Basic Set Terminology:

A is the set of students registered for this class. *(Word Description method)*

$B = \{1, 2, 3, 4, 5\}$ *(Roster method)*

$C = \{2, 4, 6, 8\}$ *(Roster method)*

$D = \{x \mid x \text{ is a counting number less than } 6\}$ *(Set-builder notation)*

$$D = \{1, 2, 3, 4, 5\}$$

$E = \{x \mid x \text{ is an even counting number less than } 10\}$ *(Set-builder notation)*

$$E = \{2, 4, 6, 8\}$$

Convert $F = \{1, 2, 3, \ldots, 19\}$ **into set-builder notation.**

$$F = \{x \mid x \text{ is a counting number less than } 20\}$$

<u>There is a special set with no elements called the empty set.</u>

Notation: $\{\ \}$ **or** ϕ**.**

Sometimes the empty set is in disguise.

$$A = \{x \mid x \text{ is greater than 5 and less than 2}\}$$

$$A = \phi$$

There are no numbers that are greater than 5 and also less than 2!

<u>Set membership:</u>

$\in$ **means is a member or element of**

$\notin$ **means is not a member or element of**

Fill-in the blanks with either $\in$ or $\notin$.

$$3 \boxed{\in} \{3,5,7\}$$

$$6 \boxed{\notin} \{3,5,7\}$$

$$15 \boxed{\in} \{1,2,3,\ldots,20\}$$

$$3 \boxed{\notin} \{x \mid x \text{ is a counting number with } 4 \le x \le 9\}$$

$$8 \boxed{\notin} \phi$$

There's a special abbreviation for the Counting Numbers or Natural numbers:

$$\mathrm{N} = \{1,2,3,4,5,6,\ldots\}$$

Cardinal Number and Cardinality:

The cardinal number or cardinality of a set, *A*, is the number of elements in the set *A*.

Notation: $n(A)$

Determine the following cardinal numbers:

$$n(\{2,4,6,8\}) = 4$$

$$n(\{x \mid x \in \mathbb{N} \text{ with } 4 \le x \le 12\}) = 9$$

$$n(\{x \mid x \in \mathbb{N} \text{ with } x \le 4 \text{ and } x > 7\}) = 0$$

$$n(\{2,2,4,6,8\}) = 4$$

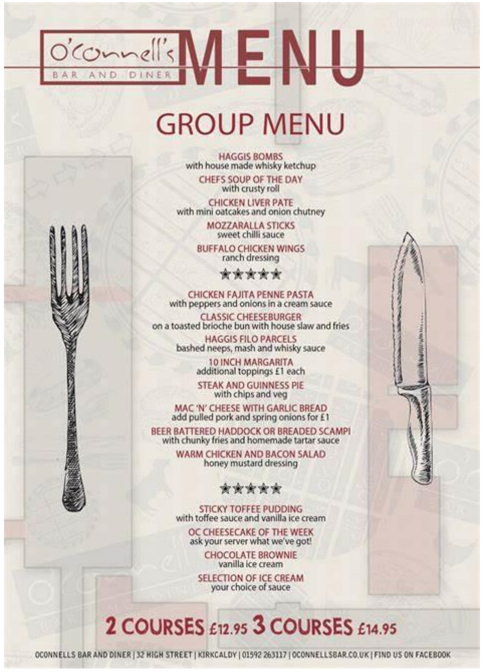
Duplicated items in the list of elements in a set are not counted as additional elements!

Subsets:

A set *A* is a subset of the set *B* if each element of *A* is also an element of *B*.

Notation: $A \subset B$ ***{****Think of B as a menu and a subset A as an order from the menu.}*

Fill-in the blanks with either $\subset$ or $\not\subset$.



$$\{3,7\} \boxed{\subset} \{3,5,7\}$$

$$\{3,6\} \boxed{\not\subset} \{3,5,7\}$$

$$\{4,5,6,\ldots,13\} \boxed{\subset} \{1,2,3,\ldots,20\}$$

$$\{1,2,3,4,5\} \boxed{\not\subset} \phi$$

$$\phi \boxed{\subset} \{a,b,c\}$$

$$\{1,2,3\} \boxed{\subset} \{1,2,3\}$$

$$\phi \boxed{\subset} \phi$$

If you're not hungry and you're not thirsty, then you don't order anything from the menu-i.e. the empty set is considered to be a subset of every set.

How many subsets does a set have?

Set A	$n(A)$	Subsets of A	# of subsets
ϕ	**0**	ϕ	**1**
$\{a\}$	**1**	$\phi, \{a\}$	**2**
$\{a,b\}$	**2**	$\phi, \{a\}, \{b\}, \{a,b\}$	4
$\{a,b,c\}$	**3**	$\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}$	8

Notice that each time a new element is added, you still have the previous subsets along with the new subsets generated by adding the new element to each of the previous subsets. This is why the number of subsets doubles.

Use inductive reasoning to complete the following:

If a set has *n* elements, then it has ______ 2^n ________ subsets.

How many subsets are there of the set $\{a, b, c, d, e\}$?

$$2^5 = \boxed{32}$$

A pizza can be ordered with some, none, or all of the following toppings:

$\{pepperoni, sausage, mushroom, onion, peppers, black\ olives, green\ olives, hamburger\}$.

How many different pizzas are possible?

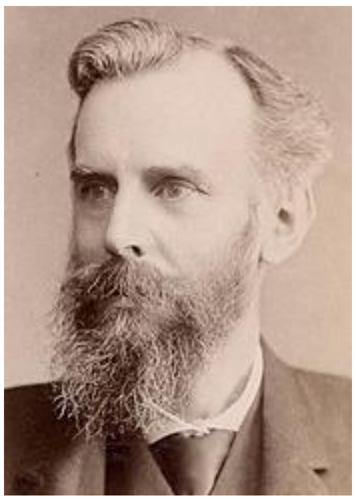
$$2^8 = \boxed{256}$$

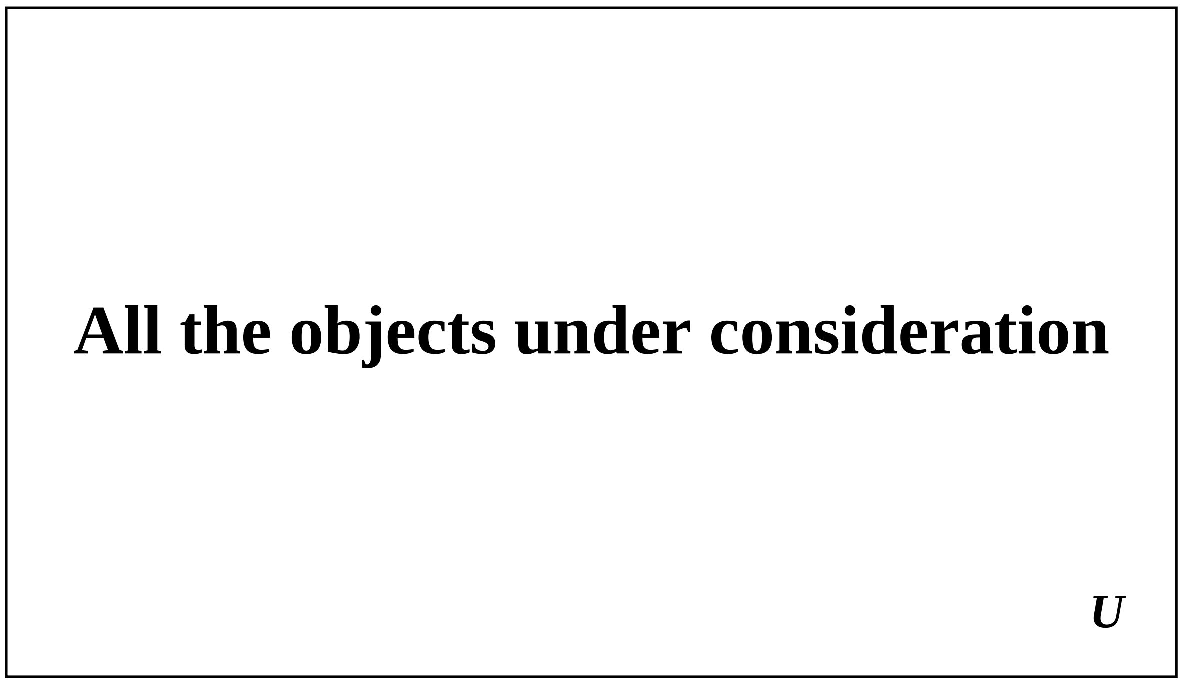
In this example, what would correspond to the empty set?

a cheese pizza!

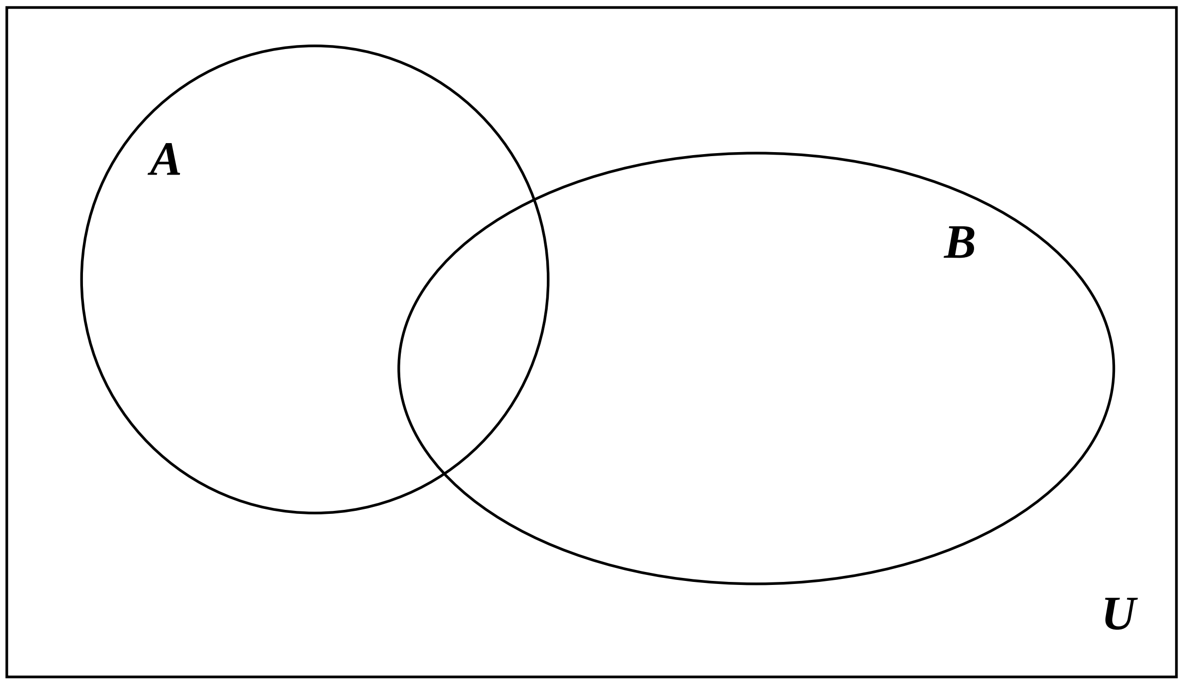
Set Operations and Venn Diagrams:

In a particular problem or situation, the set of all objects under consideration is called a universal set. It is abbreviated with the letter *U*, and represented in a Venn diagram as a large square or rectangle.





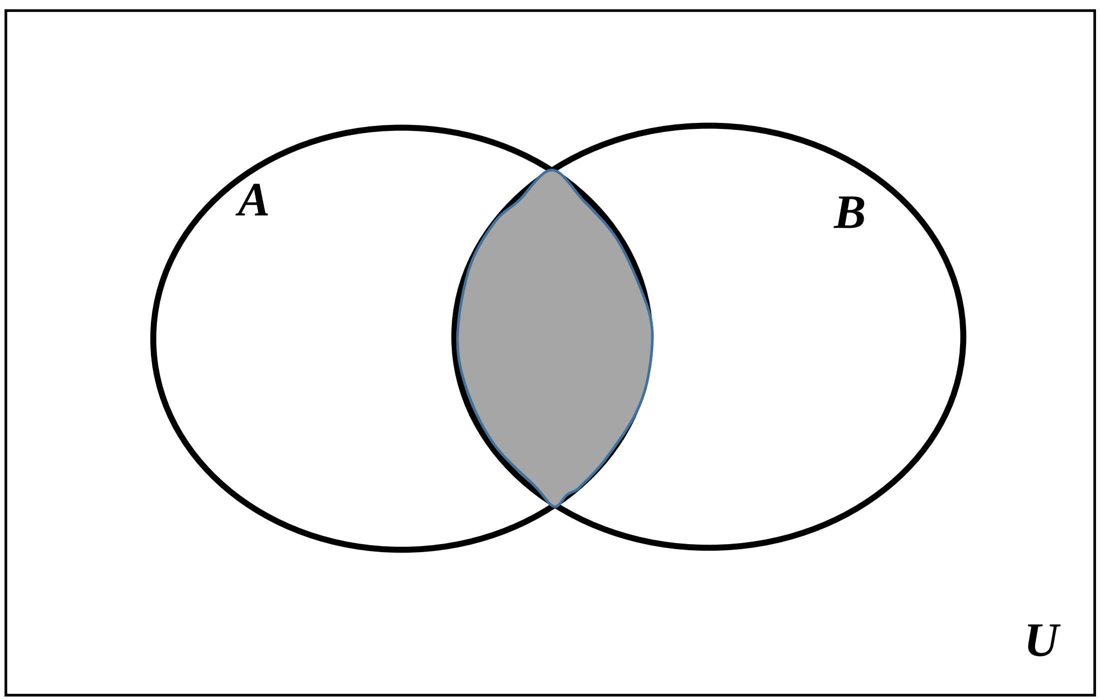
Sets of objects in a universal set are represented by circles or ovals.



Set Intersection:

The intersection of sets *A* and *B*, written as $A \bigcap B$, is the set of elements common to both set *A* *and* set *B*. In other words, it's the objects shared by the two sets.

$A \bigcap B$ is represented in a Venn diagram as the shaded region, the region of overlap of the two ovals.



Examples: $A=\{1,2,3,4,5,6\}$ $B=\{2,3,4,5,7\}$ $C=\{5,8\}$ $D=\{1,2,3\}$

List the elements in the following sets:

$A \cap B$

$\{2,3,4,5\}$

$B \cap C$

$\{5\}$

$A \cap C$

$\{5\}$

$C \cap D$

ϕ

$A \cap \phi$

ϕ

$A \cap B \cap C$

$\{5\}$

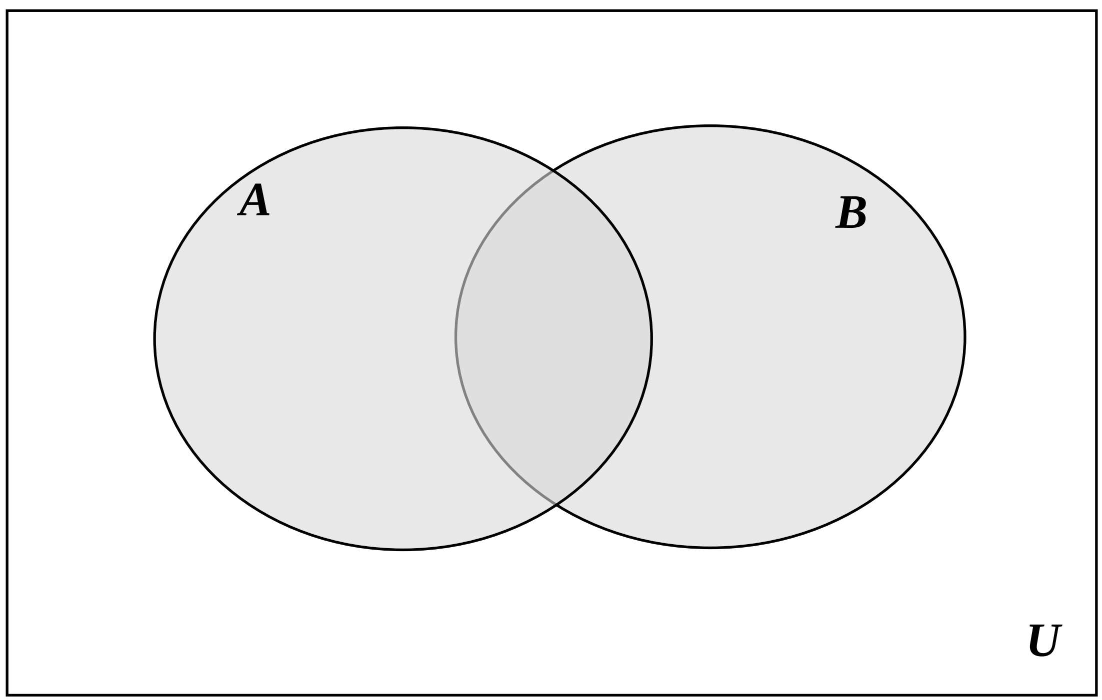
$C \cap C$

$\{5,8\}$

<u>Set Union:</u>

The union of sets *A* and *B*, written as $A\cup B$, is the set of elements that are in set *A* <u>*or*</u> in set *B*, <u>*or*</u> in both. In other words, it's the elements of both sets combined into one.

$A\cup B$ **is represented in a Venn diagram as the shaded region below. It's formed by joining the regions inside the ovals.**



Examples: $A=\{1,2,3,4,5,6\}$ $B=\{2,3,4,5,7\}$ $C=\{5,8\}$ $D=\{1,2,3\}$

List the elements in the following sets:

$A\cup B$

$\{1,2,3,4,5,6,7\}$

$C\cup D$

$\{1,2,3,5,8\}$

$\phi\cup D$

$\{1,2,3\}$

$A\cap(C\cup D)$

$\{1,2,3,5\}$

$A\cup B\cup C$

$\{1,2,3,4,5,6,7,8\}$

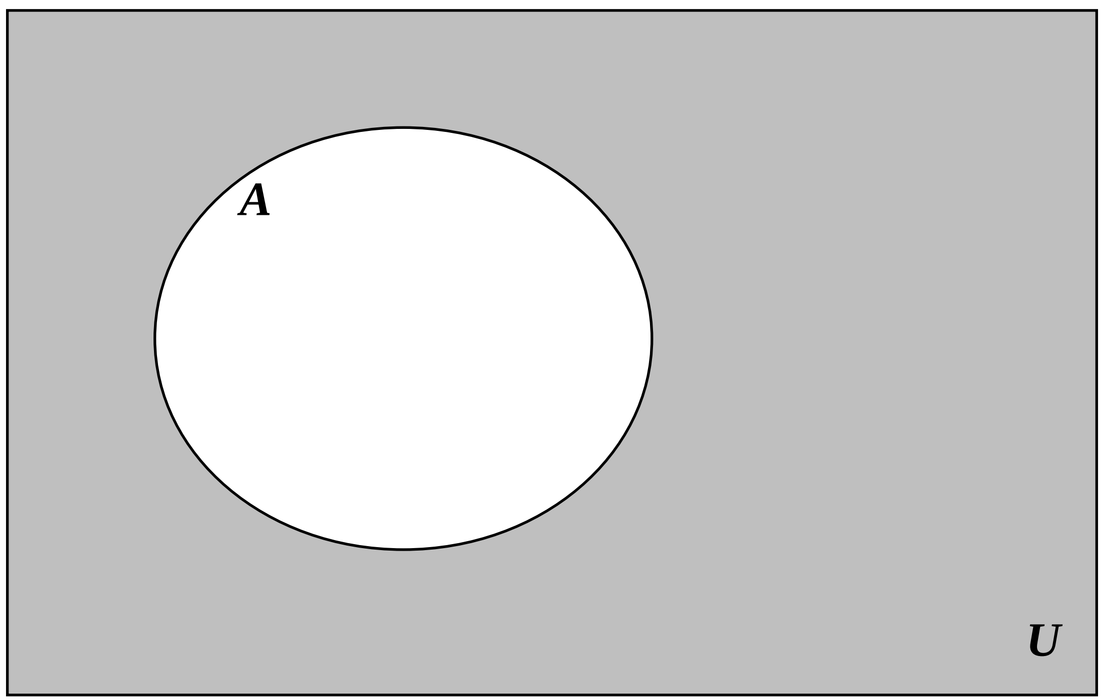
$(B\cap C)\cup(A\cap D)$

$\{1,2,3,5\}$

Set Complement:

The complement of the set *A*, written A', is the set of all objects in the universe that are *not* in the set *A*. In other words it's the opposite of *A*.

A' is represented in a Venn diagram as the shaded region below. It's the region outside of the oval.



Examples: $A=\{1,2,3,4,5,6\}$ $B=\{2,3,4,5,7\}$ $C=\{5,8\}$ $D=\{1,2,3\}$

$U=\{1,2,3,4,5,6,7,8\}$

List the elements in the following sets:

A'

$\{7,8\}$

B'

$\{1,6,8\}$

$(A\cap B)'$

$\{1,6,7,8\}$

$(A\cup B)'$

$\{8\}$

$B'\cap C$

$\{8\}$

$(A\cap B)'\cup C$

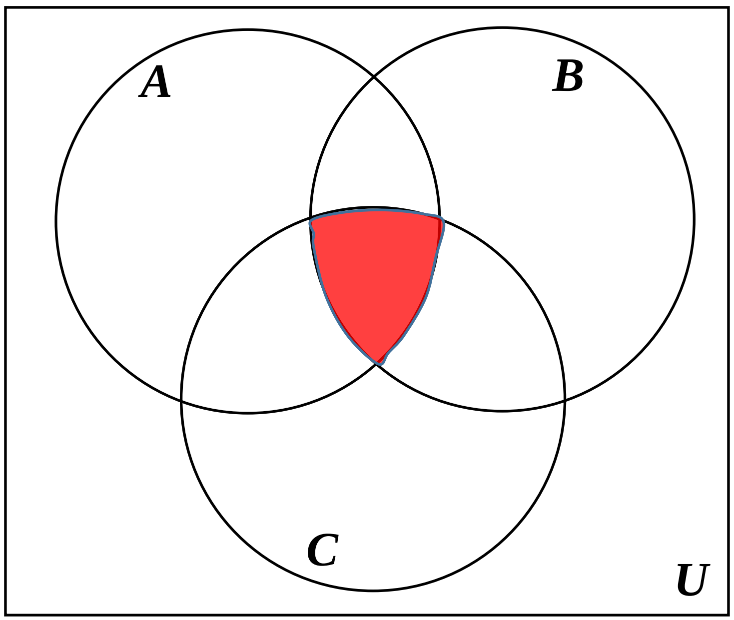
$\{1,5,6,7,8\}$

$A'\cap B'$

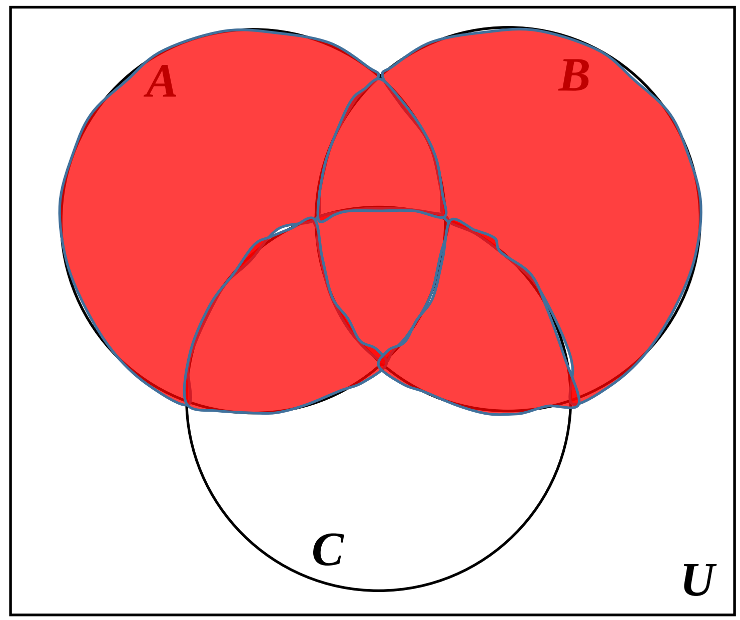
$A'\cup B'$

$\{8\}$ $\{1,6,7,8\}$

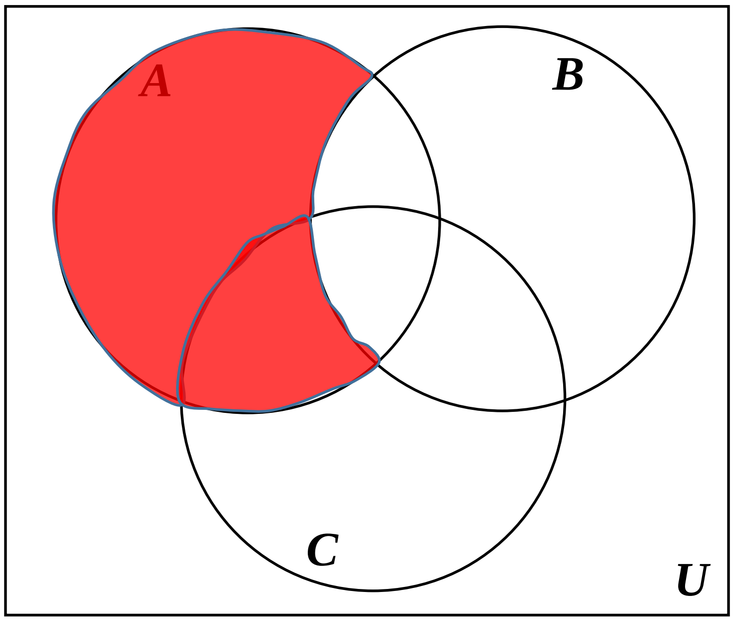
Shade the region(s) that is represented by the following set operations.



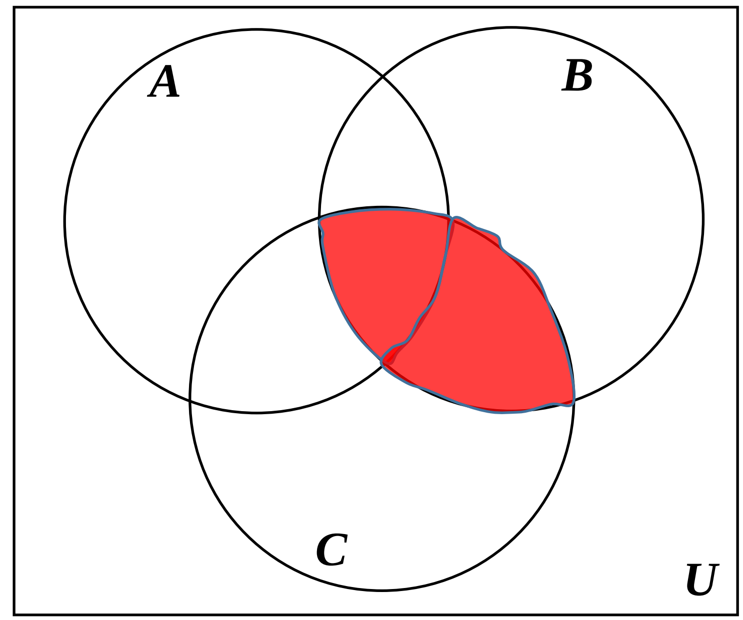
$A \cap B \cap C$



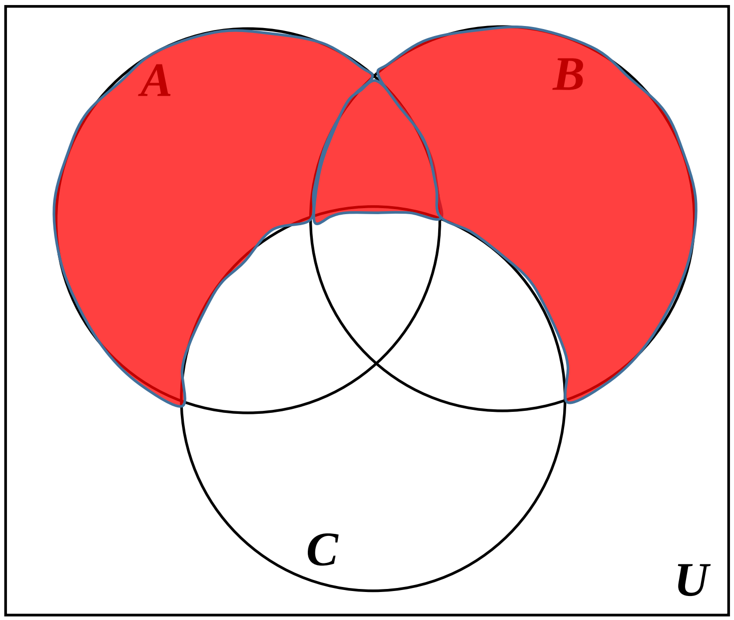
$A \cup B$



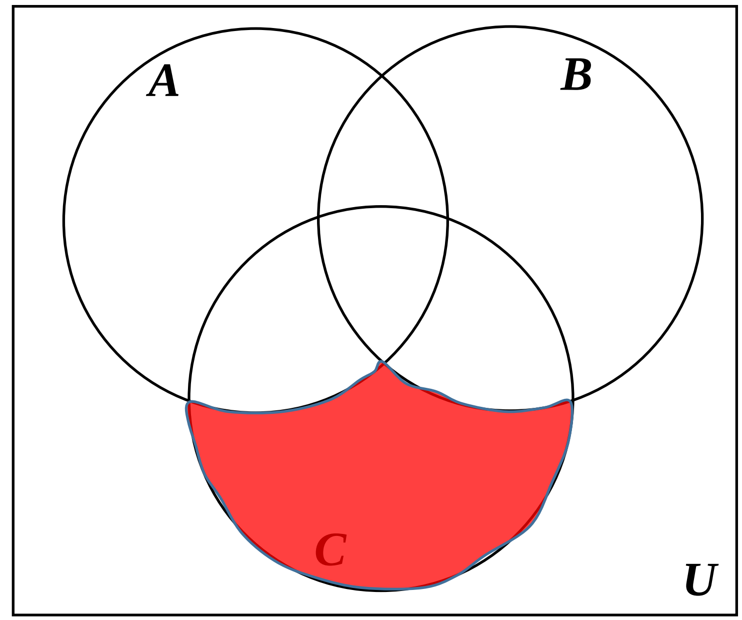
$A \bigcap B'$



$C \bigcap B$



$(A \cup B) \cap C'$



$C \cap (A \cup B)'$

<u>Analyzing Surveys Using Venn Diagrams:</u>

Examples:

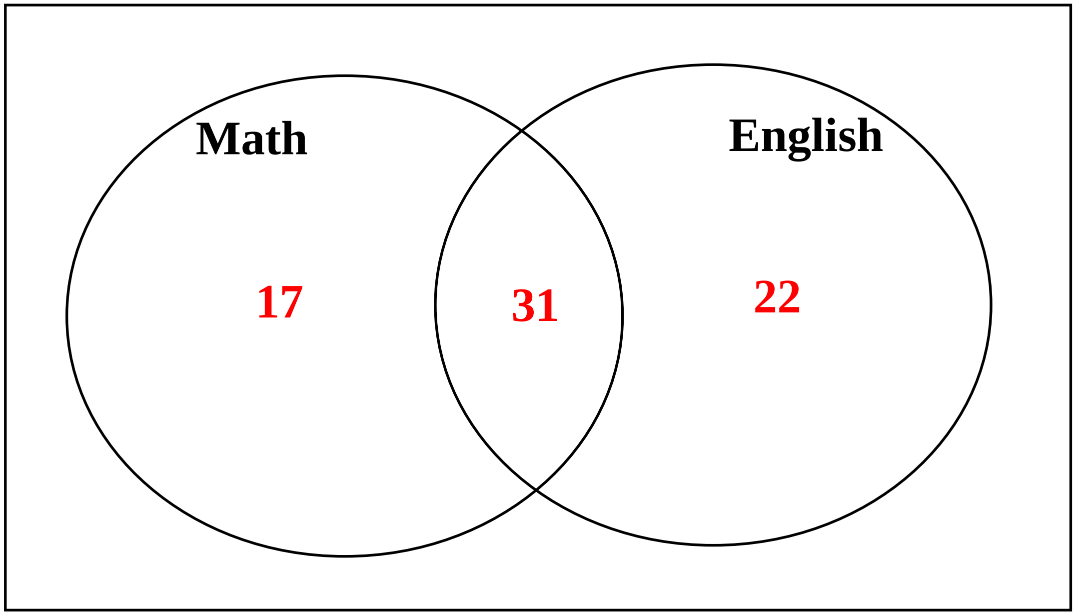
1. A survey of 100 students regarding their semester courses resulted in the following:

48 students are taking a Math class

53 students are taking an English class

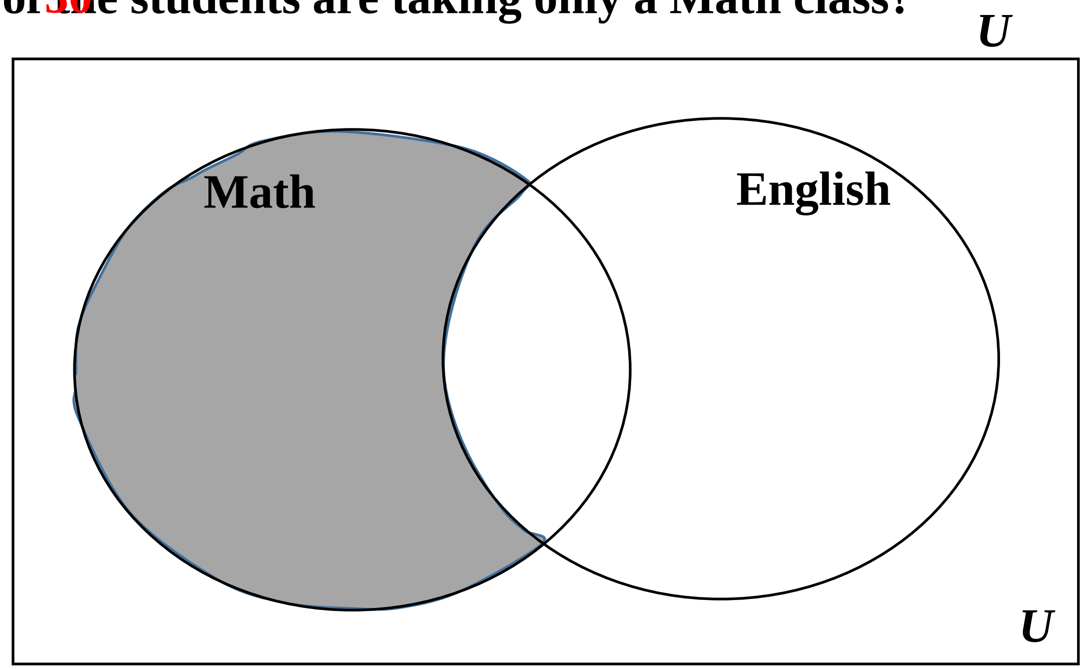
31 students are taking both Math and English classes

a) Complete the following Venn Diagram of the survey.

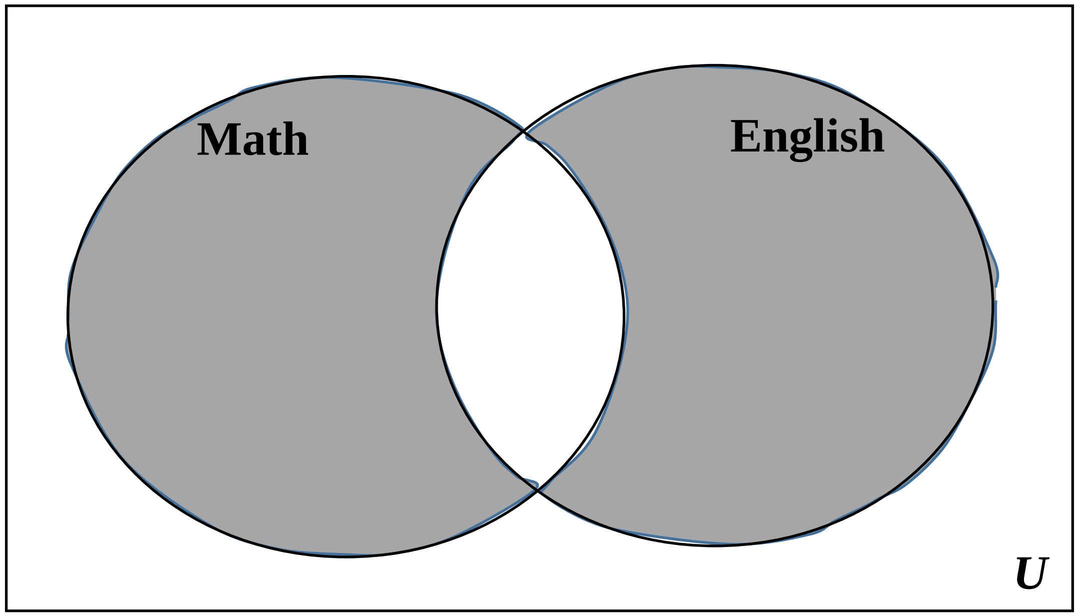


Fill-in the number at the bottom, 31, first. Then work your way up to the top. Be careful, 31 of the 53 students taking English are already accounted for, that's where the 22 comes from.

b) How many of 30 the students are taking only a Math class?

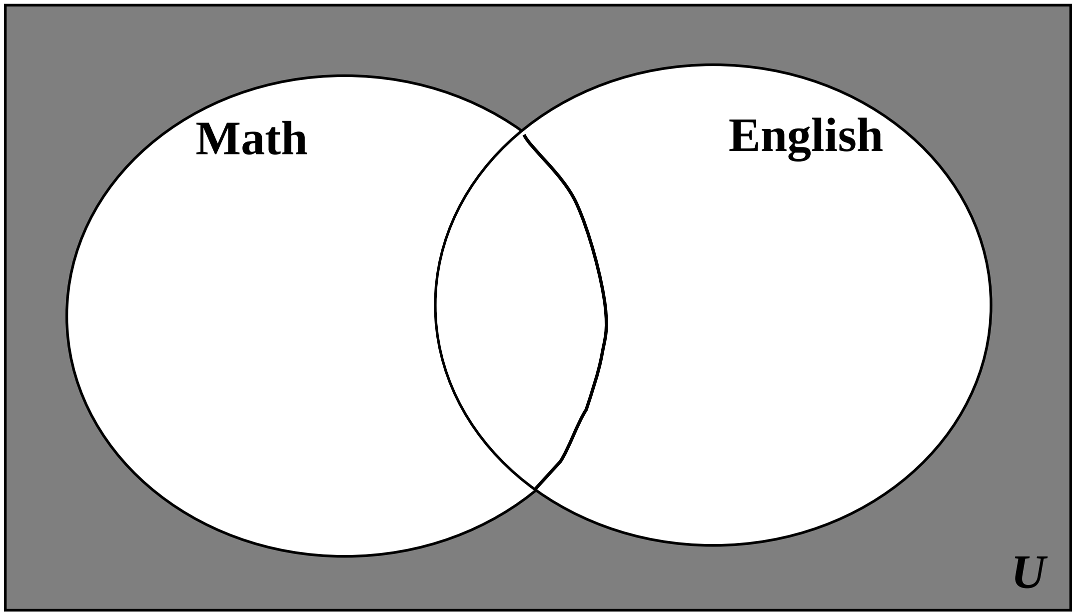


c) How many of the students are taking only one of the two classes?



17 + 22 = 39

d) How many of the students aren't taking a Math or English class?



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2. A survey of 180 students resulted in the following:

43 students were in a campus club

52 played in a campus sport

35 were in a campus tutorial program

13 were in a club and a sport

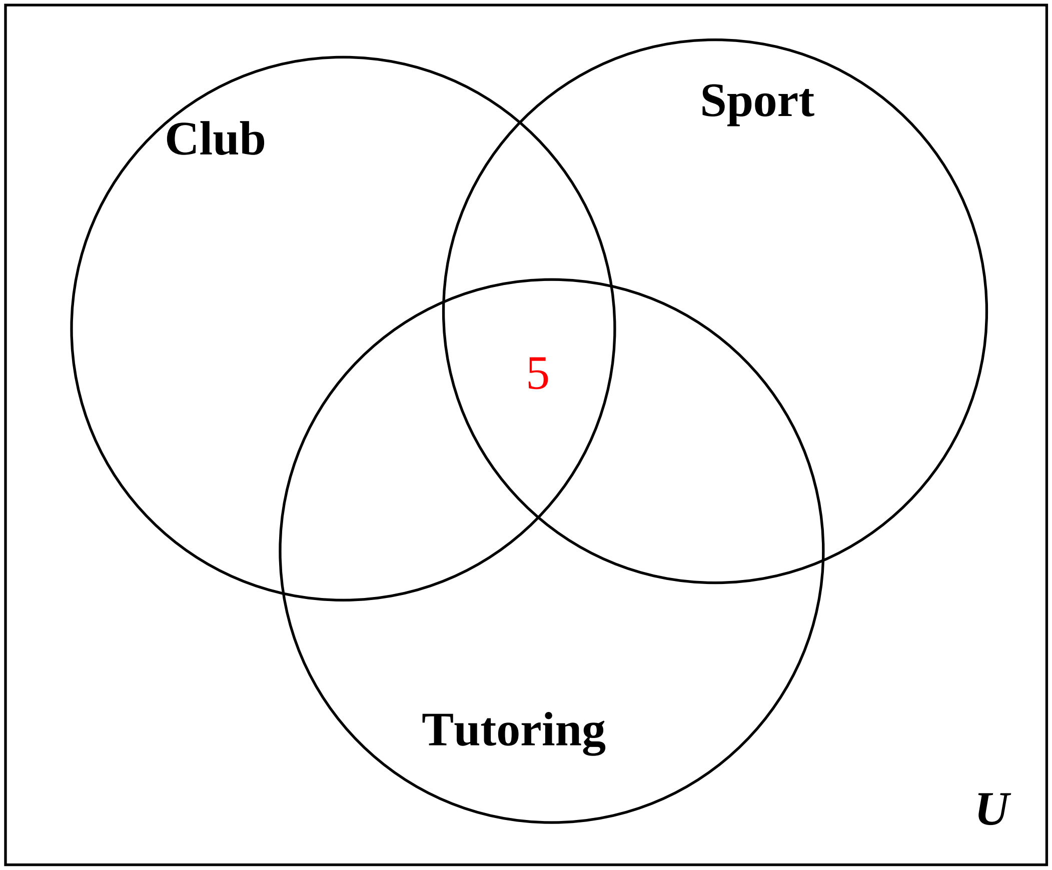
14 were in a sport and a tutorial program

12 were in a club and a tutorial program

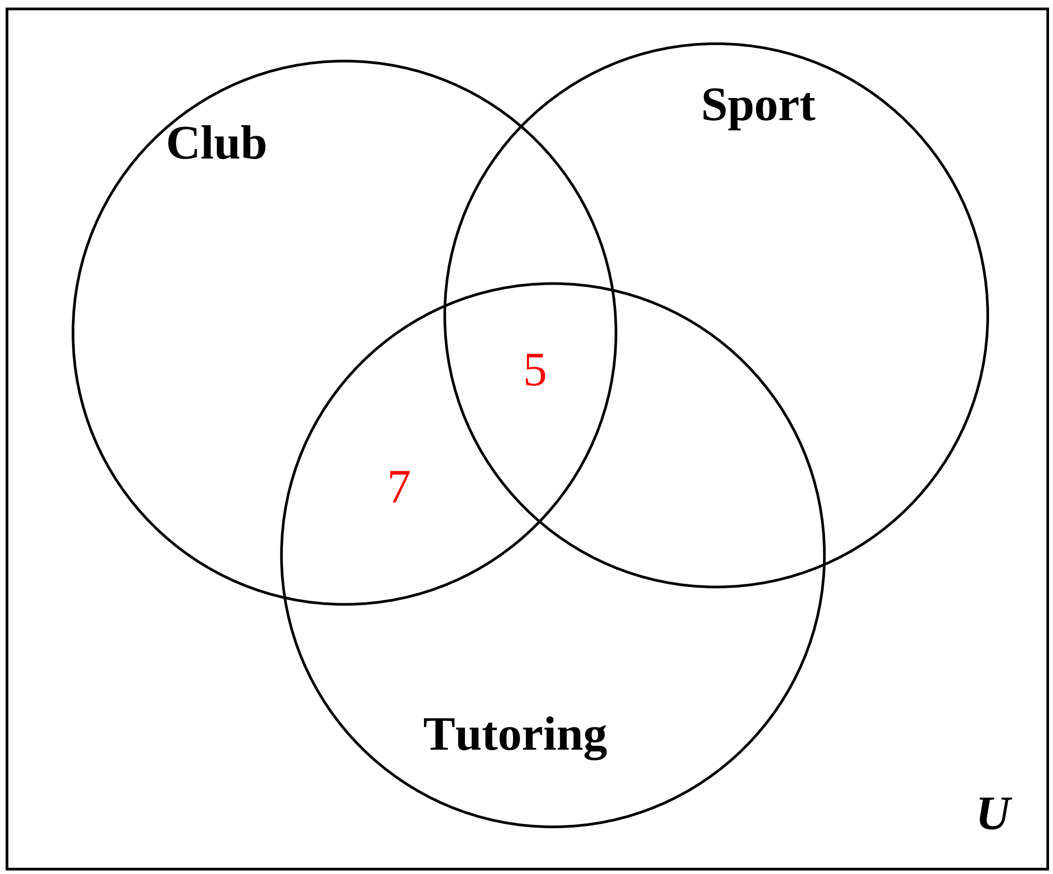
5 were in all three activities

a) Complete the following Venn Diagram of the survey.

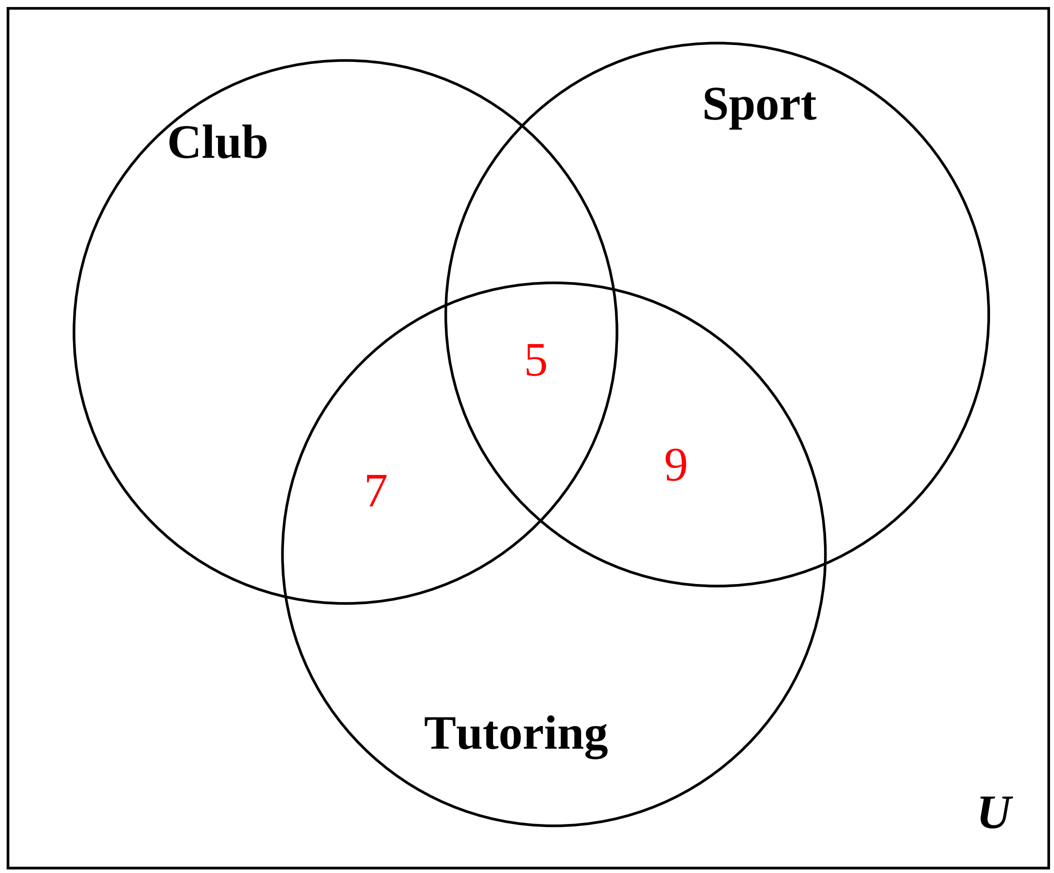
Start at the bottom with 5 were in all three activities, and put the 5 in the center region.



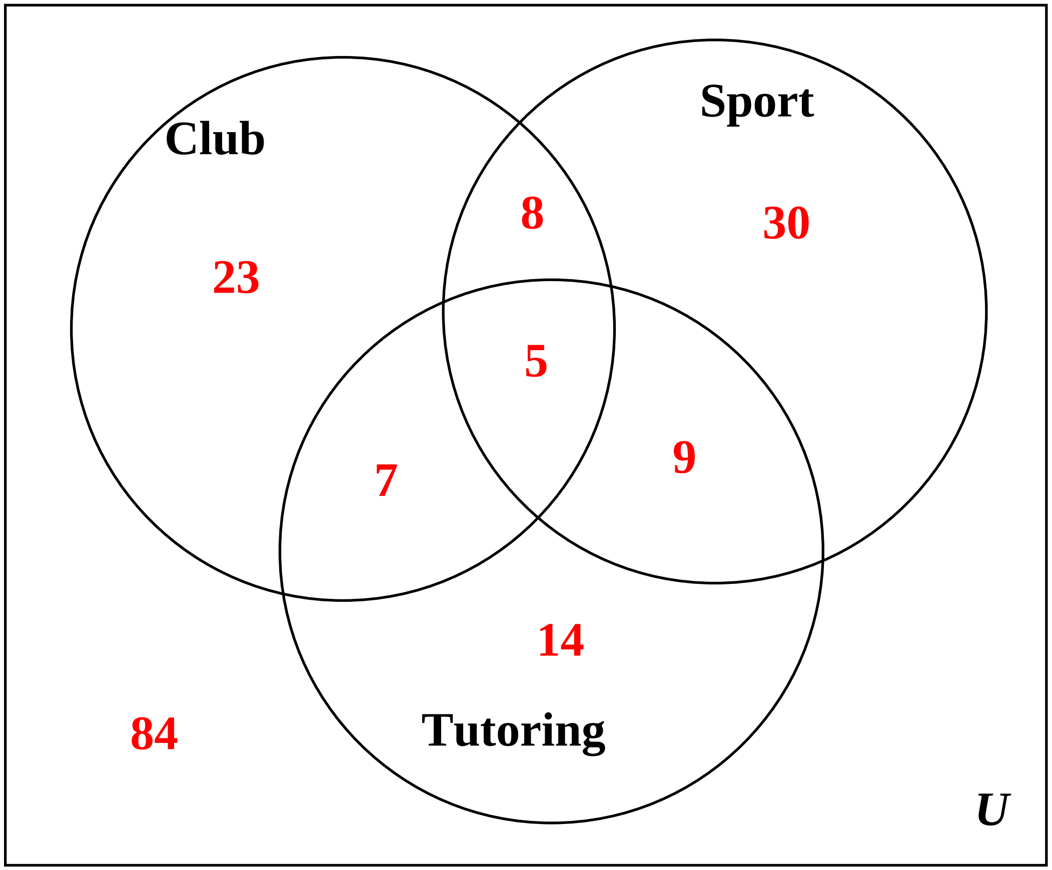
Now move up to 12 were in a club and a tutorial program. We know that the total number of people in the overlap region of club and tutorial is 12, and we've already accounted for 5 of them, we need to put the other 7 in the other part of the overlap region of club and tutorial.



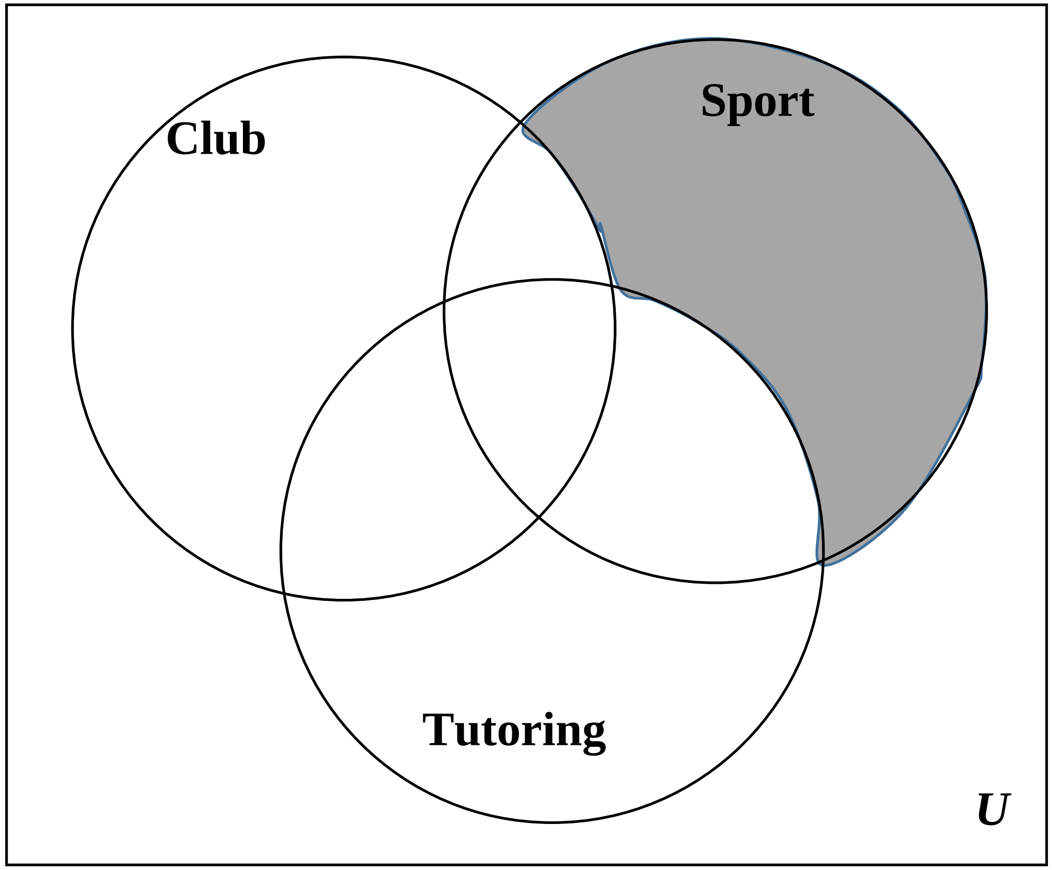
Now move up to 14 were in a sport and a tutorial program. We know that the total number of people in the overlap region of sport and tutorial is 14, and we've already accounted for 5 of them, we need to put the other 9 in the other part of the overlap region of sport and tutorial.



Continue this process of moving toward the beginning until you've used the fact that 180 students were involved in the survey, and you'll have a complete Venn diagram.



b) How many participated only in a sport?



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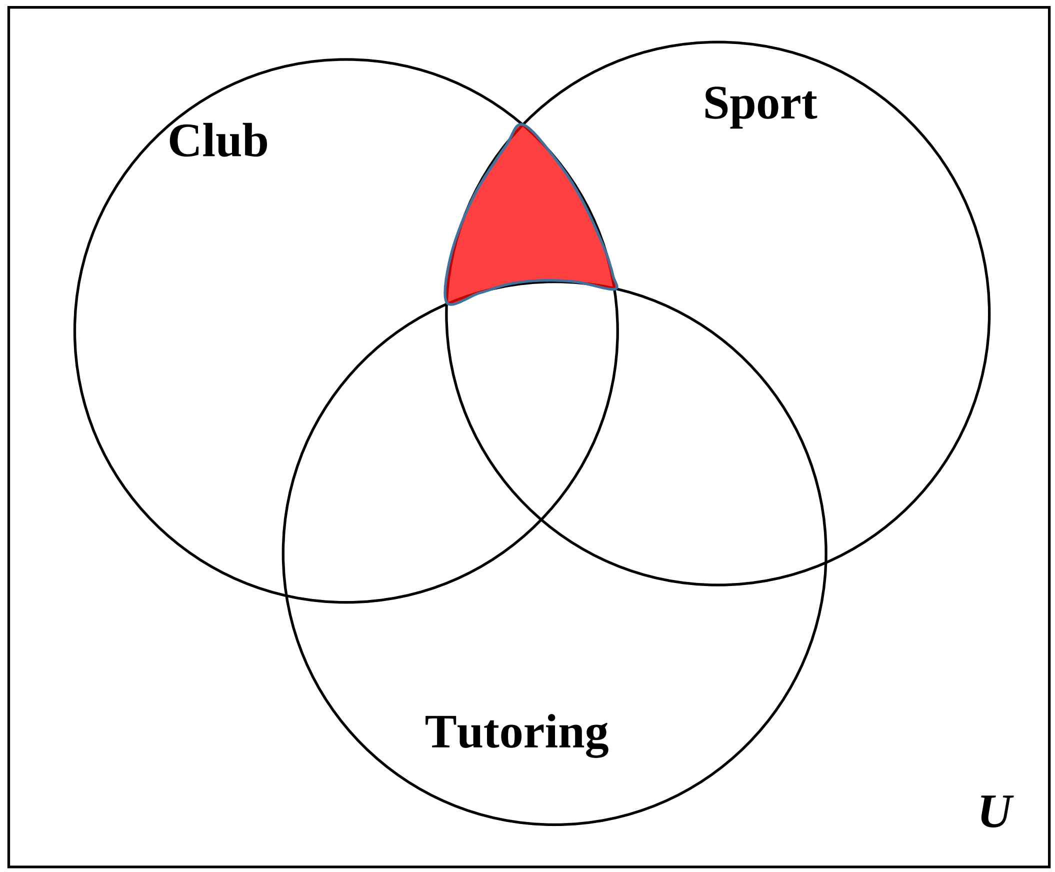
c) How many participated in a club and a sport, but not a tutoring program?



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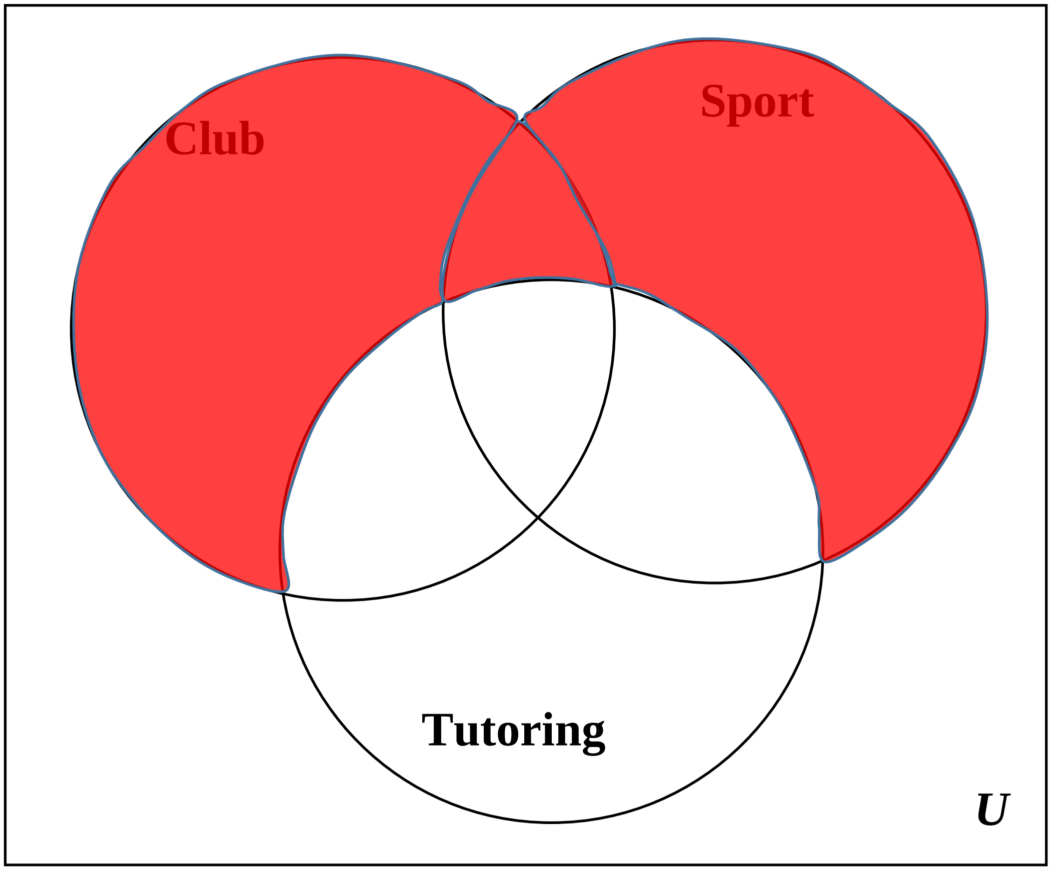
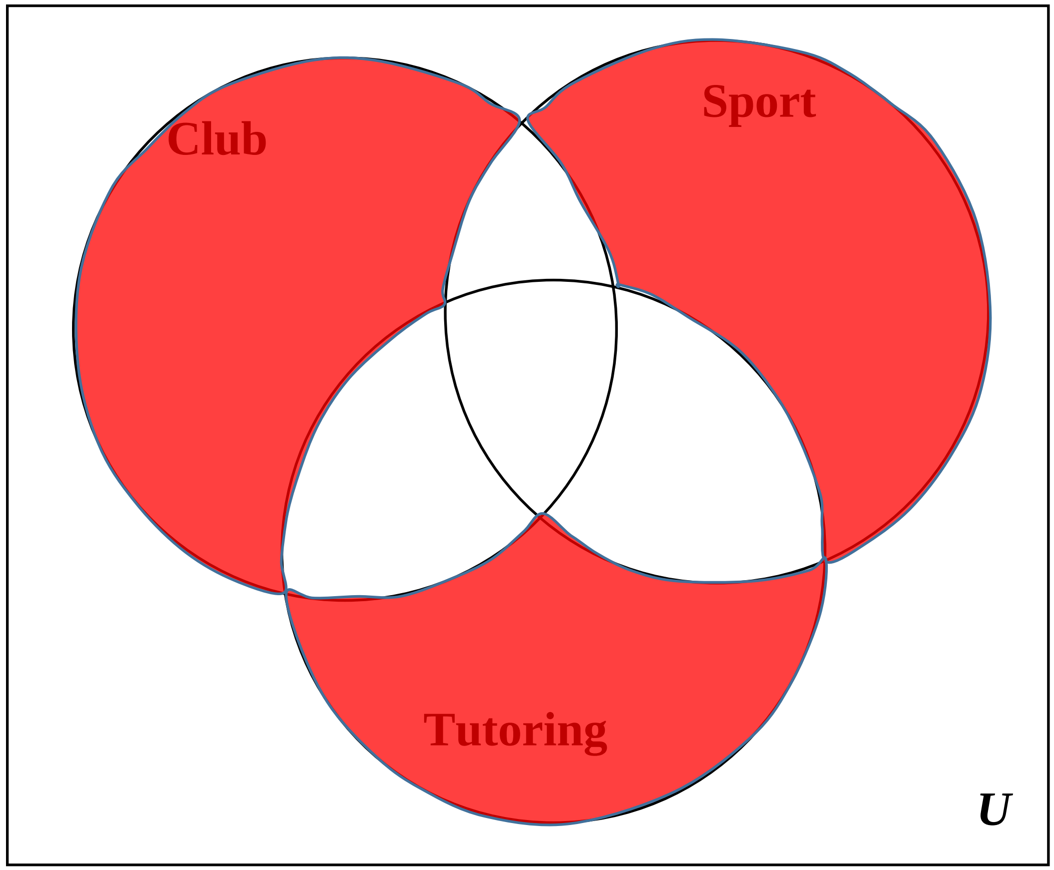
d) How many participated in a club or a sport, but not a tutoring program?



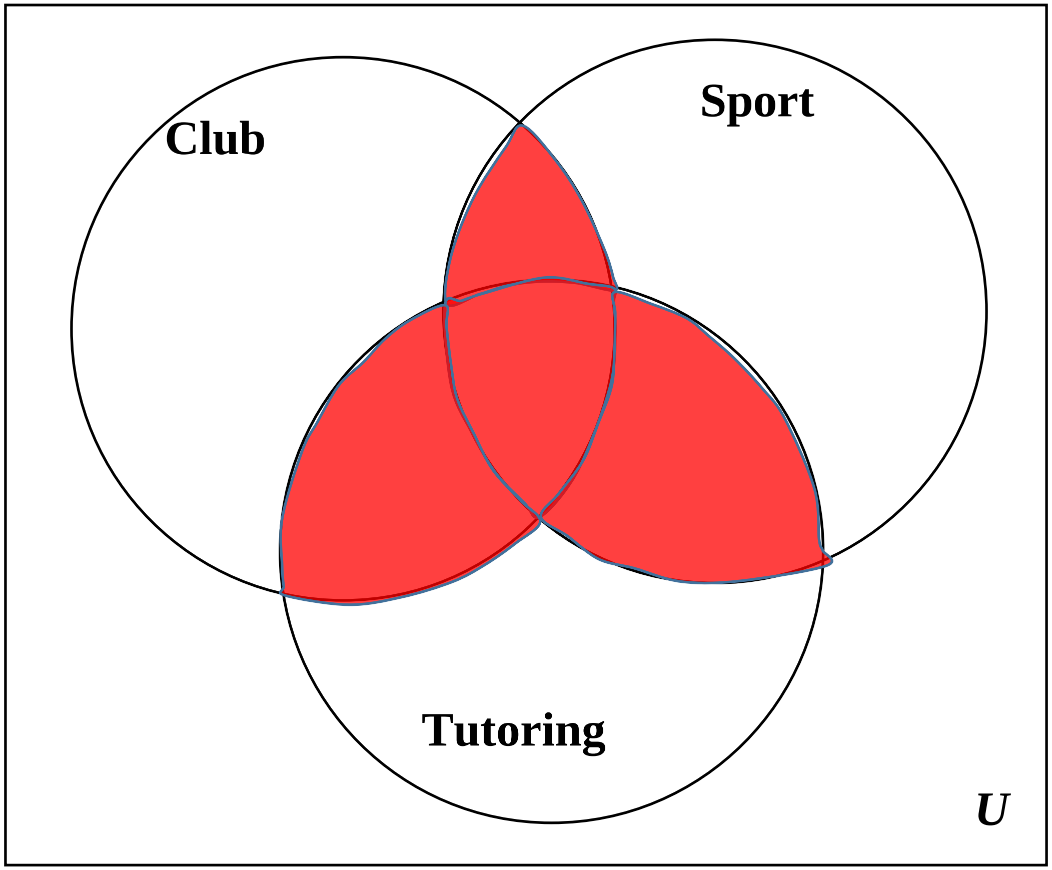
23 + 8 + 30 = 61

e) How many participated in exactly one of the three activities?



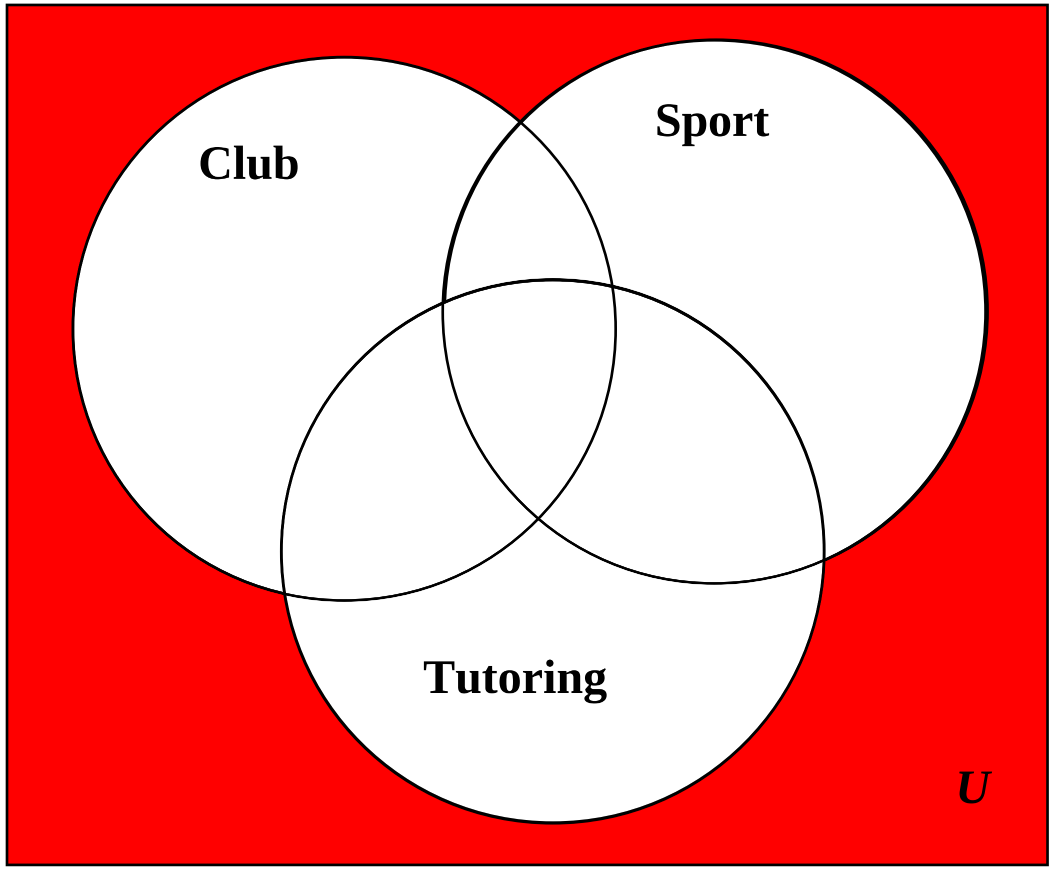
23 + 30 + 14 = 67

f) How many participated in at least two of the activities?



7 + 8 + 5 + 9 = 29

g) How many didn't participate in any of the three activities?



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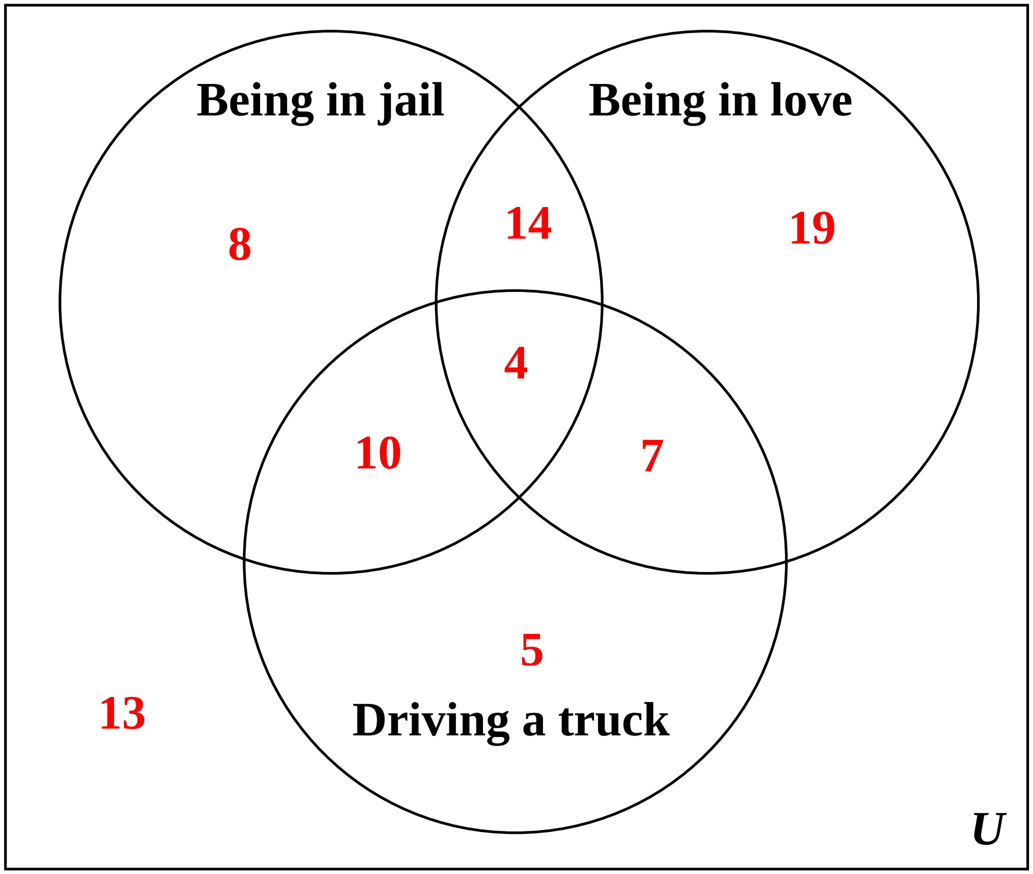
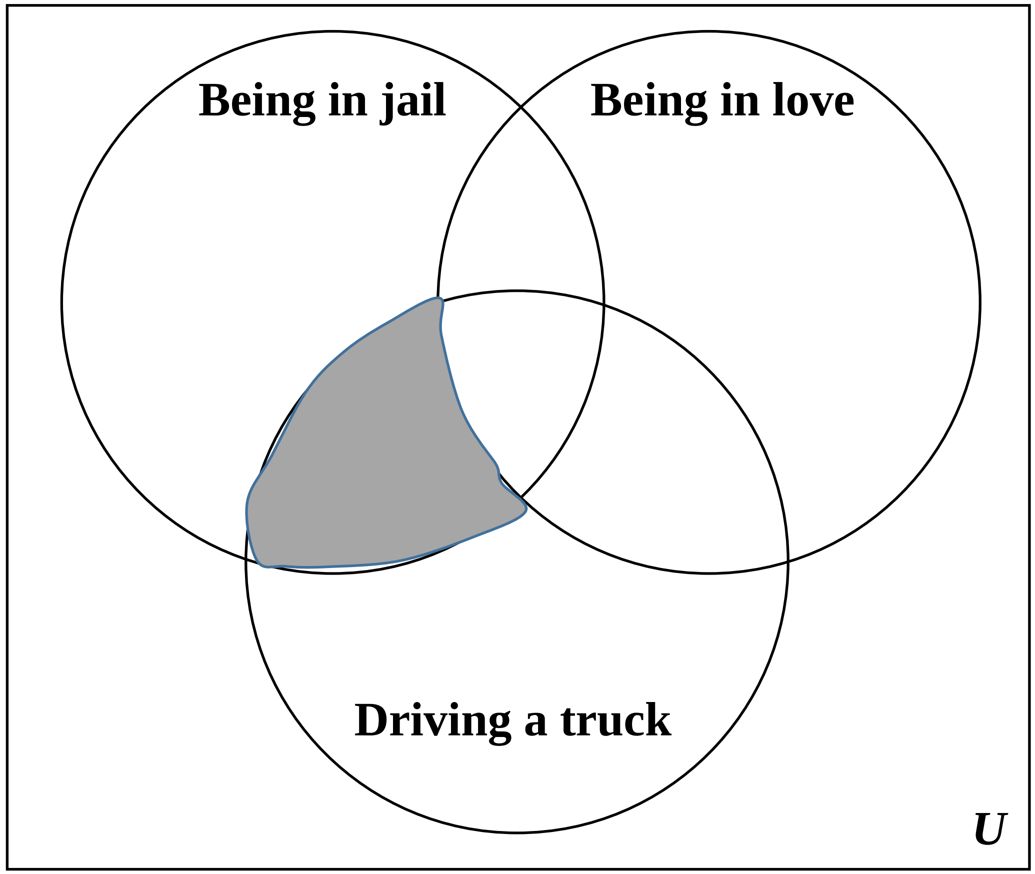
3. **A survey of 80 country music songs resulted in the following: 36 songs are about being in jail, 44 songs are about being in love, 26 songs are about driving a truck, 18 songs are about being in jail and being in love, 14 songs are about being in jail and driving a truck, 11 songs are about being in love and driving a truck, and 4 songs are about being in jail and being in love and driving a truck.**

a) Complete the following Venn Diagram.



b) How many songs are about being in jail and driving a truck, but not being in love?



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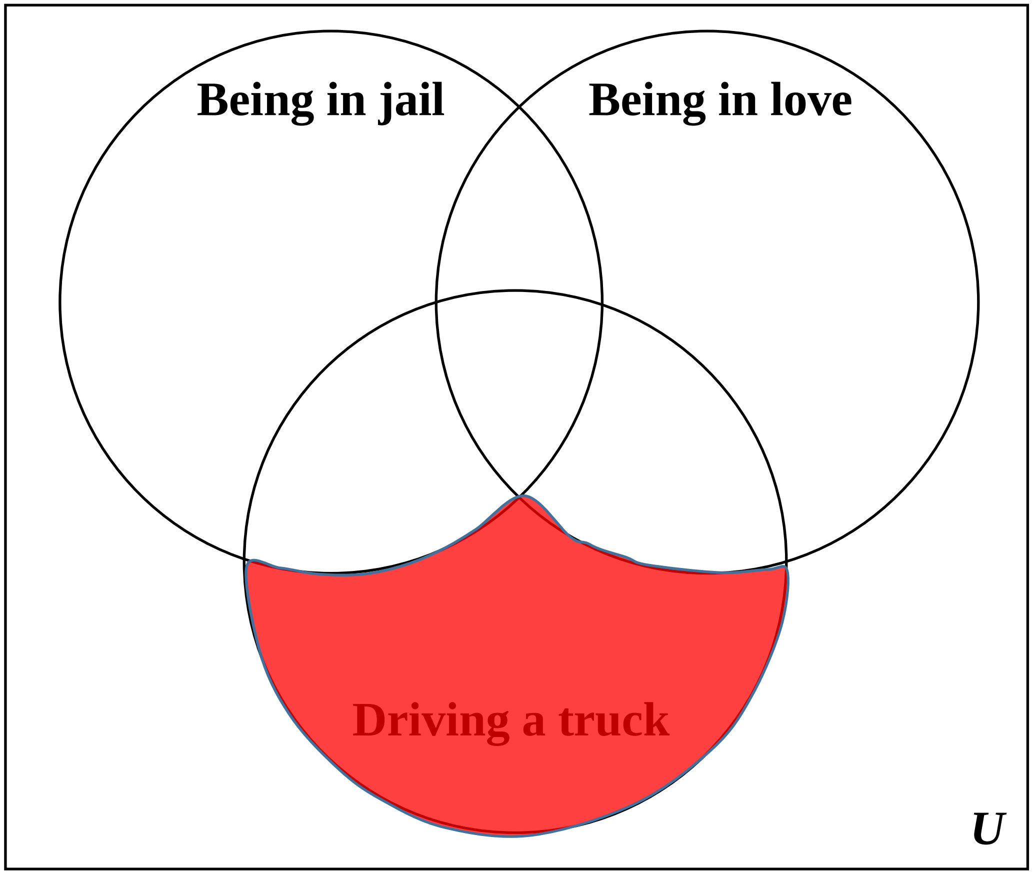
c) How many songs are only about driving a truck?



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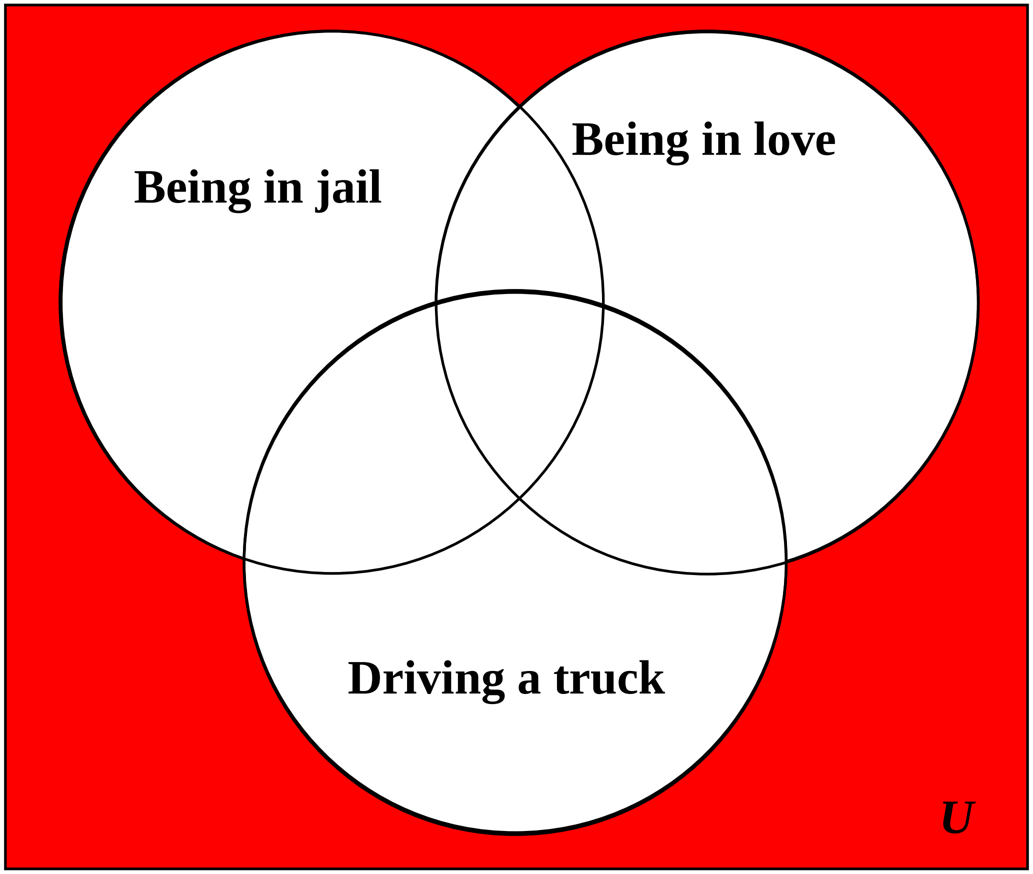
d) How many songs are not about being in jail or being in love, or driving a truck?



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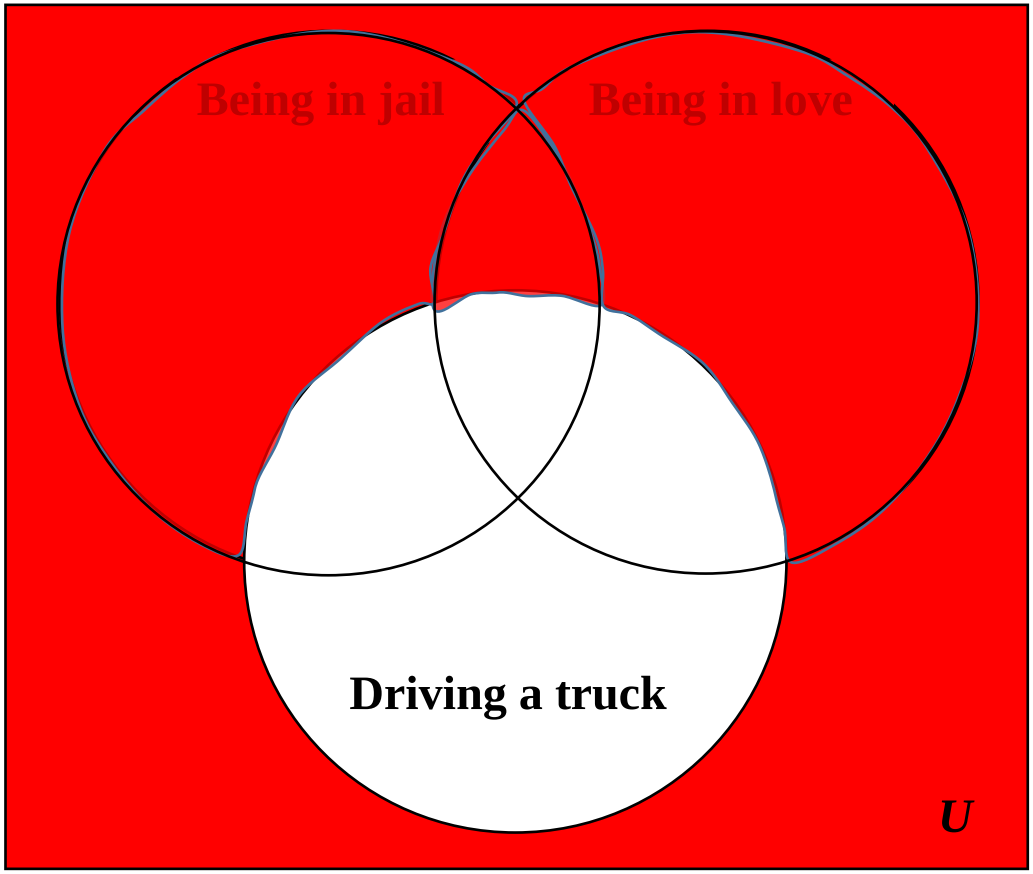
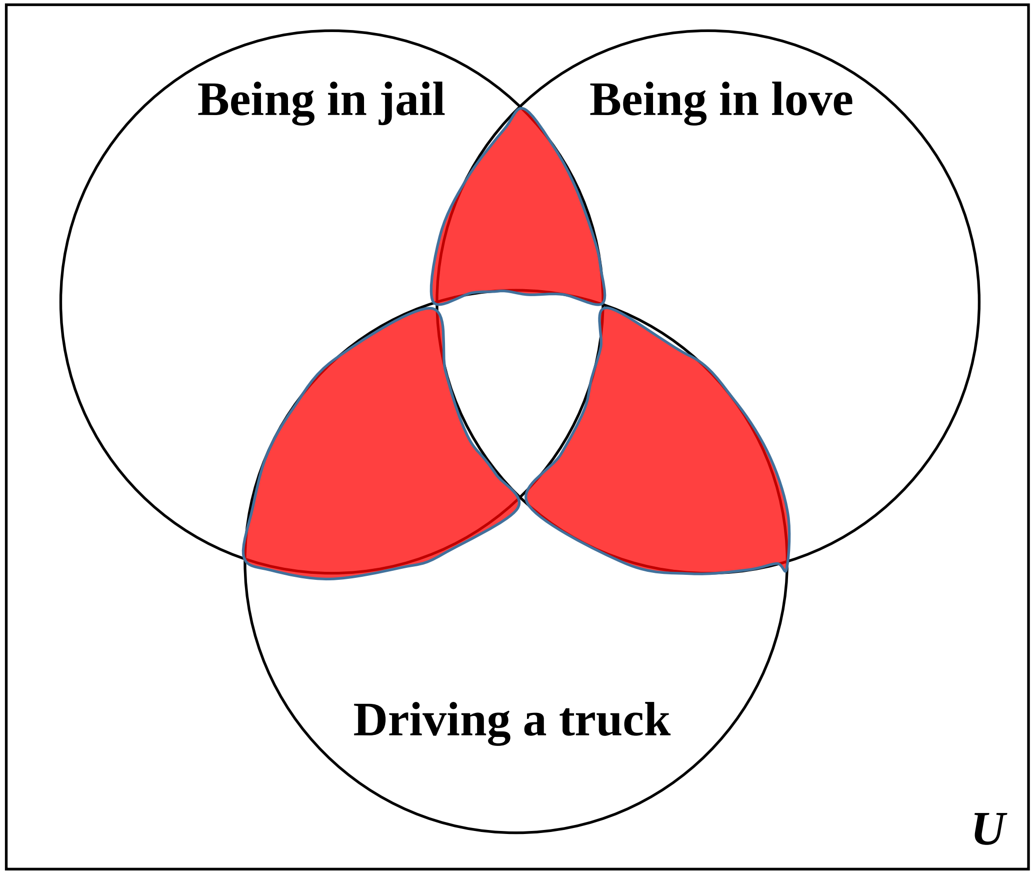
e) How many songs are not about driving a truck?



8 + 14 + 19 + 13 = 54

f) How many songs are about exactly two of the topics?



10 + 14 + 7 = 31