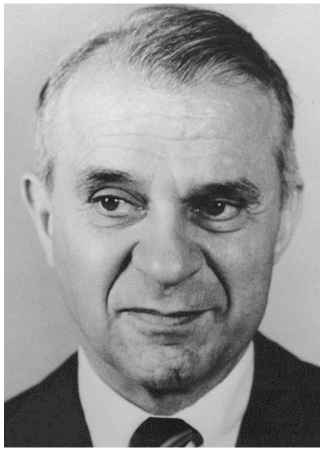
<u>Leontief Input-Output Analysis:</u>

It's a simple mathematical model of an economy.



Wassily Leontief(1905-1999)
Nobel Prize in 1973

<u>Two-Industry Example:</u>

The two industries are a natural gas company, *G*, and a water company, *W*. The natural gas and water companies <u>use</u> both natural gas and water, called inputs to the companies. The natural gas and water companies <u>produce</u> both natural gas and water, respectively, called outputs from the companies. Suppose that the production of each dollar's worth of natural gas requires $.30 worth of natural gas and $.40 worth of water. Further, suppose that the production of each dollar's worth of water requires $.40 worth of natural gas and $.20 worth of water.

The rest of the economy requires amounts of natural gas and water, as well, called final demands. Suppose that the rest of the economy requires \$10 million worth of natural gas and \$8 million worth of water. How much natural gas and water should be produced to meet both the internal and external demands?

Let x_1 **be the total output of natural gas(in millions of dollars) and** x_2 **be the total output of water(in millions of dollars). Also, let** d_1 **be the final demand for natural gas(in millions of dollars) and** d_2 **be the final demand for water(in millions of dollars). Then the following system is our mathematical model for this economy.**

$$\begin{aligned} x_1 &= \underbrace{.3x_1}_{\text{gas for gas}} + \underbrace{.4x_2}_{\text{gas for water}} + d_1 \\ x_2 &= \underbrace{.4x_1}_{\text{water for gas}} + \underbrace{.2x_2}_{\text{water for water}} + d_2 \end{aligned}$$

with $d_1 = 10$ and $d_2 = 8$**, or**

$$\begin{aligned} x_1 &= .3x_1 + .4x_2 + 10 \\ x_2 &= .4x_1 + .2x_2 + 8 \end{aligned}$$

In matrix form, you get.

Called the output matrix, *X*

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} .3 & .4 \\ .4 & .2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 10 \\ 8 \end{bmatrix}$$

Called the technology matrix, *M*

Called the final demand matrix, *D*

Or, written more compactly as

$$X = MX + D.$$

$$\begin{array}{ccc} & & \text{output} \\ & & \begin{array}{cc} G & W \end{array} \\ \text{input} & \begin{array}{c} G \\ W \end{array} & \begin{bmatrix} .3 & .4 \\ .4 & .2 \end{bmatrix} = M \end{array}$$

When constructing the technology matrix, *M*, you need to be careful about the placement of the coefficients.

Let's solve it in general,

$$
\begin{aligned}
&X = MX + D \\
&\Rightarrow X - MX = D \\
&\Rightarrow \left(I - M\right)X = D \\
&\Rightarrow X = \left(I - M\right)^{-1} D
\end{aligned}
$$

So if $\left(I - M\right)^{-1}$ **exists, then we have the solution for the simple economic model.**

In our example, $I - M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} .3 & .4 \\ .4 & .2 \end{bmatrix} = \begin{bmatrix} .7 & -.4 \\ -.4 & .8 \end{bmatrix}$**, and** $\left(I - M\right)^{-1} = \begin{bmatrix} 2 & 1 \\ 1 & 1.75 \end{bmatrix}$**.**

This means that $X = \begin{bmatrix} 2 & 1 \\ 1 & 1.75 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \end{bmatrix} = \begin{bmatrix} 28 \\ 24 \end{bmatrix}$**, so in our specific example, *G* needs to produce $28 million worth of natural gas, and *W* needs to produce $24 million worth of water to meet the needs of the entire economy.**

<u>Another example:</u>

An economy is based on two industrial sectors, agriculture-called *A* and energy-called *E*. The technology matrix is $\begin{matrix} & A & E \\ A & .4 & .2 \\ E & .2 & .1 \end{matrix}$ $\begin{bmatrix} .4 & .2 \\ .2 & .1 \end{bmatrix} = M$ **.**

1) How much input from *A* and *E* is required to produce a dollar's worth of output for *A*?

2) How much input from *A* and *E* is required to produce a dollar's worth of output for *E*?

3) Find $I - M$

4) Find $\left(I - M\right)^{-1}$

5) Find the output for *A* and *E* needed to satisfy the final demand $D = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$**, in millions of dollars.**

6) Find the output for *A* and *E* needed to satisfy the final demand $D = \begin{bmatrix} 8 \\ 5 \end{bmatrix}$**, in millions of dollars.**

7) Find the output for *A* and *E* needed to satisfy the final demand $D = \begin{bmatrix} 12 \\ 9 \end{bmatrix}$**, in millions of dollars.**