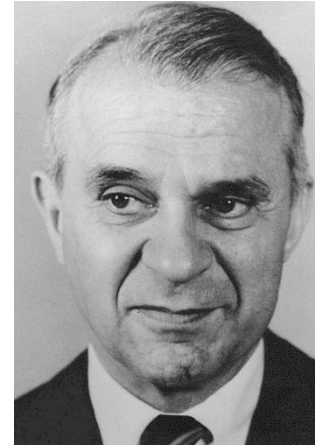


Leontief Input-Output Analysis:

It's a simple mathematical model of an economy.



Wassily Leontief(1905-1999)
Nobel Prize in 1973

Two-Industry Example:

The two industries are a natural gas company, G , and a water company, W . The natural gas and water companies use both natural gas and water, called inputs to the companies. The natural gas and water companies produce both natural gas and water, respectively, called outputs from the companies. Suppose that the production of each dollar's worth of natural gas requires \$.30 worth of natural gas and \$.40 worth of water. Further, suppose that the production of each dollar's worth of water requires \$.40 worth of natural gas and \$.20 worth of water.



The rest of the economy requires amounts of natural gas and water, as well, called final demands. Suppose that the rest of the economy requires \$10 million worth of natural gas and \$8 million worth of water. How much natural gas and water should be produced to meet both the internal and external demands?

Let x_1 be the total output of natural gas(in millions of dollars) and x_2 be the total output of water(in millions of dollars). Also, let d_1 be the final demand for natural gas(in millions of dollars) and d_2 be the final demand for water(in millions of dollars).

Then the following system is our mathematical model for this economy.

$$\begin{array}{lcl}
 x_1 = & .3x_1 + .4x_2 + d_1 & \\
 \text{gas for gas} & \text{gas for water} & \\
 x_2 = & .4x_1 + .2x_2 + d_2 & \text{with } d_1 = 10 \text{ and } d_2 = 8, \text{ or} \\
 \text{water for gas} & \text{water for water} &
 \end{array}$$

$$x_1 = .3x_1 + .4x_2 + 10$$

$$x_2 = .4x_1 + .2x_2 + 8$$

In matrix form, you get.

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} .3 & .4 \\ .4 & .2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 10 \\ 8 \end{bmatrix}$$

Called the output matrix, X
Called the final demand matrix, D

Called the technology matrix, M

Or, written more compactly as

$$X = MX + D.$$

$$\begin{matrix} & \text{output} \\ & G \quad W \\ \text{input} & \begin{matrix} G \\ W \end{matrix} \end{matrix} \begin{bmatrix} .3 & .4 \\ .4 & .2 \end{bmatrix} = M$$

When constructing the technology matrix, M , you need to be careful about the placement of the coefficients.

Let's solve it in general,

$$X = MX + D$$

$$\Rightarrow X - MX = D$$

$$\Rightarrow (I - M)X = D$$

$$\Rightarrow X = (I - M)^{-1} D$$

So if $(I - M)^{-1}$ exists, then we have the solution for the simple economic model.

In our example, $I - M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} .3 & .4 \\ .4 & .2 \end{bmatrix} = \begin{bmatrix} .7 & -.4 \\ -.4 & .8 \end{bmatrix}$, and $(I - M)^{-1} = \begin{bmatrix} 2 & 1 \\ 1 & 1.75 \end{bmatrix}$.

This means that $X = \begin{bmatrix} 2 & 1 \\ 1 & 1.75 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \end{bmatrix} = \begin{bmatrix} 28 \\ 24 \end{bmatrix}$, so in our specific example, G needs to

produce \$28 million worth of natural gas, and W needs to produce \$24 million worth of water to meet the needs of the entire economy.

Another example:

An economy is based on two industrial sectors, agriculture-called *A* and energy-called

E. The technology matrix is
$$\begin{matrix} & \begin{matrix} A & E \end{matrix} \\ \begin{matrix} A \\ E \end{matrix} & \rightarrow \begin{bmatrix} .4 & .2 \\ .2 & .1 \end{bmatrix} = M . \end{matrix}$$



1) How much input from *A* and *E* is required to produce a dollar's worth of output for *A*?

40 cents from *A* and 20 cents from *E*

2) How much input from *A* and *E* is required to produce a dollar's worth of output for *E*?

20 cents from *A* and 10 cents from *E*

3) Find $I - M$

$$I - M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} .4 & .2 \\ .2 & .1 \end{bmatrix} = \begin{bmatrix} .6 & -.2 \\ -.2 & .9 \end{bmatrix}$$

4) Find $(I - M)^{-1}$

$$(I - M)^{-1} = \begin{bmatrix} 1.8 & .4 \\ .4 & 1.2 \end{bmatrix}$$

5) Find the output for A and E needed to satisfy the final demand $D = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$, in millions of dollars.

$$X = \begin{bmatrix} 1.8 & .4 \\ .4 & 1.2 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \end{bmatrix} = \begin{bmatrix} 12.4 \\ 7.2 \end{bmatrix}$$

A needs to produce \$12.4 million worth of agriculture, and E needs to produce \$7.2 million worth of energy to meet the needs of the entire economy.

6) Find the output for A and E needed to satisfy the final demand $D = \begin{bmatrix} 8 \\ 5 \end{bmatrix}$, in millions of dollars.

$$X = \begin{bmatrix} 1.8 & .4 \\ .4 & 1.2 \end{bmatrix} \begin{bmatrix} 8 \\ 5 \end{bmatrix} = \begin{bmatrix} 16.4 \\ 9.2 \end{bmatrix}$$

A needs to produce \$16.4 million worth of agriculture, and E needs to produce \$9.2 million worth of energy to meet the needs of the entire economy.

7) Find the output for A and E needed to satisfy the final demand $D = \begin{bmatrix} 12 \\ 9 \end{bmatrix}$, in millions of dollars.

$$X = \begin{bmatrix} 1.8 & .4 \\ .4 & 1.2 \end{bmatrix} \begin{bmatrix} 12 \\ 9 \end{bmatrix} = \begin{bmatrix} 25.2 \\ 15.6 \end{bmatrix}$$

A needs to produce \$25.2 million worth of agriculture, and E needs to produce \$15.6 million worth of energy to meet the needs of the entire economy.