Simple Interest:

$$I = Prt$$

***I* is the interest earned in dollars.**

***P* is the amount of money loaned or borrowed, called the principal or present value**

***r* is the annual interest rate as a decimal.**

***t* is the amount of time in years.**

Examples:

1. You borrow $580 at 4.3% for 2 years. How much interest will you owe?

$$I = 580 \cdot .043 \cdot 2 = \boxed{\$49.88}$$

Make sure you write the interest rate as a decimal.

2. You deposit $1250 into an account paying 2.3% for 6 months. How much interest will you earn?

$$I = 1250 \cdot .023 \cdot \frac{6}{12} = \boxed{\$14.38}$$

6 months is the same as $\frac{6}{12}$ *years.*

If you add the amount of money loaned or borrowed to the amount of interest earned, you get the total amount or future value, *A*.

$$A = P + I = P + Prt$$
$$= P(1 + rt)$$

Examples:

1. You borrow $650 at 4.3% for 90 days. How much will you owe?

$$A = 650\left(1 + .043 \cdot \frac{90}{360}\right) = \boxed{\$656.99}$$

In finance and banking, a year has 360 days.

2. You deposit $930 into an account paying 2.3% for 48 weeks. How much money will be in the account?

$$A = 930\left(1 + .023 \cdot \frac{48}{52}\right) = \boxed{\$949.74}$$

More examples:

1. **What simple annual interest rate, rounded to the nearest tenth of a percent will turn $500 into $600 in 2 years?**

$$A = P(1+rt) \Rightarrow \frac{A}{P} = 1+rt \Rightarrow \boxed{r = \frac{\frac{A}{P}-1}{t}}$$

$$r = \frac{\frac{600}{500}-1}{2} = .1 = \boxed{10\%}$$

2. **How many years, rounded to the nearest tenth of a year, will it take for $200 to turn into $250 at an annual rate of 5%?**

$$A = P(1+rt) \Rightarrow \frac{A}{P} = 1+rt \Rightarrow \boxed{t = \frac{\frac{A}{P}-1}{r}}$$

$$t = \frac{\frac{250}{200}-1}{.05} = 5 = \boxed{5 \, years}$$

Present Value with Simple Interest:

If you solve the total amount/future value equation for *P*, you get the present value formula for simple interest.

$$A = P(1+rt) \Rightarrow P = \frac{A}{1+rt}$$

Example:

You would like to have $500 2 years from now by depositing money into an account paying 3.1%. How much should you deposit now?

$$P = \frac{500}{1+.031\cdot 2} = \boxed{\$470.81}$$

Compound Interest:

In compound interest, interest earns interest. There are time periods called compounding periods, and an interest rate per compounding period called *i*.

P	**First compounding period**	$P+iP=P(1+i)$	**Second compounding period**	$P(1+i)^2$	**Third compounding period**	$P(1+i)^3$	…

So if the process is allowed to continue for *n* compounding periods, then the total amount or future value will be

$$A=P(1+i)^n.$$

In most problems, you will be given the value of *r*, called the annual nominal rate, and the number of compounding periods per year, *m*. To get the value of *i*, simply divide *r* by *m*: $i=\frac{r}{m}$**.**

$$A=P\left(1+\frac{r}{m}\right)^n$$

Examples:

1. You borrow $650 at 2.5% compounded monthly for 2 years. How much will you owe? How much of what you owe is interest?

$$A = 650\left(1+\frac{.025}{12}\right)^{24} = \boxed{\$683.29}, \text{ the interest portion is } \boxed{\$33.29}.$$

The number of compounding periods per year is 12, but the number for 2 years is 24.

The interest is the difference between what you owe and what you borrowed.

2. You deposit $900 into an account paying 3.1% compounded semi-annually for 4 years. How much money will be in the account? How much of the money in the account is interest?

$$A = 900\left(1+\frac{.031}{2}\right)^{8} = \boxed{\$1{,}017.85}, \text{ the interest portion is } \boxed{\$117.85}.$$

The interest is the difference between what you deposited and what's in the account.

3. You deposit $500 into an account paying 2.1% compounded quarterly for 6 years. How much money will be in the account? How much of the money in the account is interest?

$$A = 500\left(1+\frac{.021}{4}\right)^{24} = \boxed{\$566.95}, \text{ the interest portion is } \boxed{\$66.95}.$$

More examples:

1. **What nominal rate, rounded to the nearest hundredth of a percent, compounded annually will turn $350 into $425 in 4 years?**

$$A = P(1+i)^n \Rightarrow \frac{A}{P} = (1+i)^n \Rightarrow \sqrt[n]{\frac{A}{P}} = 1+i \Rightarrow \boxed{i = \sqrt[n]{\frac{A}{P}} - 1}$$

$$i = \sqrt[4]{\frac{425}{350}} - 1 = .049736... = \boxed{4.97\%}$$

2. **Between which two compounding periods will $500 turn into $600 at 6% compounded semi-annually?**

$$A = P(1+i)^n \Rightarrow \frac{A}{P} = (1+i)^n \Rightarrow \log\left(\frac{A}{P}\right) = n\log(1+i) \Rightarrow \boxed{n = \frac{\log\left(\frac{A}{P}\right)}{\log(1+i)}}$$

$$n = \frac{\log\left(\frac{600}{500}\right)}{\log\left(1+\frac{.06}{2}\right)} = 6.168... \Rightarrow \boxed{\text{between the 6th and 7th compounding periods}}$$

Present Value with Compound Interest:

If you solve the total amount/future value equation for *P*, you get the present value formula for compound interest.

$$A = P\left(1+\frac{r}{m}\right)^{n} \Rightarrow P = \frac{A}{\left(1+\frac{r}{m}\right)^{n}} \text{ or } A\left(1+\frac{r}{m}\right)^{-n}$$

Example:

You would like to have $500 2 years from now by depositing money into an account paying 3.1% compounded weekly. How much should you deposit now?

$$P = \frac{500}{\left(1+\frac{.031}{52}\right)^{104}} = \boxed{\$469.95}$$

Comparing Compound Interest Investments:

To compare different compound interest investment schemes, you can find the simple interest rate that generates the same amount of money as the compound scheme in 1 year. This rate is called the Effective Rate.

$$P\left(1+\frac{r}{m}\right)^m = P\left(1+r_e\right)$$

$$\left(1+\frac{r}{m}\right)^m = \left(1+r_e\right)$$

$$r_e = \left(1+\frac{r}{m}\right)^m - 1$$

The larger the effective rate, the more money that will be produced by the compound investment.

Example:

Find the effective rate of 3.1% compounded quarterly, rounded to the nearest hundredth percent.

$$r_e = \left(1+\frac{.031}{4}\right)^4 - 1 = .0313622... \boxed{3.14\%}$$