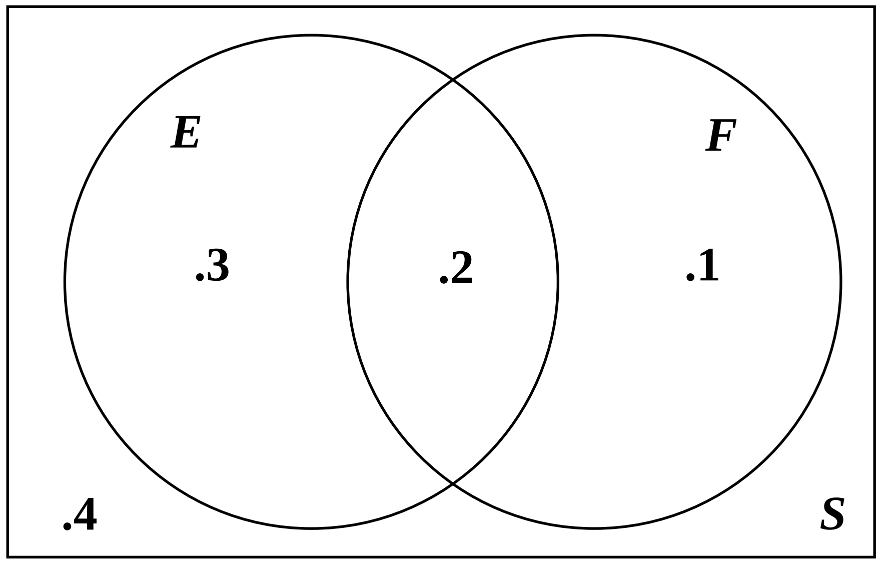
Probability Diagrams and Probability Formulas:

A probability diagram is like a specialized Venn Diagram in which the probabilities of different events in the sample space are labelled.



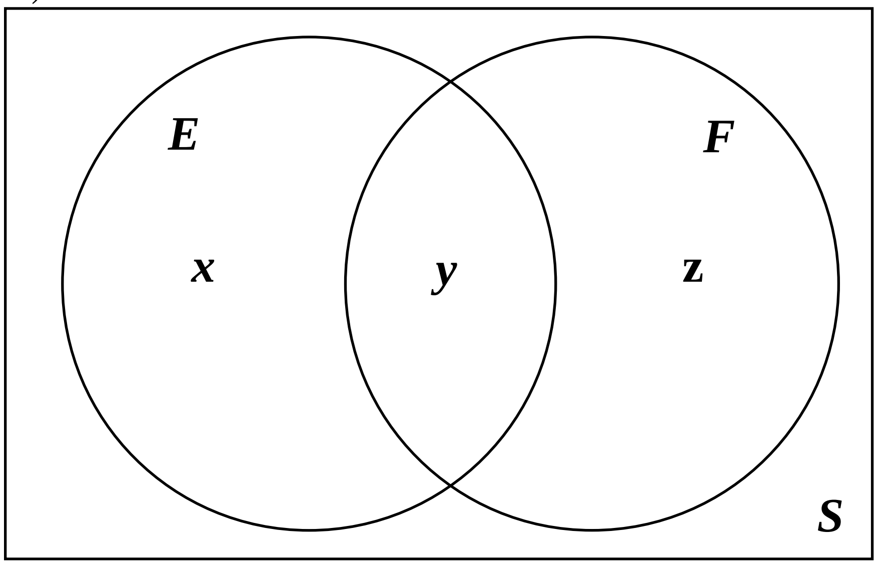
The sum of all the probabilities that make up all the disjoint regions of *S* must be 1.

$P(E) = .3 + .2 = \boxed{.5}$ $P(F) = .2 + .1 = \boxed{.3}$ $P(E \text{ and } F) = P(E \cap F) = .2$

$P(E \text{ or } F) = P(E \cup F) = .3 + .2 + .1 = \boxed{.6}$ $P(\text{not } E) = P(E') = .1 + .4 = \boxed{.5}$

$P\left((E \cup F)'\right) = .4$

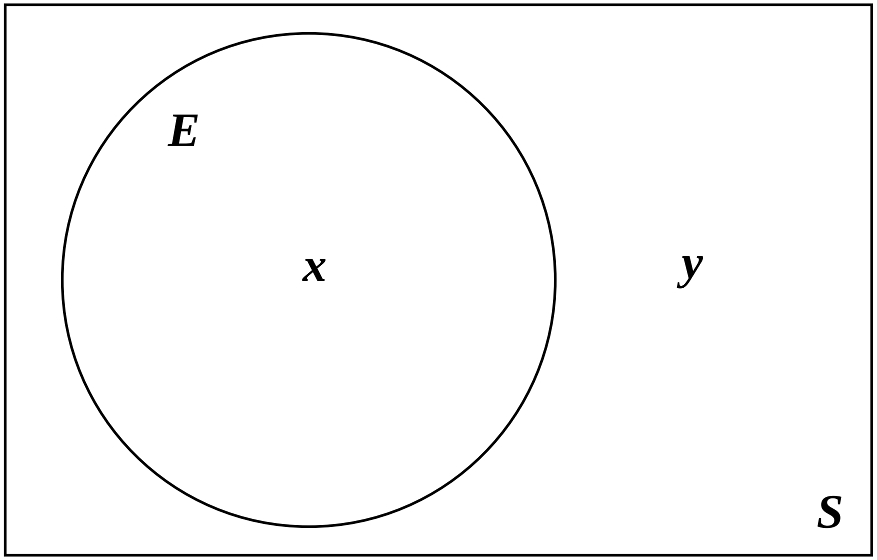
A formula for $P(E \cup F)$:



$$P(E \cup F) = x + y + z = x + y + y + z - y$$
$$= (x + y) + (y + z) - y$$
$$= P(E) + P(F) - P(E \cap F)$$

If $E \cap F = \phi$, then it's impossible for both events to occur, and they are called mutually exclusive events. In this case, $P(E \cup F) = P(E) + P(F)$

Formulas involving $P(E')$:



$$1 = P(S) = x + y = P(E) + P(E')$$

So

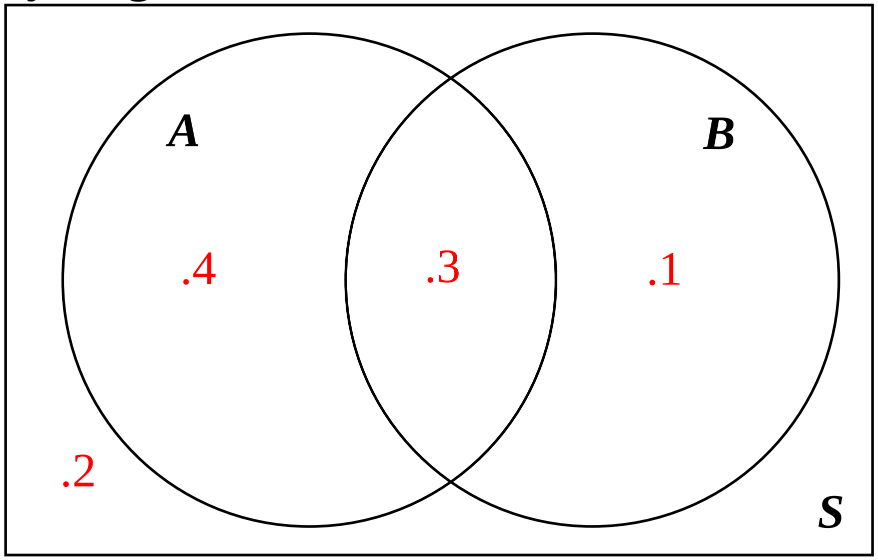
$$P(E) = 1 - P(E')$$

$$P(E') = 1 - P(E)$$

<u>Out of context example:</u>

Suppose $P(A)=.7$**,** $P(B)=.4$**, and** $P(A\cap B)=.3$**.**

Complete the probability diagram:



Find

$P(A\cup B)$

.8

$P(A')$

.3

$P\left((A\cup B)'\right)$

.2

$P(A\cap B')$

.4

$P(B\cap A')$

.1

$P\left((A\cap B)'\right)$

.7

In context examples:

1. A card is randomly selected from a standard 52-card deck.

$$P(\text{ace or a heart})$$

$$= P(\text{ace}) + P(\text{heart}) - P(\text{ace and heart})$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \boxed{\frac{4}{13}}$$

$$P(\text{ace or king})$$

$$= P(\text{ace}) + P(\text{king})$$

$$= \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \boxed{\frac{2}{13}}$$

$$P(\text{face card or a club})$$

$$= P(\text{face card}) + P(\text{club}) - P(\text{face card and club})$$

$$= \frac{12}{52} + \frac{13}{52} - \frac{3}{52} = \frac{22}{52} = \boxed{\frac{11}{26}}$$

2. A survey of North Harris students had the following results.

	Pepperoni	Sausage	Mushroom	Total
Freshman	25	15	5	45
Sophomore	30	20	5	55
Total	55	35	10	100

A student from the survey is *selected at random*.

a) $P(\text{sausage or mushroom})$

$$= P(\text{sausage}) + P(\text{mushroom})$$

$$= \frac{35}{100} + \frac{10}{100} = \frac{45}{100} = \boxed{\frac{9}{20}}$$

b) $P(\text{freshman or pepperoni})$

$$= P(\text{freshman}) + P(\text{pepperoni}) - P(\text{freshman and pepperoni})$$

$$= \frac{45}{100} + \frac{55}{100} - \frac{25}{100} = \frac{75}{100} = \boxed{\frac{3}{4}}$$

Odds and Probability:

The odds in favor of an event *E* is the ratio of the probability that *E* will occur to the probability that *E* won't occur.

Odds in favor of E: $P(E):P(E')$ **or** $P(E)$ to $P(E')$

The odds are usually expressed as a ratio of whole numbers.

Example:

If $P(E)=\frac{2}{5}$**, then find the odds in favor of *E*.**

$$\frac{2}{5} \text{ to } \frac{3}{5}\text{, multiply by 5 to get } \boxed{2 \text{ to } 3}$$

The odds against an event E is the ratio of the probability that *E* won't occur to the probability that *E* will occur, i.e. the reversal of the odds in favor.

Odds against E: $P(E'):P(E)$ **or** $P(E')$ to $P(E)$

Example:

If $P(E)=\frac{3}{7}$**, then find the odds against *E*.**

$$\frac{4}{7} \text{ to } \frac{3}{7}\text{, multiply by 7 to get } \boxed{4 \text{ to } 3}$$

Sometimes you'll want to go from odds to probability. If the odds in favor of *E* is *a* to *b*, then $\frac{P(E)}{P(E')}=\frac{a}{b}\Rightarrow\frac{P(E)}{1-P(E)}=\frac{a}{b}$**. Cross-multiplying leads to**

$$bP(E)=a-aP(E)\Rightarrow(a+b)P(E)=a\Rightarrow P(E)=\frac{a}{a+b}.$$

Example:

If the odds in favor of *E* is 4 to 7, then find $P(E)$**.**

$$P(E)=\frac{4}{4+7}=\boxed{\frac{4}{11}}$$