Odds and Probability:

The odds in favor of an event *E* is the ratio of the probability that *E* will occur to the probability that *E* won't occur.

Odds in favor of *E*: $P(E):P(\bar{E})$ **or** $P(E)\text{ to }P(\bar{E})$

The odds are usually expressed as a ratio of whole numbers.

Example:

If $P(E)=\frac{2}{5}$**, then find the odds in favor of *E*.**

$$\frac{2}{5}\text{ to }\frac{3}{5}\Rightarrow 5\cdot\left(\frac{2}{5}\text{ to }\frac{3}{5}\right)=\boxed{2\text{ to }3}$$

The odds against an event *E* is the ratio of the probability that *E* won't occur to the probability that *E* will occur, i.e. the reversal of the odds in favor.

Odds against *E*: $P\left(\bar{E}\right):P\left(E\right)$ **or** $P\left(\bar{E}\right)$ to $P\left(E\right)$

Example:

If $P\left(E\right)=\frac{3}{7}$**, then find the odds against *E*.**

$$7\cdot\left(\frac{4}{7}\text{ to }\frac{3}{7}\right)=\boxed{4\text{ to }3}$$

Sometimes you'll want to go from odds to probability. If the odds in favor of *E* is *a* to *b*, then $\frac{P\left(E\right)}{P\left(\bar{E}\right)}=\frac{a}{b}\Rightarrow\frac{P\left(E\right)}{1-P\left(E\right)}=\frac{a}{b}$**. Cross-multiplying leads to**

$$bP\left(E\right)=a-aP\left(E\right)\Rightarrow\left(a+b\right)P\left(E\right)=a\Rightarrow P\left(E\right)=\frac{a}{a+b}.$$

Examples:

If the odds in favor of *E* is 4 to 7, then find $P(E)$.

$$P(E)=\frac{4}{4+7}=\boxed{\frac{4}{11}}$$

If the odds against *E* is 3 to 2, then find $P(E)$.

$$P(E)=\frac{2}{2+3}=\boxed{\frac{2}{5}}$$

Estimating Probabilities Using Simulations:

1. Estimate the probability of rolling a double in two rolls of a fair die.

The larger the number of trials of the simulation, the more accurate the estimate of the probability.

Trial #	First Roll	Second Roll
1	4	4
2	1	3
3	5	2
4	1	1
5	3	6
6	2	3
7	1	4
8	3	3
9	4	6
10	1	5
11	2	6
12	6	3
13	5	5
14	4	3
15	6	3
16	2	5
17	2	1
18	2	2
19	3	6
20	4	3

In 20 trials of the experiment, a double occurred 5 times, so our estimate for the probability of rolling a double with this simulation would be $\frac{5}{20}$ or $\frac{1}{4}$.

Check out the link Rolling a Double.

What's the exact probability of rolling a double? $\frac{6}{36}=\boxed{\frac{1}{6}}$

Second Roll

First Roll

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

2. Estimate the probability of getting a sum of at least 8 in two rolls of a fair die.

Trial #	First Roll	Second Roll	Sum
1	4	4	8
2	1	3	4
3	5	2	7
4	1	1	2
5	3	6	9
6	2	3	5
7	1	4	5
8	3	3	6
9	4	6	10
10	1	5	6
11	2	6	8
12	6	3	9
13	5	5	10
14	4	3	7
15	6	3	9
16	2	5	7
17	2	1	3
18	2	2	4
19	3	6	9
20	4	3	7

Based upon this simulation, what's the estimate for the probability of getting a sum of at least 8 in two rolls of a fair die? Of the 20 trials, 8 of them resulted in a sum of 8 or more, so the estimate for the probability is $\frac{8}{20} = \boxed{\frac{2}{5}}$.

What's the exact probability of getting a sum of at least 8 in two rolls of a fair die?

$$\frac{15}{36} = \boxed{\frac{5}{12}}$$

	Second Roll					
First Roll	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Estimating Expected Values Using Simulations: The larger the number of trials, the more accurate the estimate.

1. Estimate the expected number of flips of a fair coin required to have tails occur.

Flip #	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10
1	H	T	H	T	H	T	H	T	T	H
2	H	T	H	T	T	T	T	T	H	T
3	T	T	T	T	H	T	H	H	T	T
4	H	H	H	T	T	T	H	T	H	T
5	H	T	T	T	T	H	T	H	H	H
6	T	T	T	T	T	H	T	H	T	T
7	H	H	T	H	T	T	H	T	T	H
# required	3	1	3	1	2	1	2	1	1	2

The average number of flips required in the 10 trials of our simulation is our estimate for the expected number of flips required to have tails occur. What's the estimate?

$$\frac{3+1+3+1+2+1+2+1+1+2}{10}=\frac{17}{10}=\boxed{1.7}$$

What’s the exact expected value?

Let *X* be the number of flips to get tails.

X	**1**	**2**	**3**	**4**	**5**	…
P(*X*)	**T** $\frac{1}{2}$	**HT** $\frac{1}{4}$	**HHT** $\frac{1}{8}$	**HHHT** $\frac{1}{16}$	**HHHHT** $\frac{1}{32}$	…

$$
\begin{array}{rcccccccccccccc}
E\left(X\right) & = & 1\cdot\frac{1}{2} & + & 2\cdot\frac{1}{4} & + & 3\cdot\frac{1}{8} & + & 4\cdot\frac{1}{16} & + & 5\cdot\frac{1}{32} & + & 6\cdot\frac{1}{64} & + & \cdots \\
 & = & \frac{1}{2} & + & \frac{1}{4} & + & \frac{1}{8} & + & \frac{1}{16} & + & \frac{1}{32} & + & \frac{1}{64} & + & \cdots \\
 & & & & \frac{1}{4} & + & \frac{1}{8} & + & \frac{1}{16} & + & \frac{1}{32} & + & \frac{1}{64} & + & \cdots \\
 & & & & & & \frac{1}{8} & + & \frac{1}{16} & + & \frac{1}{32} & + & \frac{1}{64} & + & \cdots \\
 & & & & & & & & \frac{1}{16} & + & \frac{1}{32} & + & \frac{1}{64} & + & \cdots \\
 & & & & & & & & \vdots & & & & & &
\end{array}
$$

$$E\left(X\right)=1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots=1+1=2$$

2. Estimate the expected number of flips of a fair coin required to see both heads and tails.

Flip #	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10
1	H	T	H	T	H	T	H	T	T	H
2	H	T	H	T	T	T	T	T	H	T
3	T	T	T	T	H	T	H	H	T	T
4	H	H	H	T	T	T	H	T	H	T
5	H	T	T	T	T	H	T	H	H	H
6	T	T	T	T	T	H	T	H	T	T
7	H	H	T	H	T	T	H	T	T	H
# required	3	4	3	7	2	5	2	3	2	2

The average number of flips required in the 10 trials of our simulation is our estimate for the expected number of flips required to see both heads and tails. What's the estimate? ***See the link Coin Toss.***

$$\frac{3+4+3+7+2+5+2+3+2+2}{10}=\frac{33}{10}=\boxed{3.3}$$

What's the exact expected value? After 1 flip, you have a heads or tails, so you just need the other one. From the previous calculation, the expected number of flips to get tails (or heads) is 2. So the exact expected number of flips to see both is $1+2=3$**.**

3. Estimate the expected number of rolls of a fair die needed to see all 6 numbers.

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
Roll 1	2	4	6	1	6
Roll 2	2	1	1	2	5
Roll 3	4	6	1	6	5
Roll 4	4	4	2	2	4
Roll 5	3	3	5	4	4
Roll 6	2	5	6	5	5
Roll 7	1	3	4	3	5
Roll 8	1	4	4		2
Roll 9	6	3	3		5
Roll 10	2	6			5
Roll 11	1	3			6
Roll 12	4	3			6
Roll 13	5	6			5
Roll 14		1			3
Roll 15		4			6
Roll 16		1			4
Roll 17		2			5
Roll 18					1
# of rolls	13	17	9	7	18

Average the 5 numbers of rolls for the 5 trials to get the estimate for the expected value from the simulation. $\frac{13+17+9+7+18}{5}=\frac{64}{5}=\boxed{12.8}$

See the link Collecting. The exact expected value is 14.7.