Math 1324 Review 1(answers)

1. Find the simple interest for a loan of $4,902 at 6.5% for 11 months.

$$I = Prt\text{, so } I = (4902)(.065)\left(\frac{11}{12}\right) = \boxed{\$292.08}$$

2. Find the present value using simple interest of $80,612 in 128 days at 6.77%.

$$P = \frac{A}{1+rt}\text{, so } P = \frac{80{,}612}{1+(.0677)\left(\frac{128}{360}\right)} = \boxed{\$78{,}717.19}$$

3. A couple wishes to have $40,000 in 6 years. At what rate of interest compounded annually must $25,000 be invested now to accomplish this goal?

$$40{,}000 = 25{,}000(1+r)^6\text{, so } r = \sqrt[6]{\frac{40{,}000}{25{,}000}} - 1 \approx 0.0814837... = \boxed{8.15\%}$$

4. Find the future value and the interest earned in each of the 3 years for the following investment: $4,677.23 at 4.57% compounded monthly for 3 years.

$$A = P(1+i)^n\text{, so } A = 4677.23\left(1+\frac{.0457}{12}\right)^{36} = \boxed{\$5{,}363.12}$$

Year	Value	Interest Earned
1	$4677.23\left(1+\frac{.0457}{12}\right)^{12} = \$4{,}895.51$	$\$4{,}895.51 - \$4{,}677.23 = \boxed{\$218.28}$
2	$4677.23\left(1+\frac{.0457}{12}\right)^{24} = \$5{,}123.98$	$\$5{,}123.98 - \$4{,}895.51 = \boxed{\$228.47}$
3	$4677.23\left(1+\frac{.0457}{12}\right)^{36} = \$5{,}363.12$	$\$5{,}363.12 - \$5{,}123.98 = \boxed{\$239.14}$

5. Find the present value of $1,347 in 3.5 years at 6.2% compounded semi-annually.

$$P = \frac{A}{(1+i)^n}\text{, so } P = \frac{1347}{\left(1+\frac{.062}{2}\right)^7} = \boxed{\$1{,}087.82}$$

6. Calculate the effective rate for 4.9% compounded daily.

$$r_{effective} = \left(1+\frac{r}{m}\right)^m - 1\text{, so } r_{effective} = \left(1+\frac{.049}{360}\right)^{360} - 1 = .050216849 \text{ or } \boxed{5.02\%}$$

7. Find the interest rate that makes \$3,000 grow to \$3,907 in 6 years with annual compounding.

$$i = \sqrt[n]{\frac{A}{P}} - 1, \text{ so } i = \sqrt[6]{\frac{3907}{3000}} - 1 = .045009791 \text{ or } \boxed{4.5\%}$$

Find the future value of the following annuities (8-9).

8. \$4,000 deposited at the end of each quarter for 8 years at 6% compounded quarterly

$$FV = Pmt \cdot \left[\frac{(1+i)^n - 1}{i}\right], \text{ so } FV = 4000 \cdot \left[\frac{\left(1+\frac{.06}{4}\right)^{32} - 1}{\frac{.06}{4}}\right] = \boxed{\$162,753.15}$$

9. \$11,900 deposited at the end of each month for 13 months at 7% compounded monthly

$$FV = Pmt \cdot \left[\frac{(1+i)^n - 1}{i}\right], \text{ so } FV = 11,900 \cdot \left[\frac{\left(1+\frac{.07}{12}\right)^{13} - 1}{\frac{.07}{12}}\right] = \boxed{\$160,232.02}$$

10. Find the periodic payment needed to accumulate \$57,000 at 6% compounded semi-annually for 8½ years with an ordinary annuity.

$$Pmt = FV \cdot \left[\frac{i}{(1+i)^n - 1}\right], \text{ so } Pmt = 57000 \cdot \left[\frac{\frac{.06}{2}}{\left(1+\frac{.06}{2}\right)^{17} - 1}\right] = \boxed{\$2,619.29}$$

11. Find the present value of the ordinary annuity with payments of \$1,500 quarterly for 7 years at 8% compounded quarterly.

$$PV = Pmt \cdot \left[\frac{1-(1+i)^{-n}}{i}\right], \text{ so } PV = 1500 \cdot \left[\frac{1-\left(1+\frac{.08}{4}\right)^{-28}}{\frac{.08}{4}}\right] = \boxed{\$31,921.91}$$

12. Find the periodic payment needed to amortize a loan of \$32,000 at 8.4% compounded quarterly for 10 quarters.

$$Pmt = PV \cdot \left[\frac{i}{1-(1+i)^{-n}}\right], \text{ so } Pmt = 32000 \cdot \left[\frac{\frac{.084}{4}}{1-\left(1+\frac{.084}{4}\right)^{-10}}\right] = \boxed{\$3,581.11}$$

13. Estimate the remaining balance on a loan of $100,000 at 5% compounded monthly for 30 years after 100 payments.

$$Bal = PV \cdot \left[\frac{1-(1+i)^{k-n}}{1-(1+i)^{-n}}\right], \text{ so } Bal = 100000 \cdot \left[\frac{1-\left(1+\frac{.05}{12}\right)^{100-360}}{1-\left(1+\frac{.05}{12}\right)^{-360}}\right] = \$85{,}132.00$$

Solve the following.

14. Complete the following amortization table for a loan of $5,000 at 5% compounded quarterly for 1 year.

Payment Number	Payment Amount	Interest Payment	Principal Payment	Remaining Balance
0	-	-	-	$5,000
1	$1,289.31	$62.50	$1,226.81	$3,773.19
2	$1,289.31	$3773.19 \cdot \left(\frac{.05}{4}\right) = \$47.16$	1289.31 − 47.16 $1,242.15	3773.19 −1242.15 $2,531.04
3	$1,289.31	$2531.04 \cdot \left(\frac{.05}{4}\right) = \$31.64$	1289.31 − 31.64 $1,257.67	2531.04 −1257.67 $1,273.37
4	1273.37 + 15.92 $1,289.29	$1273.37 \cdot \left(\frac{.05}{4}\right) = \$15.92$	$1,273.37	$0

15. Cathy wants to retire on $55,000 per year for 20 years. She estimates that she will be able to earn 9% compounded annually throughout her lifetime. To reach her retirement goal, Cathy will make annual contributions to her account for the next 25 years. One year after making her last contribution, she will take her first retirement check.

a) How much money will Cathy need in her account after her final payment to generate an annuity of $55,000 a year for 20 years?

This amount would be the present value of an ordinary annuity:

$$PV = Pmt \cdot \left[\frac{1-(1+i)^{-n}}{i}\right], \text{ so } PV = 55000 \cdot \left[\frac{1-(1+.09)^{-20}}{.09}\right] = \boxed{\$502{,}070.01}$$

b) How large should Cathy's annual contributions be to accumulate the amount you calculated in part a)?

$$Pmt = PV \cdot \left[\frac{i}{(1+i)^{n}-1}\right], \text{ so } Pmt = 502070.01 \cdot \left[\frac{.09}{(1+.09)^{25}-1}\right] = \boxed{\$5{,}927.56}$$

16. Estimate the annual interest rate for an ordinary annuity with payments of $300 a month compounded monthly for 20 years yielding $147,126.

We want $147{,}126 = 300 \cdot \left[\dfrac{(1+i)^{240} - 1}{i}\right]$

i	$A = 300 \cdot \left[\dfrac{(1+i)^{240}-1}{i}\right]$	**Want to be close to $147,126**
0.005412	$147,027.48	
0.005413	$147,048.65	
0.005414	$147,069.81	
0.005415	$147,090.99	
0.005416	$147,112.16	
0.005417	$147,133.34	This one is pretty close.
0.005418	$147,154.52	

So $r \approx 12 \cdot i = 12 \cdot (.005417) = \boxed{6.5\%}$.

17. The Beyes plan to purchase a home for $212,000. They will pay 20% down and finance the remainder for 30 years at 7.2% compounded monthly.

a) What are their monthly payments?

$$Pmt = PV \cdot \left[\frac{i}{1-(1+i)^{-n}}\right], \text{ so } Pmt = 169600 \cdot \left[\frac{\frac{.072}{12}}{1-\left(1+\frac{.072}{12}\right)^{-360}}\right] = \boxed{\$1{,}151.22}$$

b) How much in interest will they pay over the course of the loan?

$$\text{total interest} = n \cdot Pmt - \text{loan amount}, \text{ so}$$
$$\text{total interest} = 360 \cdot \$1151.22 - \$169{,}600 = \boxed{\$244{,}839.20}$$

18. Use the effective rate to rank the following compound interest schemes in ascending order:

Scheme #1: 6% compounded monthly
Scheme #2: 5.9% compounded weekly
Scheme #3: 5.8% compounded daily

$$r_{effective_1} = \left(1+\frac{.06}{12}\right)^{12} - 1 = .061677812$$

$$r_{effective} = \left(1+\frac{r}{m}\right)^{m} - 1, \text{ so } r_{effective_2} = \left(1+\frac{.059}{52}\right)^{52} - 1 = .060739763$$

$$r_{effective_3} = \left(1+\frac{.058}{360}\right)^{360} - 1 = .059710045$$

so in ascending order it's Scheme #3, Scheme #2, Scheme #1

19. A car stereo dealer sells a stereo system for \$600 down and monthly payments of \$30 for the next 3 years. If the interest rate is 1.25% per month on the unpaid balance, find

a) the cost of the stereo system

Cost = \$600 + *PV*

$$PV = Pmt \cdot \left[\frac{1-(1+i)^{-n}}{i}\right], \text{ so } PV = 30 \cdot \left[\frac{1-(1+.0125)^{-36}}{.0125}\right] = \$865.42, \text{ so the cost}$$

is $\boxed{\$1,465.42}$.

b) the total amount of interest paid

$$I = nPmt - PV, \text{ so } I = 36(30) - 865.42 = \boxed{\$214.58}$$