<u>Binomial Random Variable:</u>

<u>Bernoulli Trial:</u> A Bernoulli trial is an experiment with only two possible outcomes, success or failure. The probability of success is denoted by *p*, and the probability of failure is denoted by (1-*p*).

Example:

A fair die is rolled. If a 1, 2, 3, or 4 is rolled, it's called a success. If a 5 or 6 is rolled, it's considered a failure.

$$S = \{1,2,3,4,5,6\}$$

The probability of success, *p*, is $\frac{2}{3}$

The probability of failure, (1-*p*), is $\frac{1}{3}$

n **independent bernoulli trials are performed, each with the same probability of success, *p*.**

If *X* is the number of successes that occur, then *X* is called a <u>Binomial Random Variable</u>.

What are the possible values of *X*?

$X = 0,1,2,3,4,\ldots,n$

Example:

A fair die is rolled 3 times. If a 1, 2, 3, or 4 is rolled, it's called a success. If a 5 or 6 is rolled, it's considered a failure. *X* is the number of successes that occur. Let's find the probability distribution of this binomial random variable *X*.

X	**0**	**1**	**2**	**3**
outcomes	**f,f,f**	**s,f,f** **f,s,f** **f,f,s**	**s,s,f** **s,f,s** **f,s,s**	**s,s,s**
P(*X*)	$\left(\frac{1}{3}\right)^3 = \boxed{\frac{1}{27}}$	$3 \cdot \frac{2}{3} \cdot \left(\frac{1}{3}\right)^2 = \boxed{\frac{2}{9}}$	$3 \cdot \left(\frac{2}{3}\right)^2 \cdot \frac{1}{3} = \boxed{\frac{4}{9}}$	$\left(\frac{2}{3}\right)^3 = \boxed{\frac{8}{27}}$
	${}_3C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^3$	${}_3C_1 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^2$	${}_3C_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^1$	${}_3C_3 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^0$

The probability formula in general for a binomial random variable is

$$P(X = x) = {}_nC_x \cdot p^x \cdot (1 - p)^{n-x}$$

Back to our example:

Let's find the expected value of our specific binomial random variable *X*.

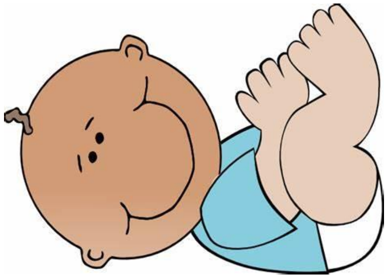
X	**0**	**1**	**2**	**3**
P(*X*)	$\frac{1}{27}$	$\frac{6}{27}$	$\frac{12}{27}$	$\frac{8}{27}$

$$E(X) = 0 \cdot \frac{1}{27} + 1 \cdot \frac{6}{27} + 2 \cdot \frac{12}{27} + 3 \cdot \frac{8}{27} = \frac{54}{27} = \boxed{2} = 3 \cdot \frac{2}{3}$$

The expected value formula in general for a Binomial random variable is

$$E(X) = np$$

So you can calculate it quickly, without creating a table!



Examples:

1. According to a study, 25% of boys 6-8 months in the United States weigh more than 20 pounds. A sample of 4 boys 6-8 months is selected at random, with replacement.

a) What is the probability that exactly 3 of them weigh more than 20 pounds?

$$P(X=3) = {}_4C_3 \cdot \left(\frac{1}{4}\right)^3 \cdot \left(\frac{3}{4}\right)^1 = 4 \cdot \frac{3}{256} = \frac{12}{256} = \boxed{\frac{3}{64}}$$

b) What is the probability that more than 2 of them weigh more than 20 pounds?

$$P(X>2) = P(X=3) + P(X=4) = {}_4C_3 \cdot \left(\frac{1}{4}\right)^3 \cdot \left(\frac{3}{4}\right)^1 + {}_4C_4 \cdot \left(\frac{1}{4}\right)^4 \cdot \left(\frac{3}{4}\right)^0 = \frac{3}{64} + \frac{1}{256} = \boxed{\frac{13}{256}}$$

c) What is the probability that at least 1 of them weighs more than 20 pounds?

$$P(X \geq 1) = 1 - P(X = 0) = 1 - {}_4C_0 \cdot \left(\frac{1}{4}\right)^0 \cdot \left(\frac{3}{4}\right)^4 = 1 - \frac{81}{256} = \boxed{\frac{175}{256}}$$

d) Would it be unusual for all 4 of them to weigh more than 20 pounds?

$$P(X = 4) = {}_4C_4 \cdot \left(\frac{1}{4}\right)^4 \cdot \left(\frac{3}{4}\right)^0 = \frac{1}{256} \approx .0039$$

The probability is so small, that I would say that it would be unusual.

e) What's the expected number of them that weigh more than 20 pounds?

$$E(X) = np = 4 \cdot \frac{1}{4} = \boxed{1}$$

2. Airlines frequently overbook flights. On a flight with 50 available seats, they will sell 52 reservations. If the probability that a person with a reserved seat will show up is 90%, then what's the probability that there will be a seat for all the people that show up?

$$
\begin{aligned}
P(X \le 50) &= 1 - P(X > 50) \\
&= 1 - P(X = 51) - P(X = 52) \\
&= 1 - {}_{52}C_{51}(.9)^{51}(.1)^{1} - {}_{52}C_{52}(.9)^{52}(.1)^{0} \\
&= 1 - 52 \cdot \left(\frac{9}{10}\right)^{51}\left(\frac{1}{10}\right) - \left(\frac{9}{10}\right)^{52} \approx .9717 \text{ or } 97.17\%
\end{aligned}
$$

So there's a strong likelihood that all arriving customers will be seated.

3. An opinion poll based upon a small sample can be unrepresentative of the population. Suppose that 40% of voters favor a certain candidate. If a random sample of 7 voters is selected, what is the probability that a majority will favor the candidate?

$$\begin{aligned} P(\text{majority favor the candidate}) &= P(X \geq 4) \\ &= P(X=4)+P(X=5)+P(X=6)+P(X=7) \\ &= {}_7C_4(.4)^4(.6)^3 + {}_7C_5(.4)^5(.6)^2 + {}_7C_6(.4)^6(.6)^1 + {}_7C_7(.4)^7(.6)^0 \\ &= \frac{22,640}{78,125} \approx .29 \text{ or } 29\% \end{aligned}$$

So there's a reasonable probability that the result of the small sample poll will wrongly indicate a majority of support for a candidate who only had 40% support.