

## **Binomial Random Variable:**

**Bernoulli Trial:** A Bernoulli trial is an experiment with only two possible outcomes, success or failure. The probability of success is denoted by  $p$ , and the probability of failure is denoted by  $(1-p)$ .



### **Example:**

A fair die is rolled. If a 1, 2, 3, or 4 is rolled, it's called a success. If a 5 or 6 is rolled, it's considered a failure.

$$S = \{1, 2, 3, 4, 5, 6\}$$

The probability of success,  $p$ , is \_\_\_\_\_

The probability of failure,  $(1-p)$ , is \_\_\_\_\_



**$n$  independent bernoulli trials are performed, each with the same probability of success,  $p$ .**

**If  $X$  is the number of successes that occur, then  $X$  is called a Binomial Random Variable.**

**What are the possible values of  $X$ ?**

**$X =$**

**Example:**

A fair die is rolled 3 times. If a 1, 2, 3, or 4 is rolled, it's called a success. If a 5 or 6 is rolled, it's considered a failure.  $X$  is the number of successes that occur. Let's find the probability distribution of this binomial random variable  $X$ .



$X$	0	1	2	3
outcomes	f,f,f	s,f,f f,s,f f,f,s	s,s,f s,f,s f,s,s	s,s,s
$P(X)$				
	${}_3C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^3$	${}_3C_1 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^2$	${}_3C_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^1$	${}_3C_3 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^0$

**The probability formula in general for a binomial random variable is**

$$P(X = x) = {}_n C_x \cdot p^x \cdot (1 - p)^{n-x}$$

**Back to our example:**

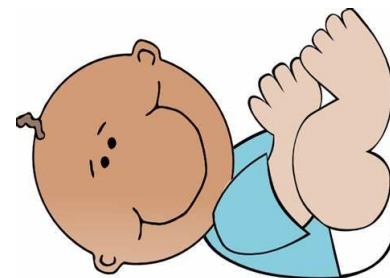
**Let's find the expected value of our specific binomial random variable  $X$ .**

$X$	0	1	2	3
$P(X)$	$\frac{1}{27}$	$\frac{6}{27}$	$\frac{12}{27}$	$\frac{8}{27}$

$$E(X) = 0 \cdot \frac{1}{27} + 1 \cdot \frac{6}{27} + 2 \cdot \frac{12}{27} + 3 \cdot \frac{8}{27} = \frac{54}{27} =$$

**The expected value formula in general for a Binomial random variable is**

$$E(X) = np$$



**Examples:**

**1. According to a study, 25% of boys 6-8 months in the United States weigh more than 20 pounds. A sample of 4 boys 6-8 months is selected at random, with replacement.**

**a) What is the probability that exactly 3 of them weigh more than 20 pounds?**

**b) What is the probability that more than 2 of them weigh more than 20 pounds?**

**c) What is the probability that at least 1 of them weighs more than 20 pounds?**

**d) Would it be unusual for all 4 of them to weigh more than 20 pounds?**

**e) What's the expected number of them that weigh more than 20 pounds?**

**2. Airlines frequently overbook flights. On a flight with 50 available seats, they will sell 52 reservations. If the probability that a person with a reserved seat will show up is 90%, then what's the probability that there will be a seat for all the people that show up?**

$$\begin{aligned}P(X \leq 50) &= 1 - P(X > 50) \\&= 1 - P(X = 51) - P(X = 52) \\&= 1 - {}_{52}C_{51} (.9)^{51} (.1)^1 - {}_{52}C_{52} (.9)^{52} (.1)^0 \\&= 1 - 52 \cdot \left(\frac{9}{10}\right)^{51} \left(\frac{1}{10}\right) - \left(\frac{9}{10}\right)^{52} \approx .9717 \text{ or } 97.17\%\end{aligned}$$



**3. An opinion poll based upon a small sample can be unrepresentative of the population. Suppose that 40% of voters favor a certain candidate. If a random sample of 7 voters is selected, what is the probability that a majority will favor the candidate?**

$$\begin{aligned}P(\text{majority favor the candidate}) &= P(X \geq 4) \\&= P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7) \\&= {}_7C_4 (.4)^4 (.6)^3 + {}_7C_5 (.4)^5 (.6)^2 + {}_7C_6 (.4)^6 (.6)^1 + {}_7C_7 (.4)^7 (.6)^0 \\&= \frac{22,640}{78,125} \approx .29 \text{ or } 29\%\end{aligned}$$

