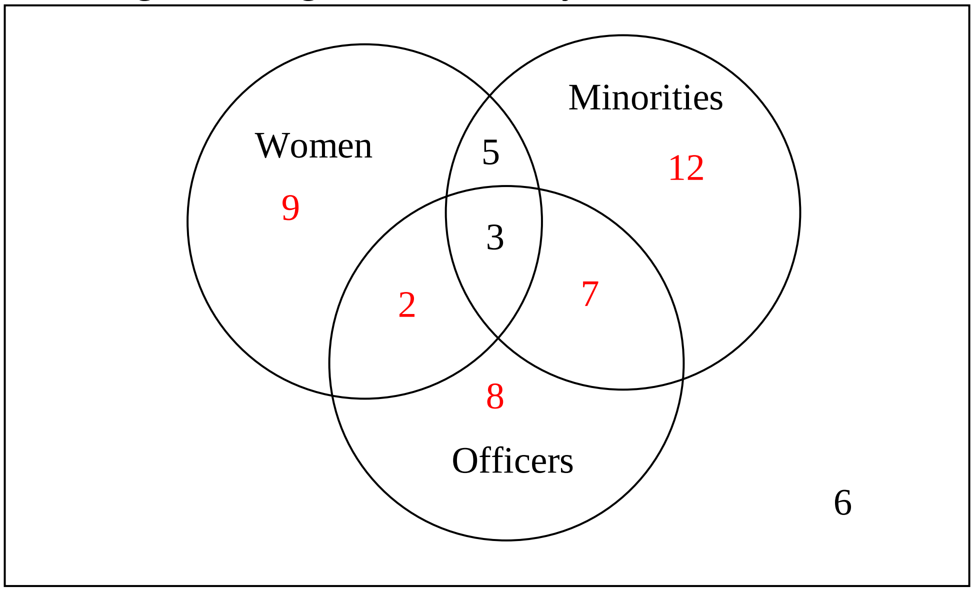
Math 1324 Review 4

A survey of a group of military personnel revealed that the respondents could be categorized as follows:

20 officers
27 minorities
19 women
5 women officers
8 minority women
10 minority officers
3 women minority officers
6 non-minority enlisted men

1. Complete the following Venn diagram of the survey:



2. How many were interviewed?

52

3. How many were enlisted minority women?

5

4. How many were male minority officers?

7

A jar contains 5 discs labeled 2,4,6,8, and 10, and another jar contains 2 blue and 3 yellow marbles. One disc is drawn, and then a marble is drawn.

5. Complete the sample space, S, for this experiment.

$(2, blue)$	$(4, blue)$	$(6, blue)$	$(8, blue)$	$(10, blue)$
$(2, yellow)$	$(4, yellow)$	$(6, yellow)$	$(8, yellow)$	$(10, yellow)$

6. List the outcomes in the event, F, that the marble is blue.

$$F = \{(2, blue), (4, blue), (6, blue), (8, blue), (10, blue)\}$$

7. List the outcomes in the event, E, that the disc shows a number greater than 5.

$$E = \{(6, blue), (6, yellow), (8, blue), (8, yellow), (10, blue), (10, yellow)\}$$

8. Are the outcomes in this sample space equally likely?

No

A single card is randomly drawn from a standard 52-card deck. Determine the following probabilities or odds:

9. probability of a red queen

$$P(\text{red queen}) = \frac{\text{\# of red queens}}{\text{\# of cards}} = \frac{2}{52} = \boxed{\frac{1}{26}}$$

10. probability of a red card or a face card

$$\begin{aligned} P(\text{red card or face card}) &= P(\text{red card}) + P(\text{face card}) - P(\text{red face card}) \\ &= \frac{26}{52} + \frac{12}{52} - \frac{6}{52} \\ &= \frac{32}{52} = \boxed{\frac{8}{13}} \end{aligned}$$

OR

$$P(\text{red card or face card}) = \frac{\text{\# of red cards or face cards}}{\text{\# of cards}} = \frac{32}{52} = \boxed{\frac{8}{13}}$$

11. probability of a black card, given that it is a 10

$$P(\text{black card} \mid 10) = \frac{P(\text{black } 10)}{\text{P}(10)} = \frac{\frac{2}{52}}{\frac{4}{52}} = \boxed{\frac{1}{2}}$$

OR

$$P(\text{black card} \mid 10) = \frac{\text{\# of black 10's}}{\text{\# of 10's}} = \frac{2}{4} = \boxed{\frac{1}{2}}$$

12. probability of a face card, given that it is a king

$$P(\text{face card} \mid \text{king}) = \boxed{1}, \text{ a king is a face card!}$$

13. the odds in favor of drawing a club

$$\begin{aligned} &P(\text{clubs}) : P(\text{clubs}') \\ &\frac{13}{52} : \frac{39}{52} \\ &\boxed{1:3} \end{aligned}$$

OR

$$\frac{P(\text{clubs})}{P(\text{clubs}')} = \frac{\frac{13}{52}}{\frac{39}{52}} = \boxed{\frac{1}{3}}$$

14. the odds in favor of drawing a face card or a 9

$$P(\text{face card or 9}):P((\text{face card or 9})')$$

$$\frac{16}{52}:\frac{36}{52}$$

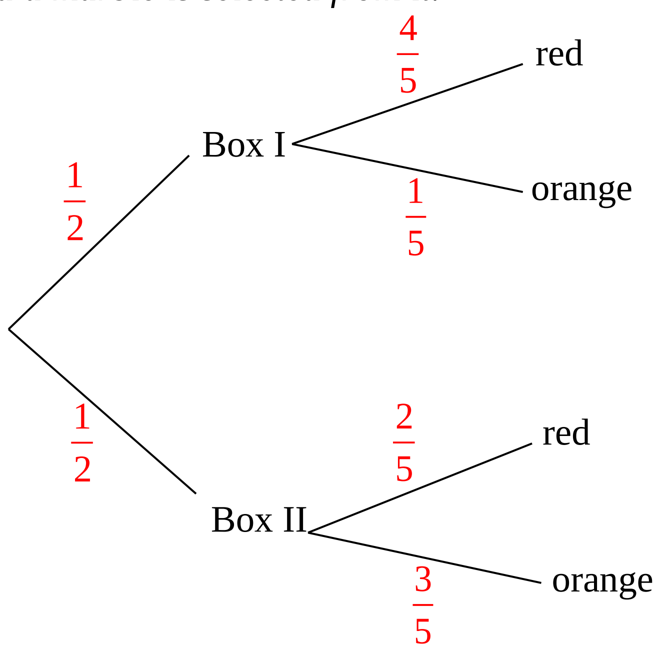
$$\boxed{4:9}$$

OR

$$\frac{P(\text{face card or 9})}{P((\text{face card or 9})')}=\frac{\frac{16}{52}}{\frac{36}{52}}=\boxed{\frac{4}{9}}$$

One orange and four red marbles are placed in Box I. Two red and three orange marbles are placed in Box II. A box is chosen at random, and a marble is selected from it.

15. Complete the following probability tree



16. $P(red)$

$$P(red)=\frac{1}{2}\cdot\frac{4}{5}+\frac{1}{2}\cdot\frac{2}{5}=\boxed{\frac{3}{5}}$$

17. $P(orange)$

$$P(orange)=\frac{1}{2}\cdot\frac{1}{5}+\frac{1}{2}\cdot\frac{3}{5}=\boxed{\frac{2}{5}}$$

OR

$$P(orange)=1-P(orange')=1-P(red)=1-\frac{3}{5}=\boxed{\frac{2}{5}}$$

18. $P(Box\ I|red)$

$$P(Box\ I|red)=\frac{P(Box\ I\ and\ red)}{P(red)}=\frac{\frac{1}{2}\cdot\frac{4}{5}}{\frac{3}{5}}=\boxed{\frac{2}{3}}$$

19. $P(Box\ I|orange)$

$$P(Box\ I|orange)=\frac{P(Box\ I\ and\ orange)}{P(orange)}=\frac{\frac{1}{2}\cdot\frac{1}{5}}{\frac{2}{5}}=\boxed{\frac{1}{4}}$$

20. $P(Box\ II|red)$

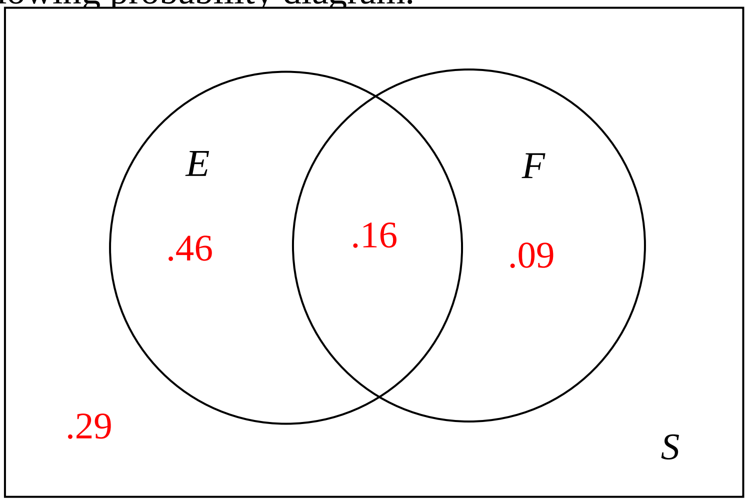
$$P(Box\ II|red)=\frac{P(Box\ II\ and\ red)}{P(red)}=\frac{\frac{1}{2}\cdot\frac{2}{5}}{\frac{3}{5}}=\boxed{\frac{1}{3}}$$

21. $P(Box\ II|orange)$

$$P(Box\ II|orange)=\frac{P(Box\ II\ and\ orange)}{P(orange)}=\frac{\frac{1}{2}\cdot\frac{3}{5}}{\frac{2}{5}}=\boxed{\frac{3}{4}}$$

Suppose that $P(E)=.62$, $P(F)=.25$, *and* $P(E\cap F)=.16$.

22. Complete the following probability diagram:



23. $P(E\cup F)$

$.46+.16+.09=\boxed{.71}$

24. $P(E\cap F')$

$\boxed{.46}$

25. $P(E'\cap F')$

$\boxed{.29}$

26. $P(E\mid F)$

$$P(E\mid F)=\frac{P(E\cap F)}{P(F)}=\frac{.16}{.25}=\boxed{\frac{16}{25}}$$

27. $P(F\mid E)$

$$P(F\mid E)=\frac{P(F\cap E)}{P(E)}=\frac{.16}{.62}=\boxed{\frac{8}{31}}$$

28. Are E and F independent?

$\boxed{No}$

The table gives the results of a survey question which asked: "Are federal income taxes too high, about right, too low, or don't know?". If a respondent is chosen at random, determine the following:

	Too High	About Right	Too Low	Don't Know	Total
Male	289	192	6	10	497
Female	257	153	3	14	427
Total	546	345	9	24	924

29. $P(\text{Female})$

$$\frac{427}{924}=\boxed{\frac{61}{132}}$$

30. $P(\text{About Right})$

$$\frac{345}{924}=\boxed{\frac{115}{308}}$$

31. $P(\text{Male or Too Low})$

$$\frac{500}{924}=\boxed{\frac{125}{231}}$$

32. $P(\text{Female and Don't Know})$

$$\frac{14}{924}=\boxed{\frac{1}{66}}$$

33. $P(\text{Female}\mid\text{Don't Know})$

$$\frac{14}{24}=\boxed{\frac{7}{12}}$$

34. Are *Female* and *Don't Know* independent?

$\boxed{No}$

35. Three cards are drawn at random from an ordinary 52-card deck.

a) How many different 3-card hands are possible?

$$C_{52,3}=\frac{52!}{49!\cdot 3!}=\frac{52\cdot 51\cdot 50}{3\cdot 2\cdot 1}=26\cdot 17\cdot 50=\boxed{22{,}100}$$

b) What is the probability that the 3-card hand has ***exactly two*** kings?

$$P(2\text{ kings})=\frac{C_{4,2}\cdot C_{48,1}}{22{,}100}=\frac{\frac{4!}{2!\cdot 2!}\cdot 48}{22{,}100}=\frac{6\cdot 48}{22{,}100}=\boxed{\frac{72}{5525}}$$

c) What is the probability that the 3-card hand has ***exactly two*** face cards?

$$P(2\text{ face cards})=\frac{C_{12,2}\cdot C_{40,1}}{22{,}100}=\frac{66\cdot 40}{22{,}100}=\boxed{\frac{132}{1105}}$$

d) What is the probability that the 3-card hand contains ***at least 1*** black card?

$$P(\text{at least 1 black card})=1-P(\text{no black cards})=1-\frac{C_{26,3}}{22{,}100}=1-\frac{\frac{26!}{23!\cdot 3!}}{22{,}100}=1-\frac{\frac{26\cdot 25\cdot 24}{3\cdot 2\cdot 1}}{22{,}100}$$

$$=1-\frac{2600}{22{,}100}=1-\frac{26}{221}=1-\frac{2}{17}=\boxed{\frac{15}{17}}$$

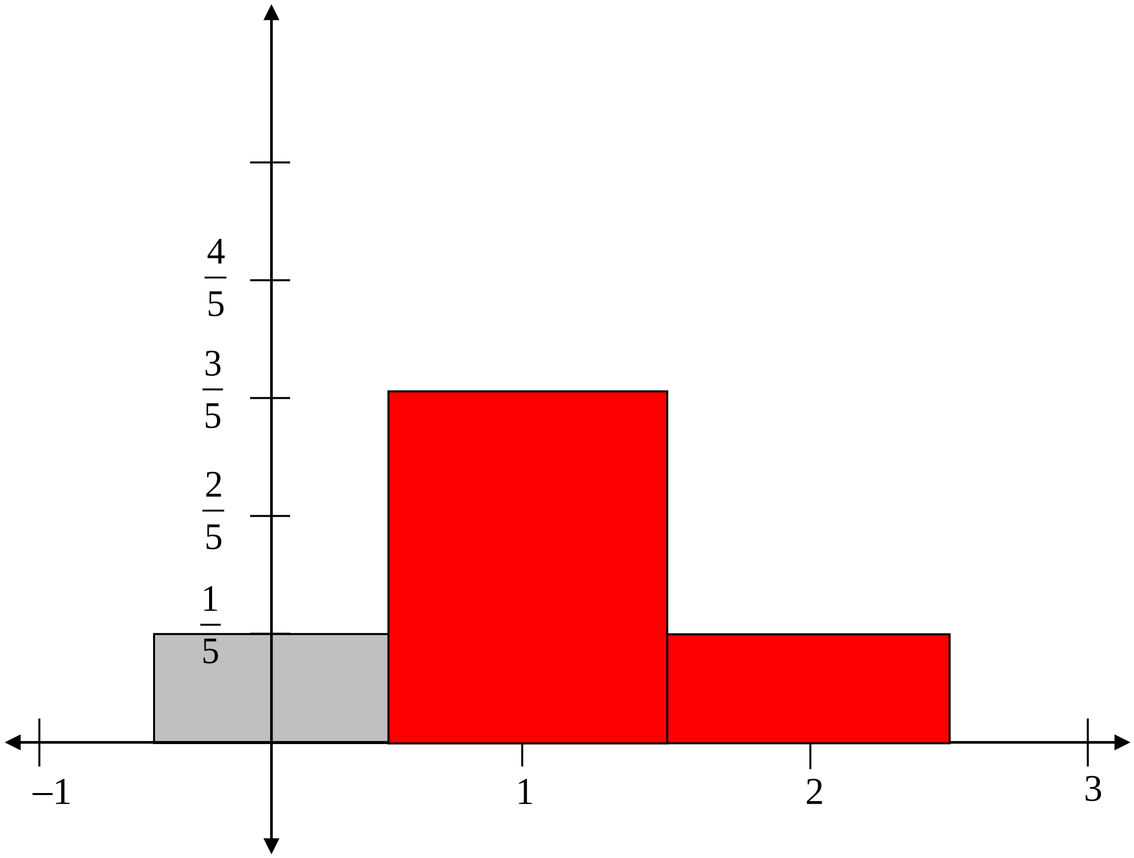
36. Two names are randomly drawn from a hat without replacement. Three of the names in the hat are Aggies, two are Longhorns, and one is a Cougar. Let the random variable, x, be the total number of Aggies selected.
The sample space for this experiment is

(L_1,A_1)	(L_1,A_2)	(L_1,A_3)	(L_1,C_1)
(L_2,A_1)	(L_2,A_2)	(L_2,A_3)	(L_2,C_1)
(A_1,C_1)	(A_2,C_1)	(A_3,C_1)	(L_1,L_2)
(A_1,A_2)	(A_1,A_3)	(A_2,A_3)	

a) Complete the probability distribution for the random variable:

x	0	1	2
$P(x)$	$\frac{1}{5}$	$\boxed{\frac{3}{5}}$	$\boxed{\frac{1}{5}}$

b) Complete the histogram for the random variable:



c) Find the expected value of the random variable.

$$E(x) = 0 \cdot \tfrac{1}{5} + 1 \cdot \tfrac{3}{5} + 2 \cdot \tfrac{1}{5} = \boxed{1}$$