Math 1324 Review 3(answers)

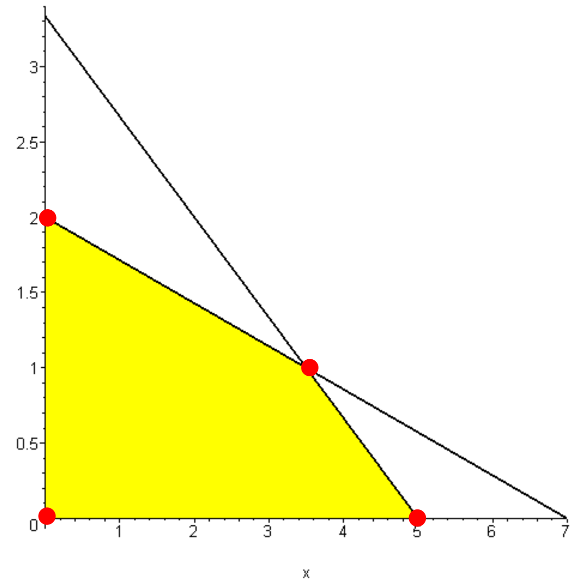
Use the method of corner points to solve the following linear programming problems.

1. Maximize and Minimize $z = 2x + 4y$

Subject to $2x + 7y \le 14$

$2x + 3y \le 10$

$x, y \ge 0$



Corner point	$z = 2x + 4y$
$(0,0)$	0
$(0,2)$	8
$(5,0)$	10
$(\frac{7}{2},1)$	11

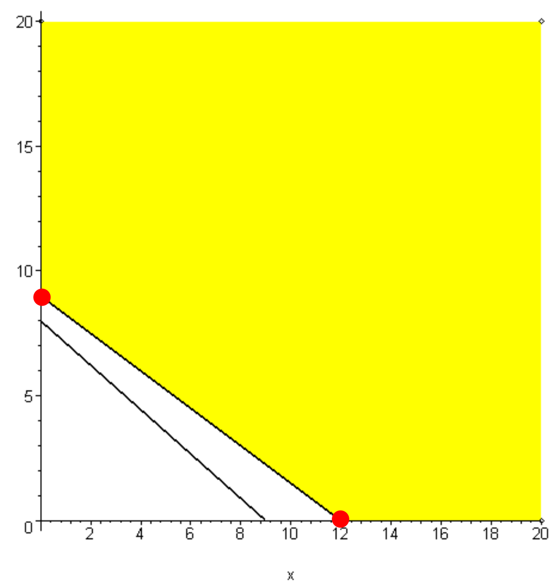
Maximum is 11, and minimum is 0.

2. Maximize and Minimize $z = 10x + 2y$

Subject to $8x + 9y \ge 72$

$6x + 8y \ge 72$

$x, y \ge 0$



Corner point	$z = 10x + 2y$
$(0,9)$	18
$(12,0)$	120

Minimum is 18, and there is no maximum.

Use the Simplex Method to solve the following linear programming problems.

3. Maximize $z = x_1 + 3x_2$

Subject to
$$x_1 + x_2 \le 10$$
$$5x_1 + 2x_2 \le 20$$
$$x_1 + 2x_2 \le 36$$
$$x_1, x_2 \ge 0$$

$$\left[\begin{array}{cccccc|c} 1 & \textcircled{1} & 1 & 0 & 0 & 0 & 10 \\ 5 & 2 & 0 & 1 & 0 & 0 & 20 \\ 1 & 2 & 0 & 0 & 1 & 0 & 36 \\ \hline -1 & -3 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \leftarrow \qquad \left[\begin{array}{cccccc|c} 1 & 1 & 1 & 0 & 0 & 0 & 10 \\ 3 & 0 & -2 & 1 & 0 & 0 & 0 \\ -1 & 0 & -2 & 0 & 1 & 0 & 16 \\ \hline 2 & 0 & 3 & 0 & 0 & 1 & 30 \end{array}\right]$$

$\uparrow$

$-2R_1 + R_2 \to R_2$

$-2R_1 + R_3 \to R_3$

$3R_1 + R_4 \to R_4$

Maximum is 30 and it occurs when $x_1 = 0$ *and* $x_2 = 10$.

4. Maximize $z = 5x_1 + 4x_2 + x_3$

Subject to
$$-2x_1 + x_2 + 2x_3 \le 3$$
$$x_1 - x_2 + x_3 \le 1$$
$$x_1, x_2, x_3 \ge 0$$

$$\left[\begin{array}{cccccc|c} -2 & 1 & 2 & 1 & 0 & 0 & 3 \\ \textcircled{1} & -1 & 1 & 0 & 1 & 0 & 1 \\ \hline -5 & -4 & -1 & 0 & 0 & 1 & 0 \end{array}\right] \leftarrow \qquad \left[\begin{array}{cccccc|c} 0 & -1 & 4 & 1 & 2 & 0 & 5 \\ 1 & -1 & 1 & 0 & 1 & 0 & 1 \\ \hline 0 & -9 & 4 & 0 & 5 & 1 & 5 \end{array}\right]$$

$\uparrow$ (column 1 of the first tableau) $\qquad \uparrow$ (column 2 of the second tableau)

$2R_2 + R_1 \to R_1$

$5R_2 + R_3 \to R_3$

The problem has no solution.

Solve the following linear programming word problems.

5. A bakery makes cakes and cookies. Each batch of cakes requires 2 hours in the oven and 3 hours in the decorating room. Each batch of cookies needs 1 ½ hours in the oven and ⅔ of an hour in the decorating room. The oven is available no more than 15 hours, while the decorating room can be used no more than 13 hours. A batch of cookies makes a profit of $20, and a batch of cakes makes a profit of $30. How many batches of each should be made to maximize profit?

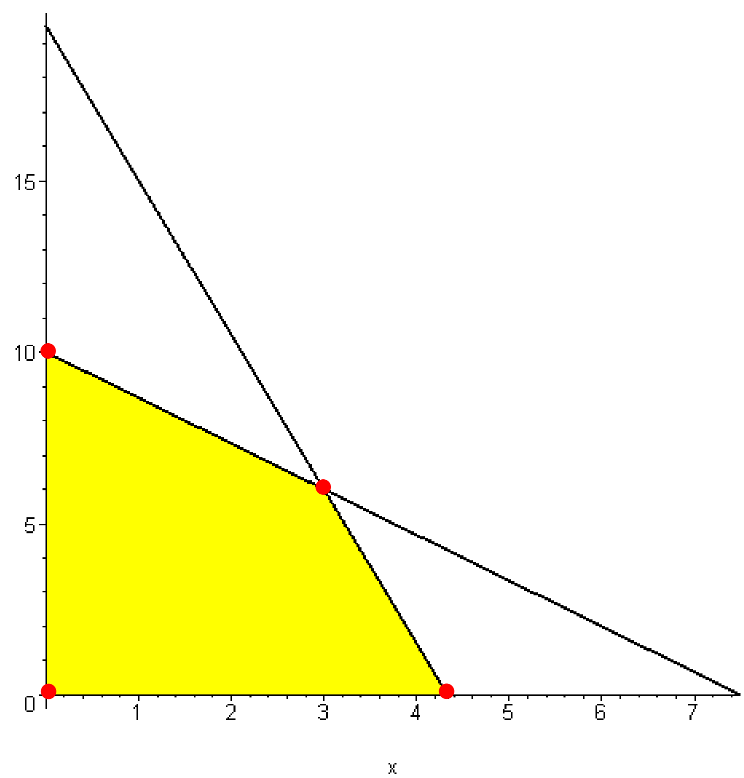
$$x = \#\ \textit{of batches of cakes}$$

$$y = \#\ \textit{of batches of cookies}$$

Oven constraint	$2x + \frac{3}{2}y \le 15$
Decorating room constraint	$3x + \frac{2}{3}y \le 13$
Nonnegative constraint	$x, y \ge 0$

You can use the Corner Point Method or the Simplex Method:

Corner point	$P = 30x + 20y$
$(0,0)$	0
$(0,10)$	200
$(\frac{13}{3},0)$	130
$(3,6)$	210



To maximize profit, 3 batches of cakes and 6 batches of cookies should be made.

$$\left[\begin{array}{ccccc|c} 2 & \frac{3}{2} & 1 & 0 & 0 & 15 \\ \textcircled{3} & \frac{2}{3} & 0 & 1 & 0 & 13 \\ \hline -30 & -20 & 0 & 0 & 1 & 0 \end{array}\right] \leftarrow \left[\begin{array}{ccccc|c} 2 & \frac{3}{2} & 1 & 0 & 0 & 15 \\ 1 & \frac{2}{9} & 0 & \frac{1}{3} & 0 & \frac{13}{3} \\ \hline -30 & -20 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\tfrac{1}{3}R_2 \to R_2$$

$$-2R_2 + R_1 \to R_1$$
$$30R_2 + R_3 \to R_3$$

$$\left[\begin{array}{ccccc|c} 0 & \textcircled{\frac{19}{18}} & 1 & -\frac{2}{3} & 0 & \frac{19}{3} \\ 1 & \frac{2}{9} & 0 & \frac{1}{3} & 0 & \frac{13}{3} \\ \hline 0 & -\frac{40}{3} & 0 & 10 & 1 & 130 \end{array}\right] \leftarrow \left[\begin{array}{ccccc|c} 0 & 1 & \frac{18}{19} & -\frac{12}{19} & 0 & 6 \\ 1 & \frac{2}{9} & 0 & \frac{1}{3} & 0 & \frac{13}{3} \\ \hline 0 & -\frac{40}{3} & 0 & 10 & 1 & 130 \end{array}\right]$$

$$\tfrac{18}{19}R_1 \to R_1$$

$$-\tfrac{2}{9}R_1 + R_2 \to R_2$$
$$\tfrac{40}{3}R_1 + R_3 \to R_3$$

$$\left[\begin{array}{ccccc|c} 0 & 1 & \frac{18}{19} & -\frac{12}{19} & 0 & 6 \\ 1 & 0 & -\frac{4}{19} & \frac{27}{57} & 0 & 3 \\ \hline 0 & 0 & \frac{240}{19} & \frac{30}{19} & 1 & 210 \end{array}\right]$$

So using Simplex we also come to the conclusion of maximizing profit by making 3 batches of cakes and 6 batches of cookies.

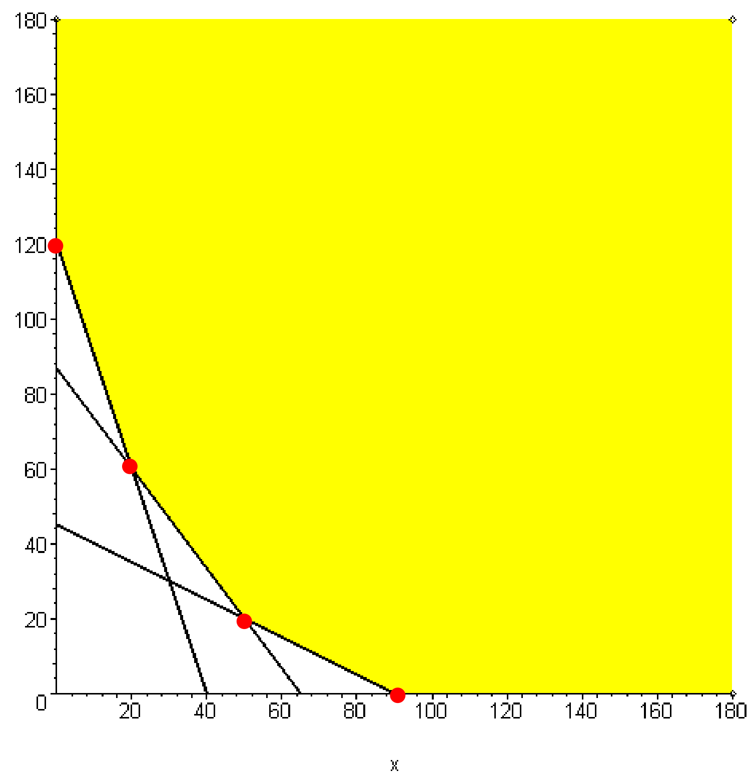
6. An oil company needs at least 9,000, 12,000, and 26,000 barrels of high grade, medium grade, and low grade oil, respectively. It owns two oil refineries, A and B. Refinery A produces 100, 300, and 400 barrels of oil, respectively per day. Refinery B produces 200, 100, and 300 barrels of oil, respectively per day. Find the number of days that each refinery should run in order to meet the requirements and minimize costs if refinery A costs $30,000 per day to operate and refinery B costs $20,000 per day to operate.

$$x = \text{\# of days for refinery } A$$
$$y = \text{\# of days for refinery } B$$

High grade constraint	$100x + 200y \geq 9{,}000$
Medium grade constraint	$300x + 100y \geq 12{,}000$
Low grade constraint	$400x + 300y \geq 26{,}000$
Nonnegative constraint	$x, y \geq 0$

The Simplex Method is not an option for this problem.

Corner point	$C = 30{,}000x + 20{,}000y$
$(0,120)$	$2{,}400{,}000$
$(90,0)$	$2{,}700{,}000$
$(50,20)$	$1{,}900{,}000$
$(20,60)$	$1{,}800{,}000$



For the minimum cost, refinery A should run for 20 days, and refinery B should run for 60 days.

7. In solving a standard maximization problem using Simplex, you are given the following initial tableau:

$$\left[\begin{array}{cccccc|c} 4 & 2 & 3 & 1 & 0 & 0 & 9 \\ 5 & 4 & 1 & 0 & 1 & 0 & 10 \\ \hline -6 & -7 & -5 & 0 & 0 & 1 & 0 \end{array}\right]$$

a) What is the problem being solved?

Maximize $z = 6x_1 + 7x_2 + 5x_3$
subject to $4x_1 + 2x_2 + 3x_3 \le 9$
$5x_1 + 4x_2 + x_3 \le 10$
$x_1, x_2, x_3 \ge 0$

b) After several steps of Simplex, the following tableau results. What is the solution?

$$\left[\begin{array}{cccccc|c} 3 & 0 & 1 & 2 & -1 & 0 & 8 \\ 11 & 1 & 0 & -1 & 3 & 0 & 21 \\ \hline 47 & 0 & 0 & 13 & 11 & 1 & 227 \end{array}\right]$$

The maximum value is 227 when $x_1 = 0$, $x_2 = 21$, and $x_3 = 8$.