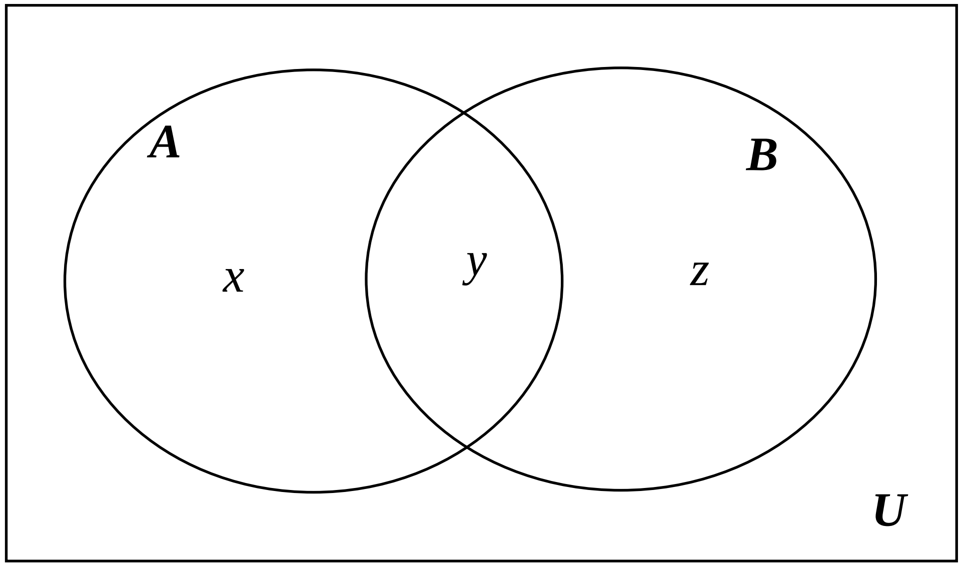
Counting Formula for the Union of Two Sets:



$$
\begin{aligned}
n(A \cup B) &= x + y + z \\
&= (x + y) + (y + z) - y \\
&= n(A) + n(B) - n(A \cap B)
\end{aligned}
$$

So $n(A \cup B) = n(A) + n(B) - n(A \cap B)$**.**

Examples:

If $n(A)=10$**,** $n(B)=19$**, and** $n(A\cap B)=5$**, then what's** $n(A\cup B)$**?**

$$n(A\cup B)=10+19-5=\boxed{24}$$

If $n(A\cup B)=27$**,** $n(A)=12$**, and** $n(B)=23$**, then what's** $n(A\cap B)$**?**

$$n(A\cap B)=12+23-27=\boxed{8}$$

<u>Counting Methods:</u>

Example:

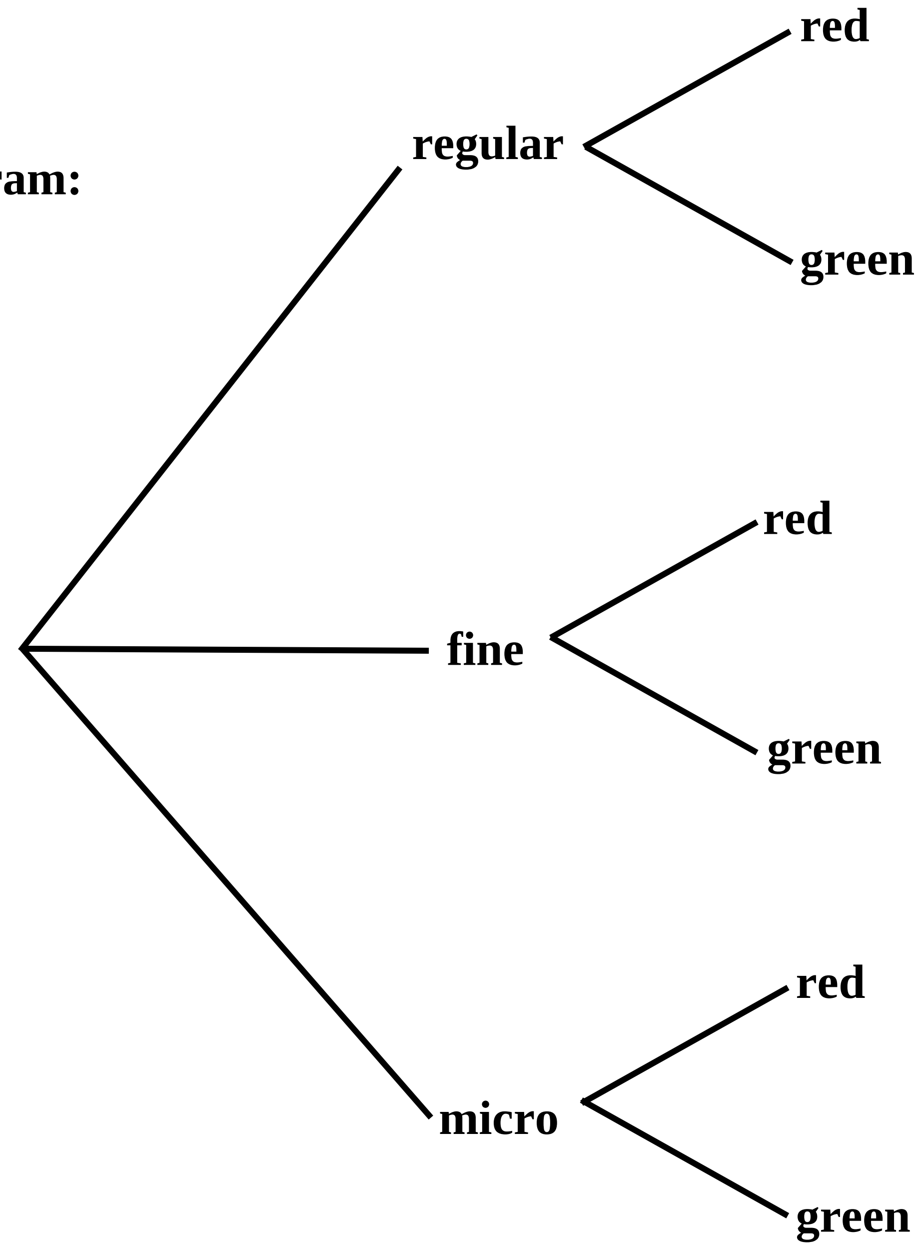
A pen has tip options of regular tip, fine tip, or micro tip, and it has ink color options of red ink or green ink. How many different pens are possible?

Using a table:

	regular	fine	micro
red			
green			

The number of pens possible is the number of cells in the table: $3 \times 2 = 6$**.**

Using a Tree Diagram:



The number of pens possible is the number of branch tips on the right: $3 \times 2 = 6$.

The Fundamental Counting Principle:

If a sequence of decisions is to be made, then the number of different ways of making all the decisions is the product of the number of options for each decision.

Examples:

1. A meal consists of 1 of 8 appetizers, 1 of 10 entrees, and 1 of 5 desserts. How many different meals are possible?

$$8 \cdot 10 \cdot 5 = \boxed{400}$$

2. In a race with 5 horses, how many different first, second, and third place finishes are possible?

$$5 \cdot 4 \cdot 3 = \boxed{60}$$

3. In a certain small state, license plates consist of three letters followed by two digits.

a) How many different plates are possible?

$$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 = \boxed{1,757,600}$$

b) How many if letters can't repeat?

$$26 \cdot 25 \cdot 24 \cdot 10 \cdot 10 = \boxed{1,560,000}$$

c) How many if digits can't repeat?

$$26 \cdot 26 \cdot 26 \cdot 10 \cdot 9 = \boxed{1,581,840}$$

d) How many if no repeats?

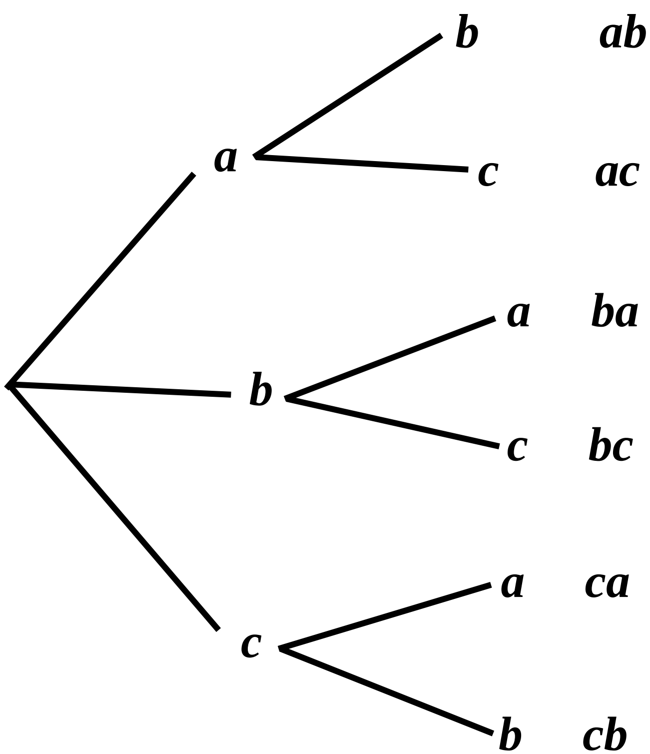
$$26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 = \boxed{1,404,000}$$

Permutations:

A permutation is an arrangement of objects in a particular order.

Example:

Find all the permutations of the objects $\{a,b,c\}$ of size 2.



There are 6 permutations of size 2 from the 3 objects. If we just wanted to know how many, we could have used the Fundamental Counting Principle.

$$3\cdot 2=6$$

In general, the number of permutations of size *r* from *n* objects is abbreviated as ${}_nP_r$.

So far, we know that ${}_3P_2 = 6$. There's a nice formula for the value of ${}_nP_r$ in general, but it involves things called factorials.

Factorials:

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 2 \cdot 1 \textbf{ or } n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-1) \cdot n$$

So $1! = 1$**.**

$$2! = 2 \cdot 1 = 2$$

$$3! = 3 \cdot 2 \cdot 1 = 6$$

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 4 \cdot 3! = 4 \cdot 6 = 24$$

$$5! = 5 \cdot 4! = 5 \cdot 24 = 120$$

$$6! = 6 \cdot 5! = 6 \cdot 120 = 720$$

By special definition, $0! = 1$**.**

Most scientific calculators have a key for factorials: *n!* or *x!; check out the manual.*

$$ {}_nP_r = \frac{n!}{(n-r)!} $$ Most scientific calculators have a key for the number of permutations.

Let's check it out for ${}_3P_2$, which we already know is equal to 6.

$$ {}_3P_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = \frac{6}{1} = 6 $$

Examples:

1. Five solo singers are to perform their acts at a nightclub on Saturday night. How many different orders of their appearances are possible?

$$ {}_5P_5 = \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{120}{1} = \boxed{120} $$

Or

Fundamental Counting Principle

$$ 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{120} $$

2. From a group of 6 people, a president, vice-president, and secretary will be selected, how many different selections are possible?

$$_6P_3 = \frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{720}{6} = \boxed{120}$$

Or

Fundamental Counting Principle

$$6 \cdot 5 \cdot 4 = \boxed{120}$$

3. In a race with 8 horses, how many different first, second, and third place finishes are possible?

$$_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 8 \cdot 7 \cdot 6 = \boxed{336}$$

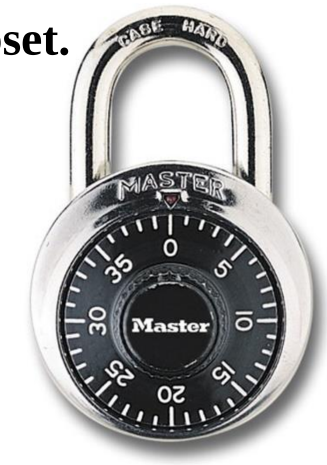
Or

Fundamental Counting Principle

$$8 \cdot 7 \cdot 6 = \boxed{336}$$

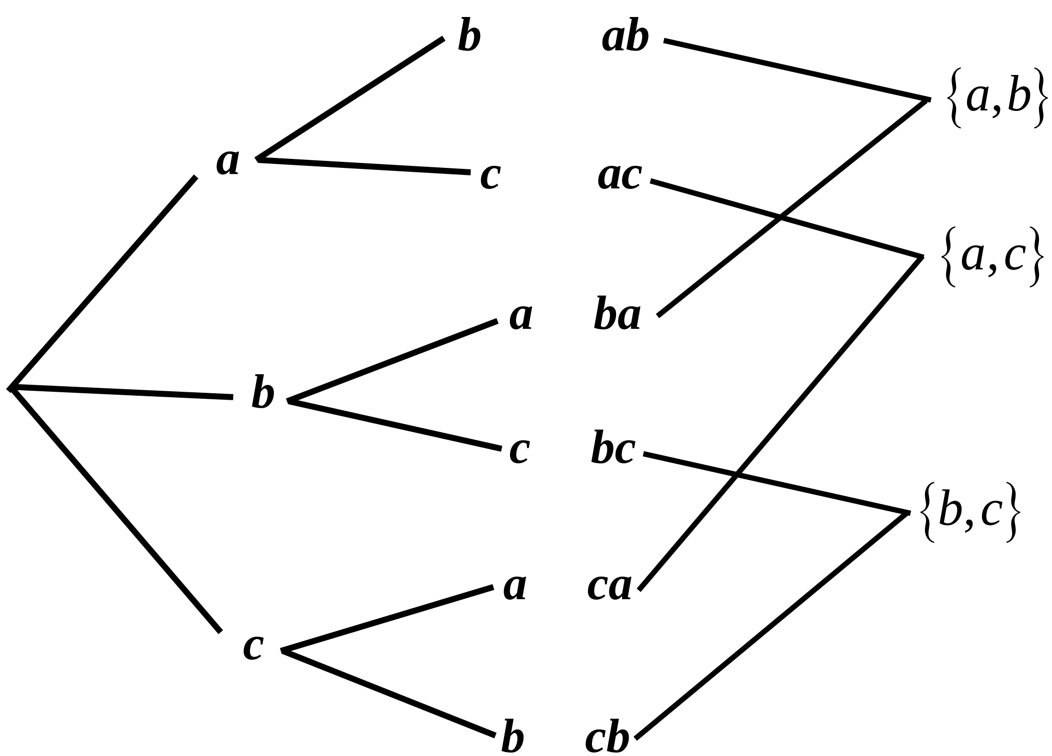
Combinations:

A combination is a selection of objects without regard to order, i.e. a subset.



Example:

Find all the combinations of the objects $\{a,b,c\}$ of size 2.



There are 6 permutations of size 2 from the 3 objects, but only 3 combinations of size 2 from the 3 objects.

In general, the number of combinations of size *r* from *n* objects is abbreviated as $_nC_r$. So far, we know that $_3C_2 = 3$, and $_3C_2 = 3 = \frac{6}{2} = \frac{_3P_2}{2!}$. This is true in general, and leads to a nice formula for $_nC_r$. $_nC_r = \frac{n!}{r!\cdot(n-r)!}$

Most scientific calculators have a key for determining the number of combinations.

Examples:

1. A three-person committee is to be selected from a group of 10 people. How many different committees are possible?

$$_{10}C_3 = \frac{10!}{3!\cdot 7!} = \frac{10\cdot 9\cdot 8}{3\cdot 2\cdot 1} = \boxed{120}$$

2. In a certain lottery, you must select 6 numbers from the numbers 1-50. How many different lottery selections are possible?

$$_{50}C_6 = \frac{50!}{6!\cdot 44!} = \frac{50\cdot 49\cdot 48\cdot 47\cdot 46\cdot 45}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1} = \boxed{15{,}890{,}700}$$

3. A group consists of 7 men and 8 women. A committee of 4 people will be selected.

a) How many different 4-person committees are possible?

$${}_{15}C_4 = \frac{15!}{4!\cdot 11!} = \frac{15\cdot 14\cdot 13\cdot 12}{4\cdot 3\cdot 2\cdot 1} = \boxed{1{,}365}$$

b) How many different 4-person committees consisting of 4 women are possible?

$${}_{8}C_4 = \frac{8!}{4!\cdot 4!} = \frac{8\cdot 7\cdot 6\cdot 5}{4\cdot 3\cdot 2\cdot 1} = \boxed{70}$$

c) How many different 4-person committees consisting of 3 women and 1 man are possible?

${}_{8}C_3 = 56$	${}_{7}C_1 = 7$
Which 3 women?	**Which 1 man?**

$56\cdot 7 = \boxed{392}$

d) How many different 4-person committees consisting of 2 women and 2 men are possible?

$_8C_2 = 28$	$_7C_2 = 21$
Which 2 women?	**Which 2 men?**

$28 \cdot 21 = \boxed{588}$

e) How many different 4-person committees have at least 1 man?

= the number of committees with at least one man

= the total number of committees − the number of committees with no men

$= 1{,}365 - 70 = \boxed{1{,}295}$

Sometimes it's easier to calculate quantities indirectly!

Calculator Advice:

Most calculators have keys for factorial, number of permutations, and number of combinations. The factorial key will usually look like $x!$, $n!$, or just !. The number of permutations key will look like ${}_nP_r$, and the number of combinations key will look ${}_nC_r$. To use the factorial key, just enter the number and then press the factorial key. To use the others, enter the n value, press the appropriate key, then enter the r value, and press the = key.

Some calculators have these operations hidden. Press the math key and look under the probability heading or press the prob key and select the operation that you want.