Probability:

Experiment:

Any process that produces random results.

Example:

Flip a coin twice and record the results.

Heads/Heads, Heads/Tails, Tails/Heads, Tails/Tails

Sample Space:

The set of all possible outcomes of an experiment. It's abbreviated with the letter *S*, and it's like the universal set for an experiment.

Example:

From the previous experiment

$$S = \{HH, HT, TH, TT\}$$

Event:

Any subset of the sample space

Example:

From the previous experiment

$$S = \{HH, HT, TH, TT\}$$

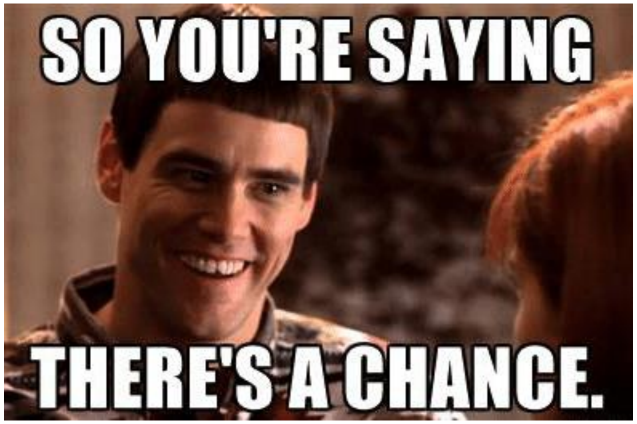
E **is the event that heads occurs.** $E = \{HH, HT, TH\}$

F **is the event that tails occurs.** $F = \{HT, TH, TT\}$

G **is the event of getting the same result on both flips.** $G = \{HH, TT\}$

J **is the event of getting different results on the two flips.** $J = \{HT, TH\}$

A probability is a number between 0 and 1(inclusive) that indicates the likelihood that an event will occur.



A probability of 1 means the event must occur.

A probability of 0 means the event won't occur.

A probability of ½ means the event is just as likely to occur as not to occur.

The closer the probability value is to 1, the more likely the event will occur, and the closer the probability value is to 0, the less likely the event will occur.

Theoretical Probability and the Equally Likely assumption:

In many experiments, the outcomes in the sample space all have the same probability of occurring. Certain conditions in the experiments will allow you to make this equally likely assumption.

When you can assume equally likely outcomes, the probability of an event is determined using counting.

$$P(E) = \frac{n(E)}{n(S)}$$

Examples:

1. A *fair* die is rolled. $S = \{1, 2, 3, 4, 5, 6\}$

a) $P(\text{rolling a 1})$

$\frac{1}{6}$

b) $P(\text{rolling a 2 or a 3})$

$\frac{2}{6} = \frac{1}{3}$

c) $P(\text{rolling an odd number})$

$\frac{3}{6} = \frac{1}{2}$

d) $P(\text{rolling a 7})$

$\frac{0}{6} = 0$

2. A card is *randomly selected* from a standard 52-card deck.

a) $P(\text{selecting an ace})$

$$\frac{4}{52} = \frac{1}{13}$$

b) $P(\text{selecting a red card})$

$$\frac{26}{52} = \frac{1}{2}$$

c) $P(\text{selecting a club})$

$$\frac{13}{52} = \frac{1}{4}$$

d) $P(\text{selecting a face card})$

J Q K ♦ ♠ ♥ ♣

$$\frac{12}{52} = \frac{3}{13}$$

e) $P(\text{selecting an ace or a diamond})$

A♥ **A**♣ **A**♠

A♦
2♦
3♦
4♦
5♦
6♦
7♦
8♦
9♦
10♦
J♦
Q♦
K♦

$$\frac{16}{52} = \frac{4}{13}$$

3. A *fair coin* is flipped twice. $S = \{HH, HT, TH, TT\}$

a) $P(\text{heads occurs})$

$\frac{3}{4}$

b) $P(\text{tails occurs})$

$\frac{3}{4}$

c) $P(\text{same result on both flips})$

$\frac{2}{4} = \frac{1}{2}$

d) $P(\text{different result on the two flips})$

$\frac{2}{4} = \frac{1}{2}$

e) $P(\text{two heads occur})$

$\frac{1}{4}$

f) $P(\text{three heads occur})$

$\frac{0}{4} = 0$

4. A survey of North Harris students had the following results.

	Pepperoni	Sausage	Mushroom	Total
Freshman	25	15	5	45
Sophomore	30	20	5	55
Total	55	35	10	100

A student from the survey is *selected at random*.

a) $P(\text{freshman})$

$$\frac{45}{100}=\frac{9}{20}$$

b) $P(\text{sausage})$

$$\frac{35}{100}=\frac{7}{20}$$

c) $P(\text{freshman and pepperoni})$

$$\frac{25}{100}=\frac{1}{4}$$

d) $P(\text{sausage or mushroom})$

$$\frac{45}{100}=\frac{9}{20}$$

e) $P(\text{freshman or pepperoni})$

$$\frac{75}{100}=\frac{3}{4}$$

Empirical Probability:

The experiment is performed a bunch of times, *n*, and the results are recorded.

$$P(E) = \frac{\text{\# of times } E \text{ occurs}}{n}$$

Example:

A coin is flipped 1,000 times with the following results: 450 heads and 550 tails. Find the empirical probability of flipping a tail.

$$\frac{550}{1000} = \frac{11}{20}$$

Finding Probabilities Using Counting Techniques:

In the case of the equally likely assumption, $P(E) = \frac{n(E)}{n(S)}$**.**

Examples:

1. Allen, Bob, Carl, and David will be seated in a row of 4 chairs at random.

a) How many different ways can they be seated?

$$4 \cdot 3 \cdot 2 \cdot 1 = \boxed{24}$$

b) How many different ways can Carl be seated in the first chair and Allen in the fourth chair?

$$1 \cdot 2 \cdot 1 \cdot 1 = \boxed{2}$$

c) What's the probability that Carl will be seated in the first chair and Allen in the fourth chair?

$$\frac{2}{24} = \frac{1}{12}$$

2. A group consists of 4 men and 5 women. Three people will be selected at random to attend a conference.

a) How many different selections of 3 people from the group are possible?

$${}_9C_3 = \boxed{84}$$

b) What's the probability that the 3 people selected are all women?

$$\frac{{}_5C_3}{{}_9C_3} = \frac{10}{84} = \frac{5}{42}$$

c) What's the probability that the 3 people selected are all men?

$$\frac{{}_4C_3}{{}_9C_3} = \frac{4}{84} = \frac{1}{21}$$

d) What's the probability that the 3 people selected will consist of 2 women and 1 man?

$$\frac{{}_5C_2 \cdot {}_4C_1}{{}_9C_3} = \frac{10 \cdot 4}{84} = \frac{40}{84} = \frac{10}{21}$$

3. Three cards will be randomly selected from a 52-card deck without replacement.

a) What's the probability that it will consist of all hearts?

$$\frac{{}_{13}C_3}{{}_{52}C_3} = \frac{286}{22{,}100} = \frac{11}{850}$$

b) What's the probability that it will consist of exactly 2 aces?

$$\frac{{}_4C_2 \cdot {}_{48}C_1}{{}_{52}C_3} = \frac{6 \cdot 48}{22{,}100} = \frac{288}{22{,}100} = \frac{72}{5{,}525}$$

c) What's the probability that it will consist of 2 aces and a king?

$$\frac{{}_4C_2 \cdot {}_4C_1}{{}_{52}C_3} = \frac{6 \cdot 4}{22{,}100} = \frac{24}{22{,}100} = \frac{6}{5{,}525}$$