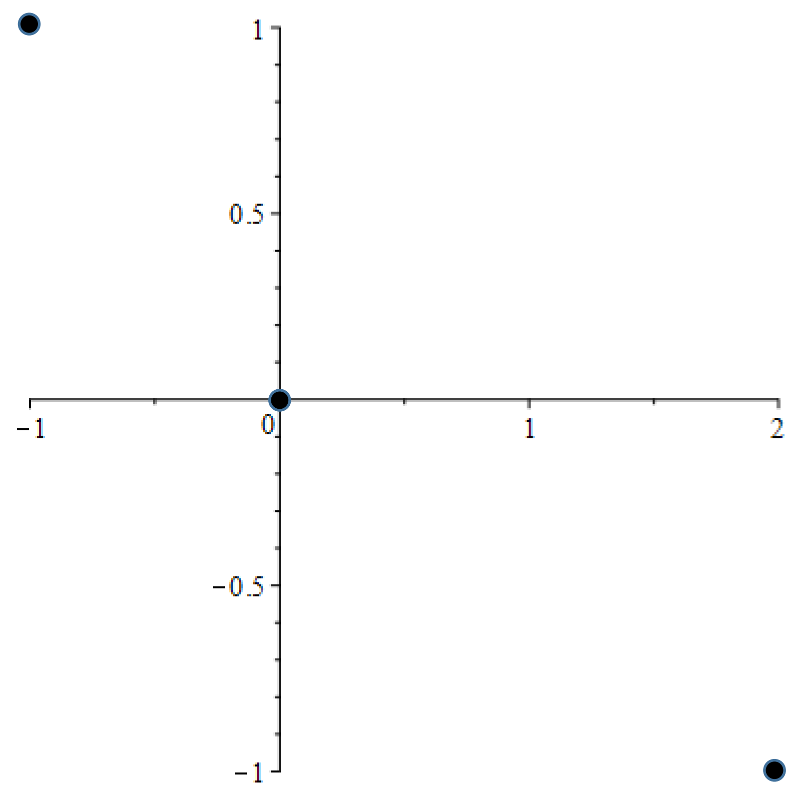
The graph of $g(x) = f(-x)$ **is the graph of** $f(x)$ **reflected about the *y*-axis.**

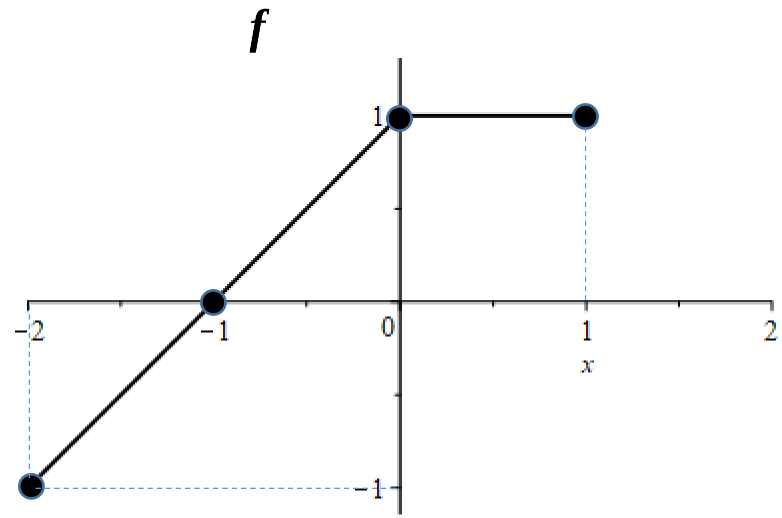
For reflection about the *y*-axis, the non-zero *x*-coordinates change, but the *y*-coordinates remain the same.

$f = \{(0,0),(1,1),(-2,-1)\}$

$g(x) = f(-x),\ g = \{(\boxed{0},0),(\boxed{-1},1),(\boxed{2},-1)\}$

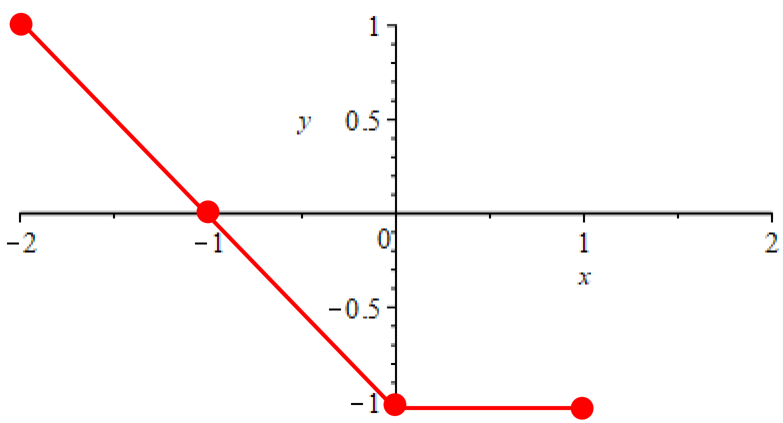




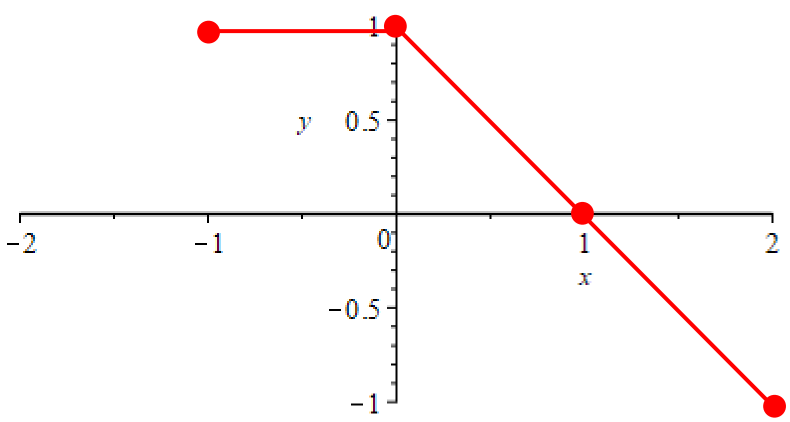


$g(x) = -f(x)$

$h(x) = f(-x)$



Reflected about the *x*-axis.



Reflected about the *y*-axis.

Vertical Stretch/Compress:

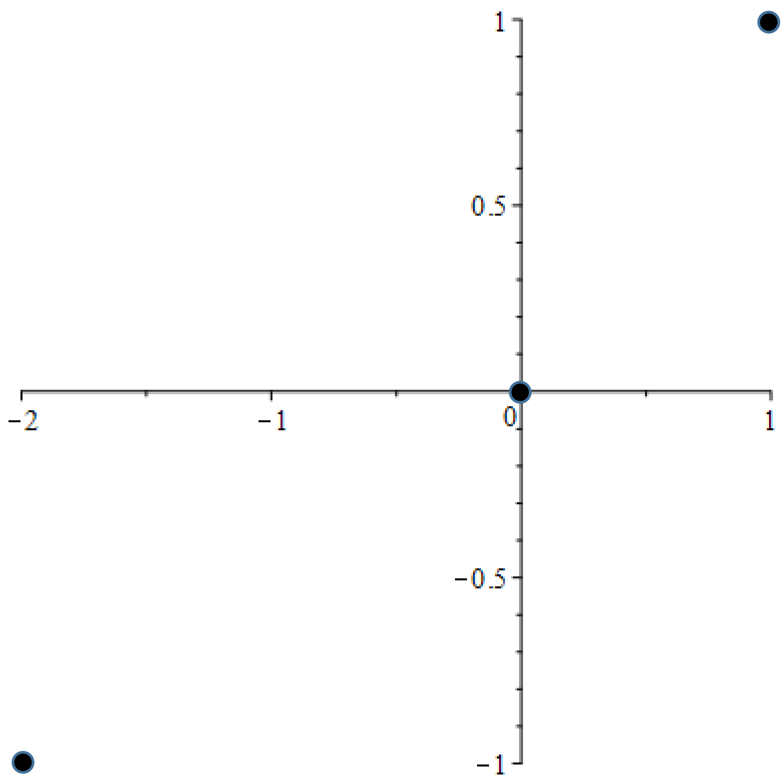
For $c > 1$, the graph of $g(x) = cf(x)$ is the graph of $f(x)$ *stretched* away from the *x*-axis by a factor of *c*.

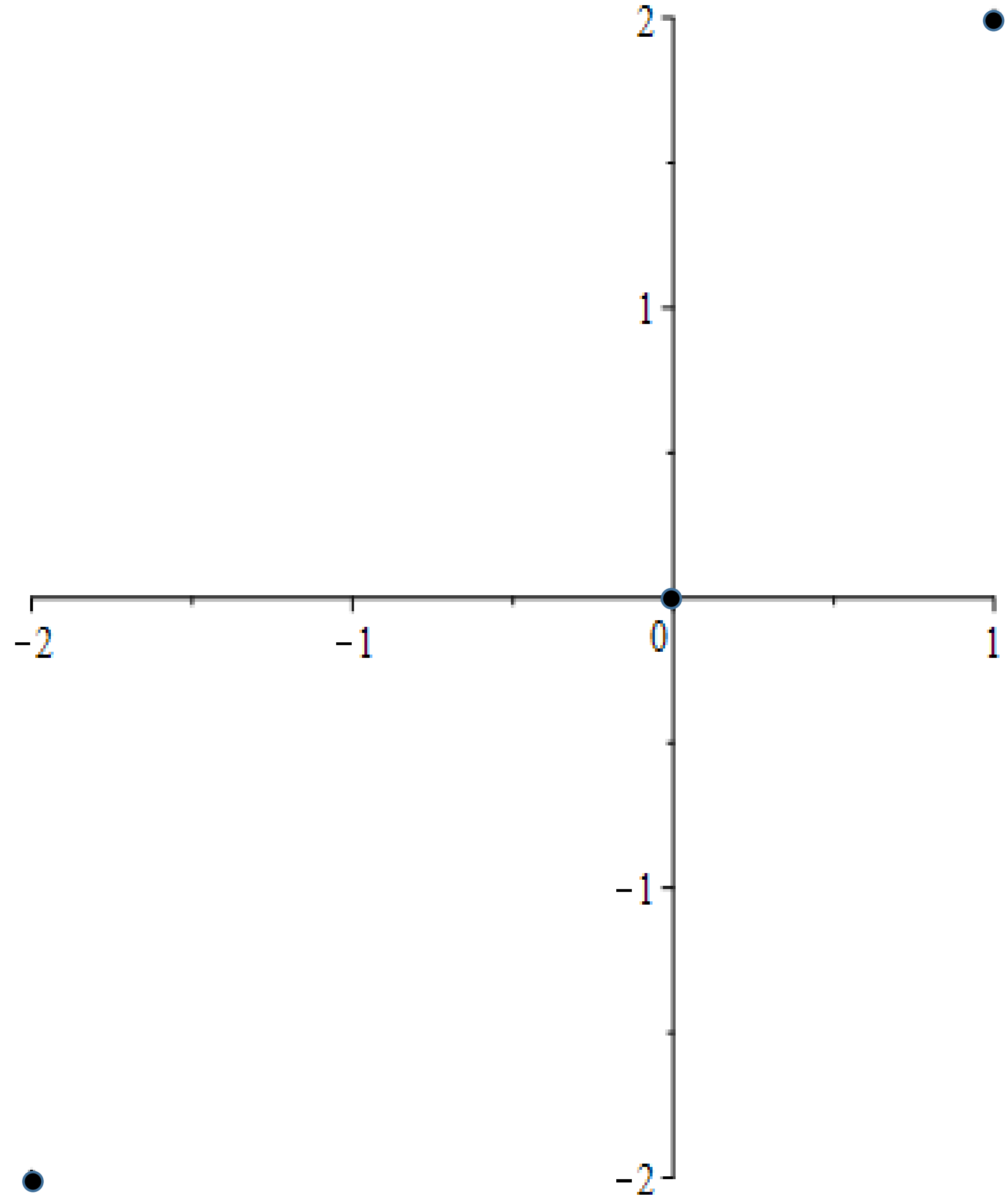
For $0 < c < 1$, the graph of $g(x) = cf(x)$ is the graph of $f(x)$ *compressed* toward the *x*-axis by a factor of *c*.

For a vertical stretch/compress, the non-zero *y*-coordinates change, but the *x*-coordinates remain the same.

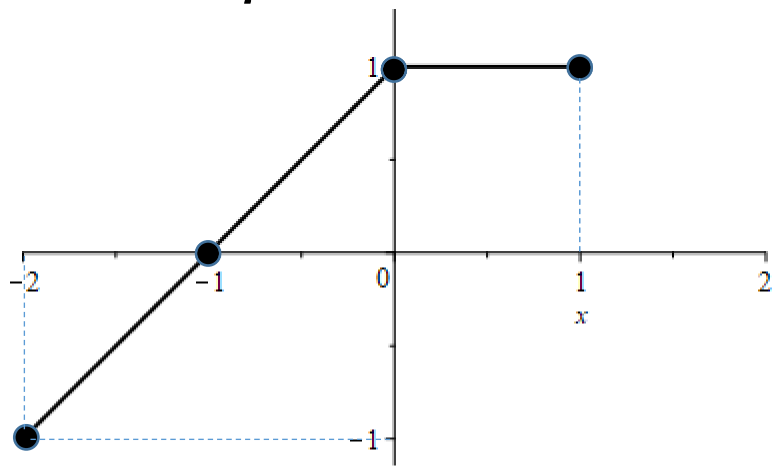
$f = \{(0,0),(1,1),(-2,-1)\}$

$g(x) = 2f(x)$, $g = \{(0,\boxed{0}),(1,\boxed{2}),(-2,\boxed{-2})\}$

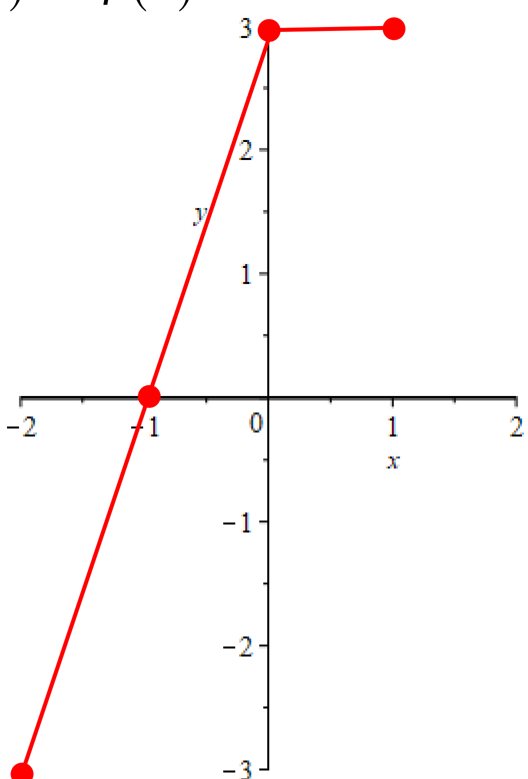




f

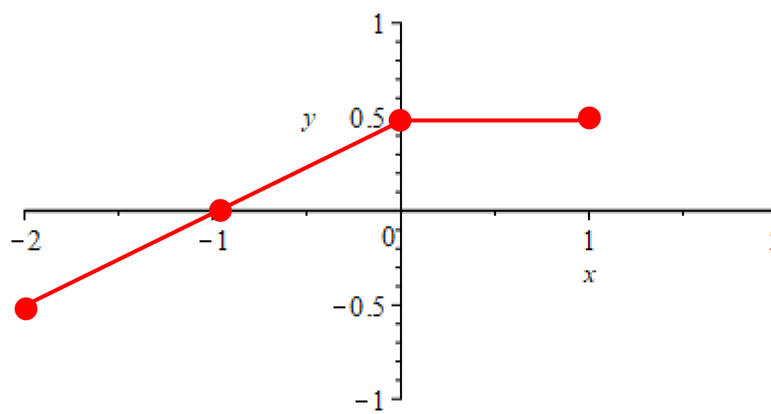


$g(x) = 3f(x)$



Stretched vertically by a factor of 3.

$h(x) = \frac{1}{2} f(x)$



Compressed vertically by a factor of $\frac{1}{2}$.

Horizontal Stretch/Compress:

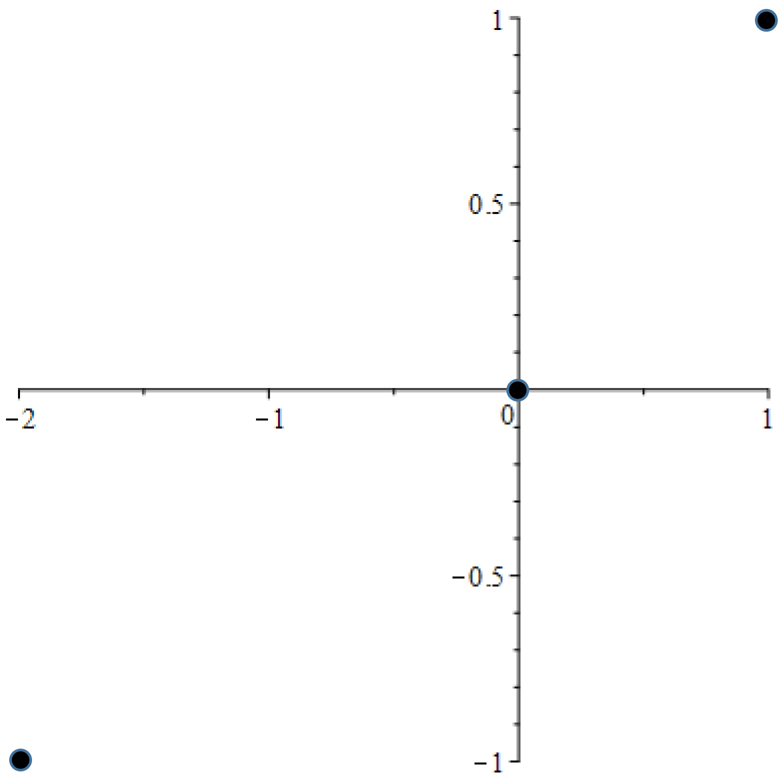
For $c > 1$**, the graph of** $g(x) = f(cx)$ **is the graph of** $f(x)$ ***compressed*** **toward the *y*-axis by a factor of** $\frac{1}{c}$**.**

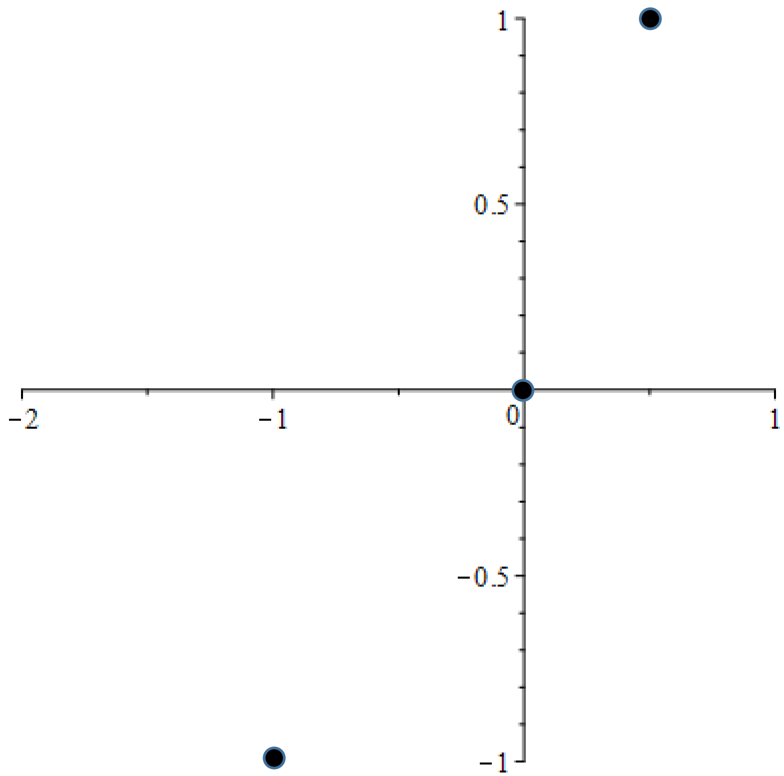
For $0 < c < 1$**, the graph of** $g(x) = f(cx)$ **is the graph of** $f(x)$ ***stretched*** **away from the *y*-axis by a factor of** $\frac{1}{c}$**.**

For a horizontal stretch/compress, the non-zero *x*-coordinates change, but the *y*-coordinates remain the same.

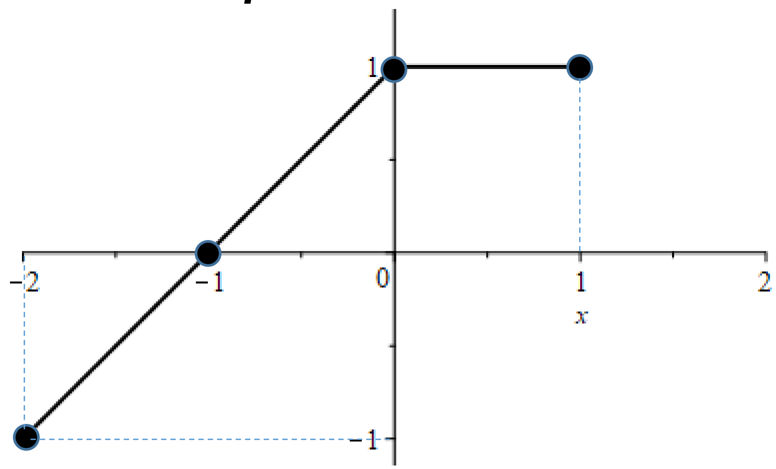
$f=\{(0,0),(1,1),(-2,-1)\}$

$g(x)=f(2x),\ g=\{(\boxed{0},0),(\boxed{\tfrac{1}{2}},1),(\boxed{-1},-1)\}$

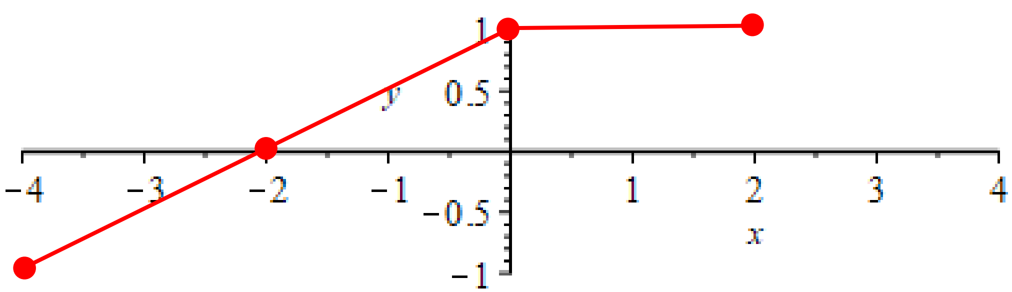




f

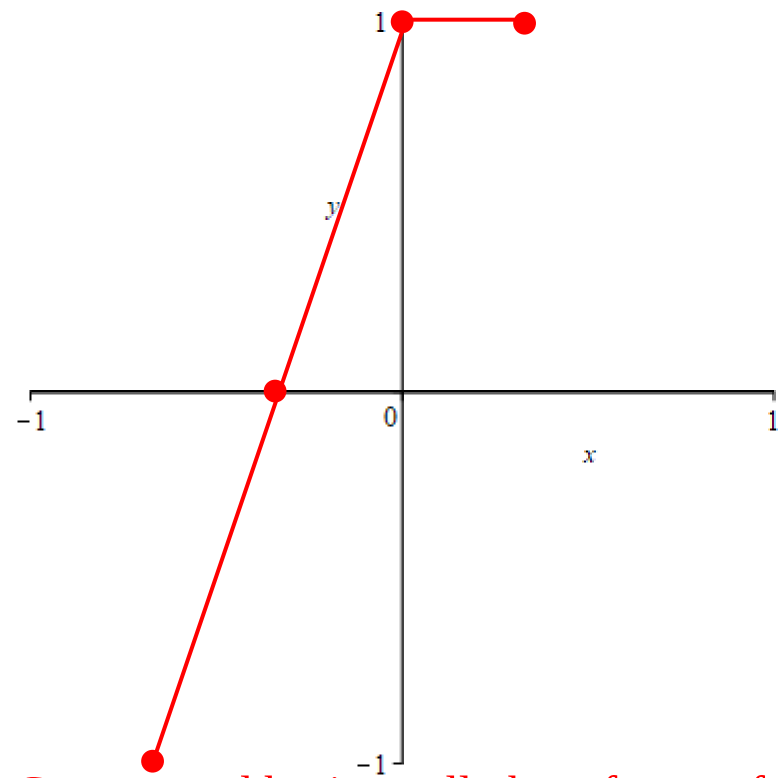


$g(x) = f\left(\frac{1}{2}x\right)$



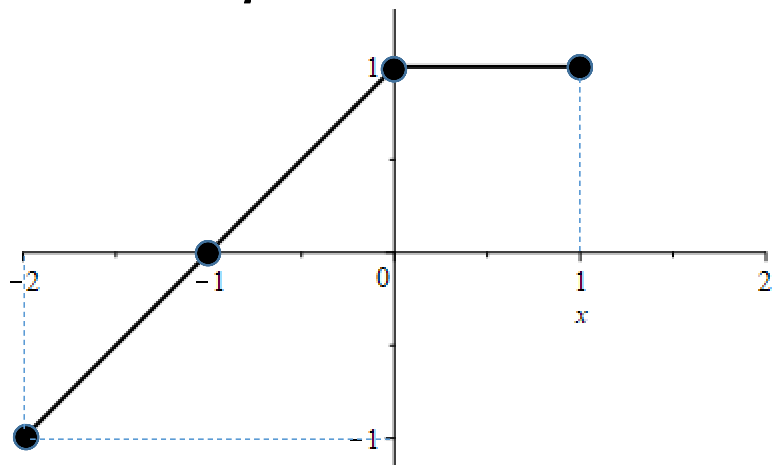
Stretched horizontally by a factor of 2.

$h(x) = f(3x)$

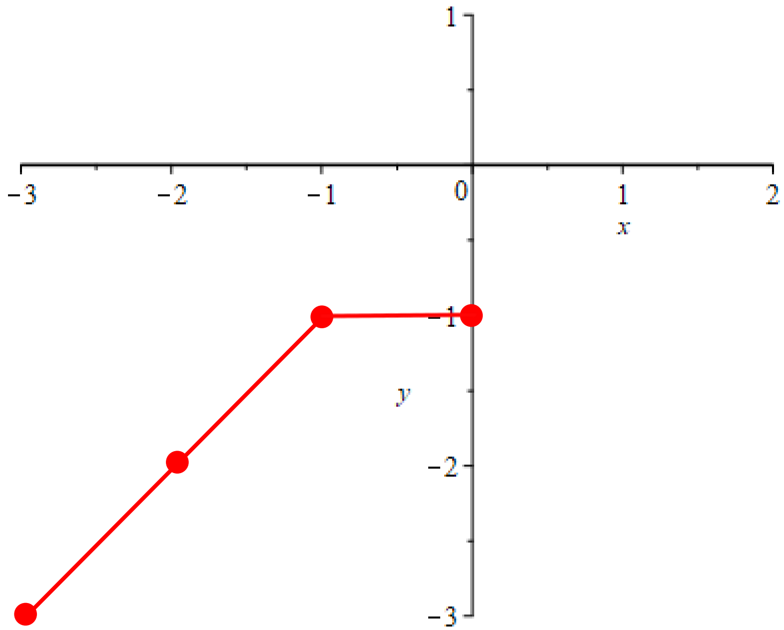


Compressed horizontally by a factor of $\frac{1}{3}$.

f

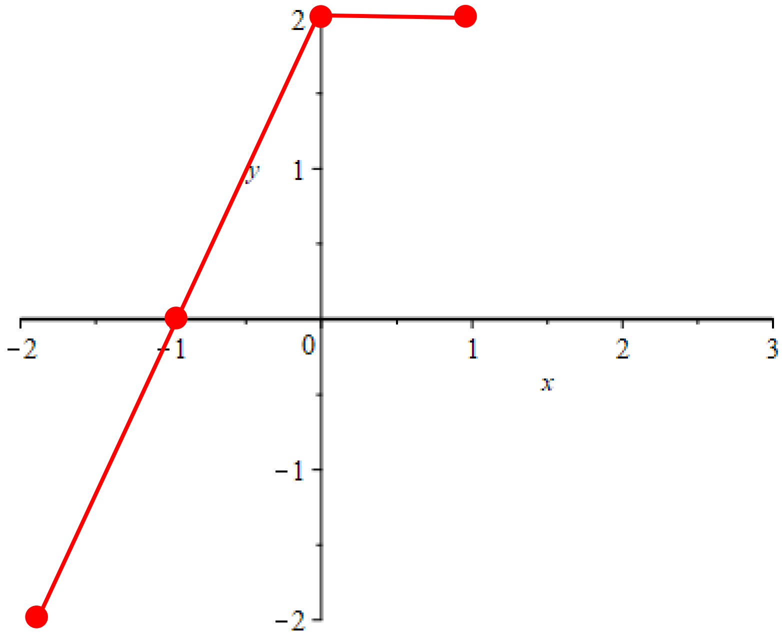


$g(x) = f(x+1) - 2$



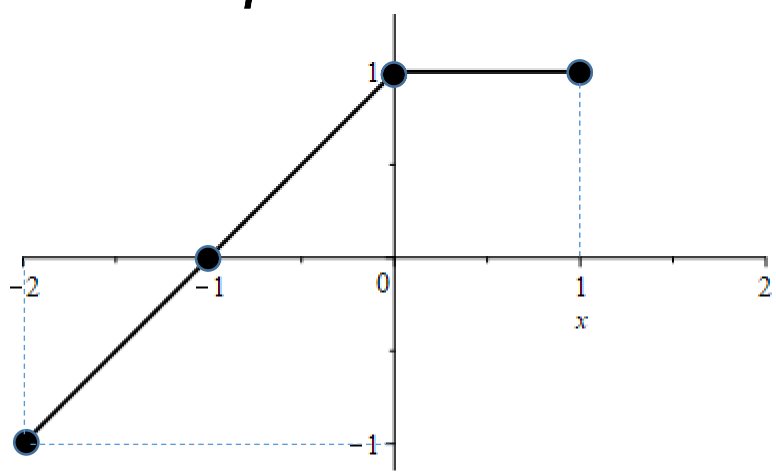
Shifted left 1 unit and down 2 units.

$h(x) = 2f(x-1)$

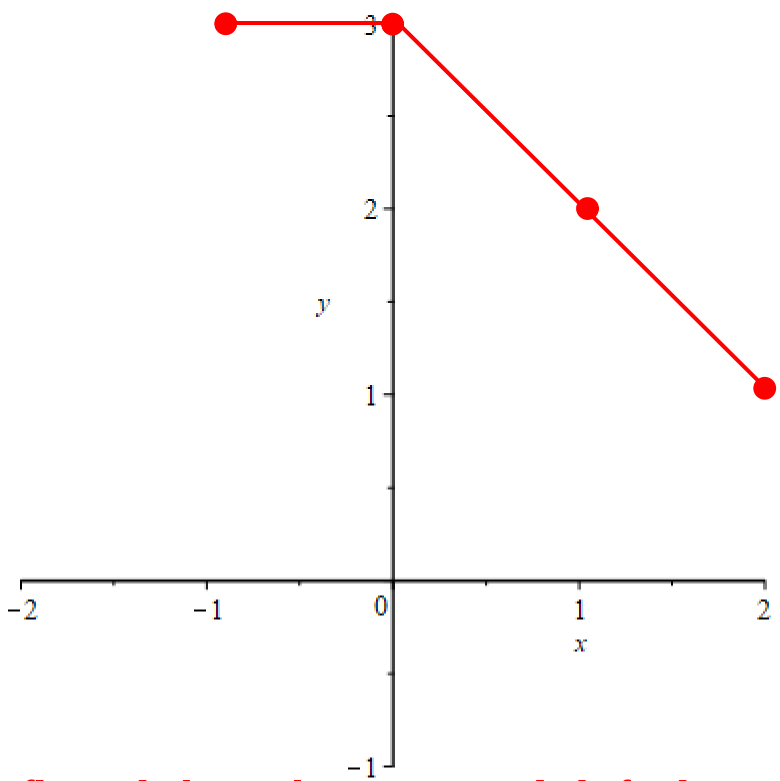


Shifted right 1 unit and stretched vertically by a factor of 2.

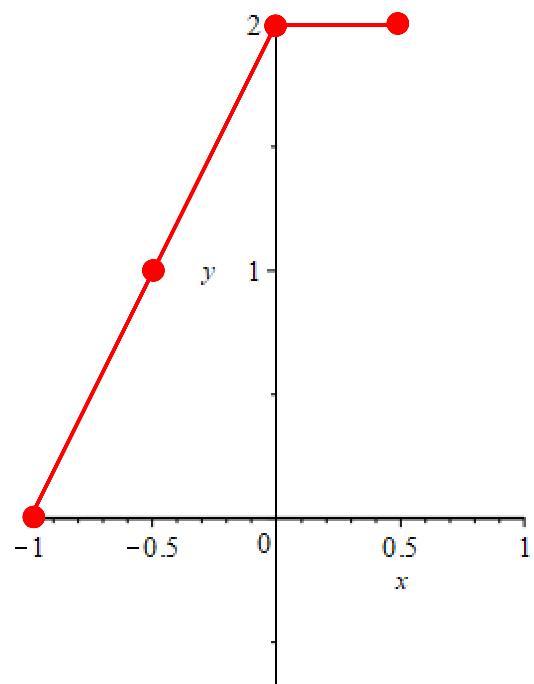
f



$g(x) = f(-x) + 2$



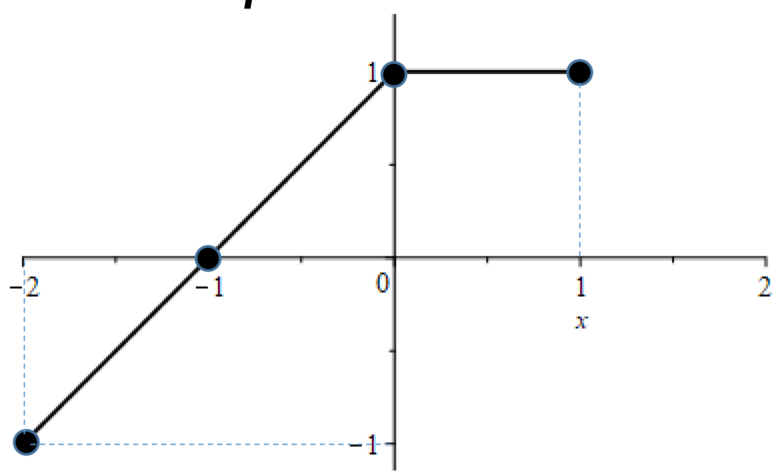
$h(x) = f(2x) + 1$



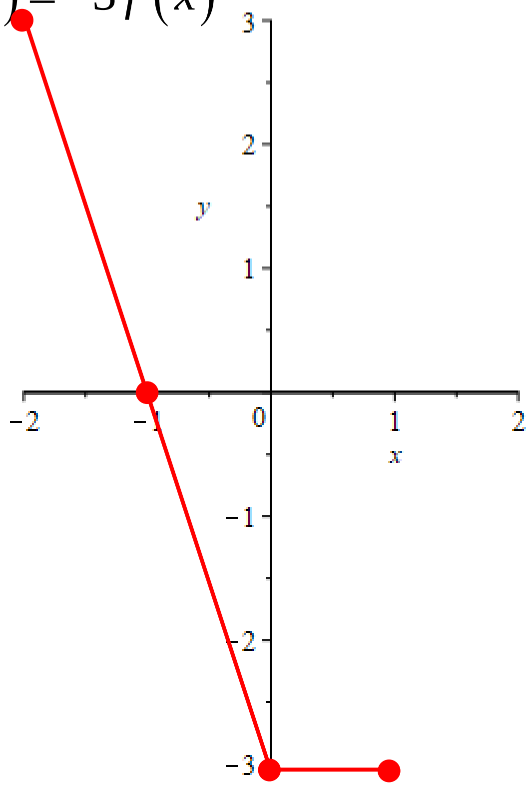
Reflected about the y-axis and shifted up 2 units.

Compressed horizontally by a factor of $\frac{1}{2}$ and shifted up 1 unit.

f

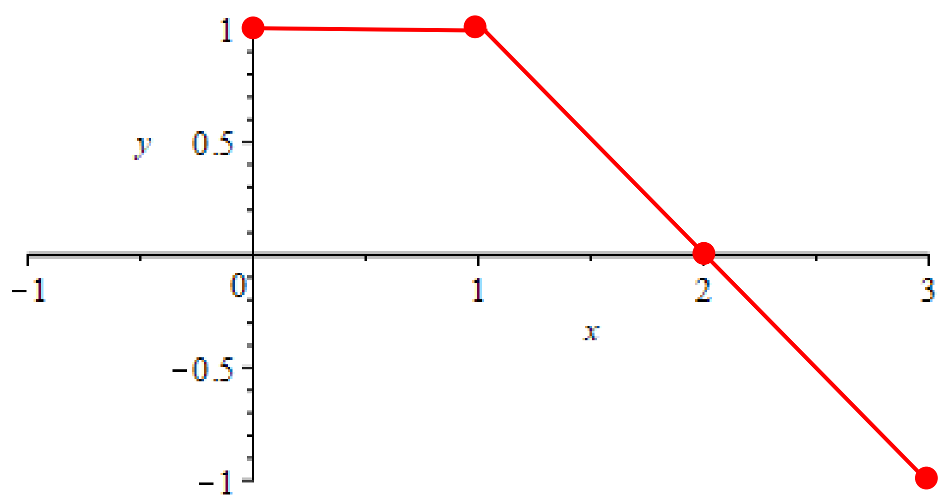


$g(x) = -3f(x)$



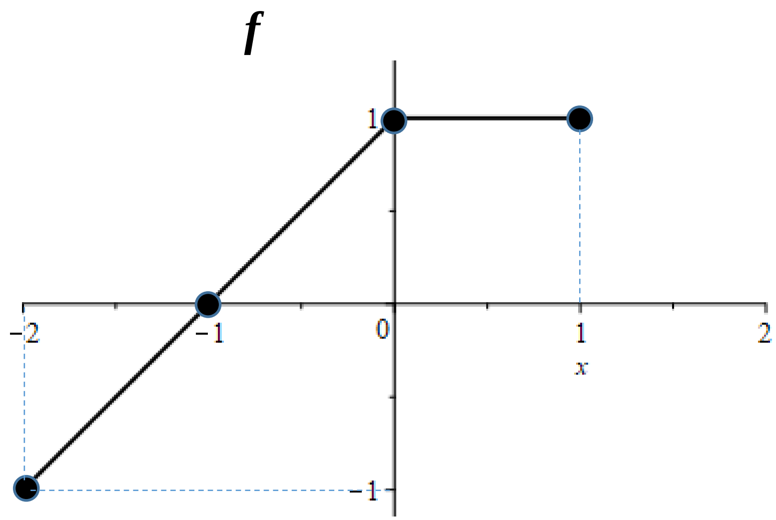
Stretched vertically by a factor of 3 and reflected about the *x*-axis.

$h(x) = f(1-x) = f(-x+1)$ or $f(-(x-1))$

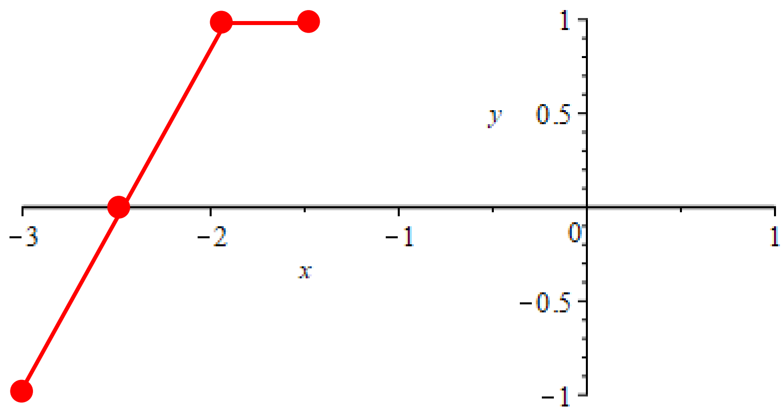


Shifted left 1 unit and reflected about the *y*-axis.

Or, reflected about the *y*-axis and shifted right 1 unit.



$g(x) = f(2x+4)$ or $f(2(x+2))$



Shift 4 units to the left and compress horizontally by factor of $\frac{1}{2}$. Or, compress horizontally by a factor of $\frac{1}{2}$ and shift 2 units to the left.