<u>*Van Hiele Theory of Geometric Development:*</u>

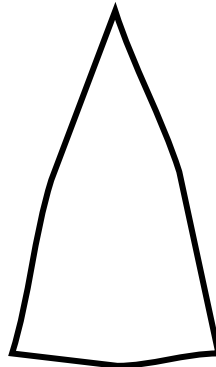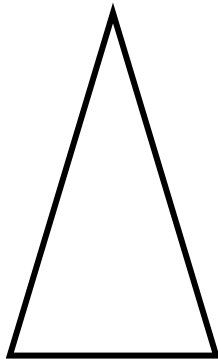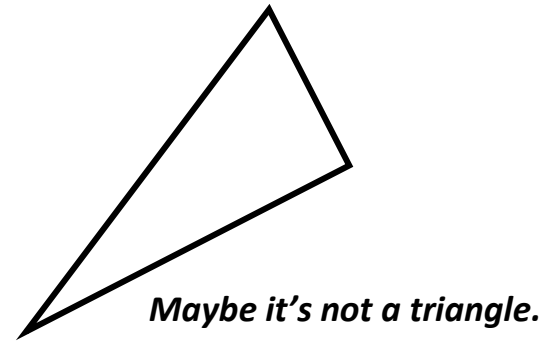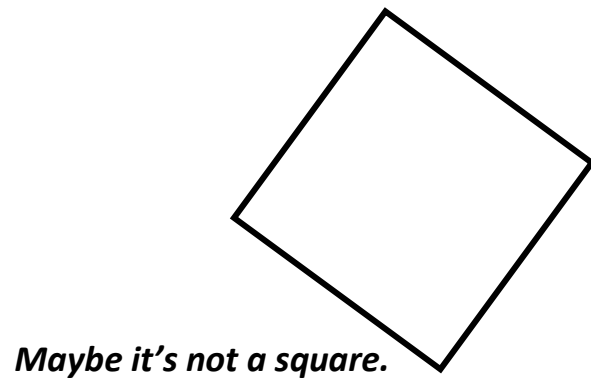<u>Level 0:</u> **Shapes are recognized as a whole without regard to the details of their components-sides, angles, vertices,…. Shapes are compared to everday objects-door, stop sign, yield sign,….**
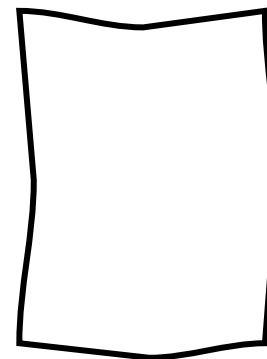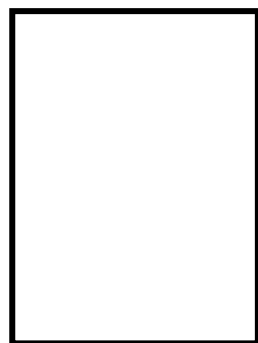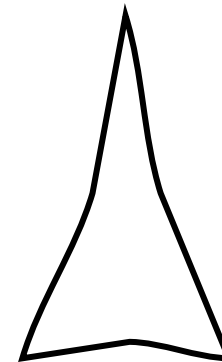
The orientation of a figure has an effect on perception.

Maybe it's not a square.

Maybe it's not a triangle.

Level 1: **The components of shapes are recognized, and used to describe and characterize figures. The orientation of figures doesn't have an effect on perception. The fact that one figure can fall into several different categories might not be recognized, e.g. a square is also a rectangle, a rhombus, a parallelogram, and a quadrilateral.**

Level 2: The fact that one figure can fall into several different categories is recognized. The use and understanding of informal deduction is possible, e.g. an isosceles triangle is a triangle with at least 2 congruent sides, and an equilateral triangle is a triangle with 3 congruent sides, so therefore, every equilateral triangle is also an isosceles triangle.

Level 3: The idea of formal deduction is understood. Proofs are constructed using axioms, postulates, theorems, and the rules of basic logic. This is a middle school to high school level of geometry.

Level 4: The study of geometry at this level is abstract-no pictures or diagrams. The axioms and postulates themselves are the object of study. This is a collegiate level of geometry.
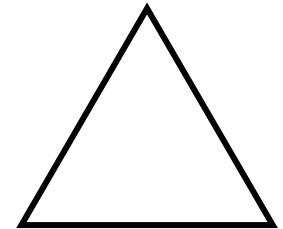
Geometric Glossary:

Triangle: It's a closed figure made up of three line segments joined at their endpoints called sides that form three angles called vertices.
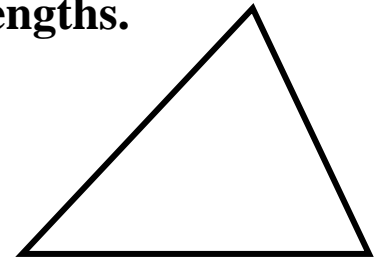
Isosceles Triangle: It's a triangle with at least two sides of the same length.
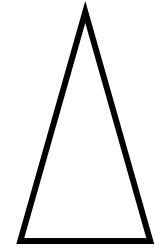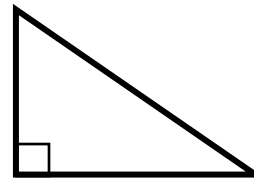
Equilateral Triangle: It's a triangle with all three sides the same length.

Scalene Triangle: It's a triangle with all three sides having different lengths.

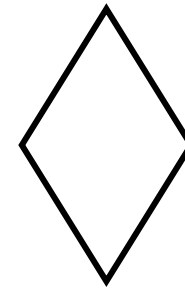**Right Triangle:** It's a triangle with one right angle.

Quadrilateral: It's a closed figure made up of four line segments joined at their endpoints called sides that form four angles called vertices.

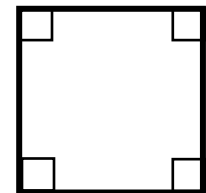**Rectangle:** It's a quadrilateral with four right angles.

**Rhombus:** It's a quadrilateral with all four sides the same length.

Parallelogram: It's a quadrilateral with two pairs of parallel sides.

Square: It's a quadrilateral with all four sides the same length and four right angles.

Here is a variety of shapes. Sides in a shape with the same length are indicated. Right angles are indicated. Assume that sides that appear parallel actually are.
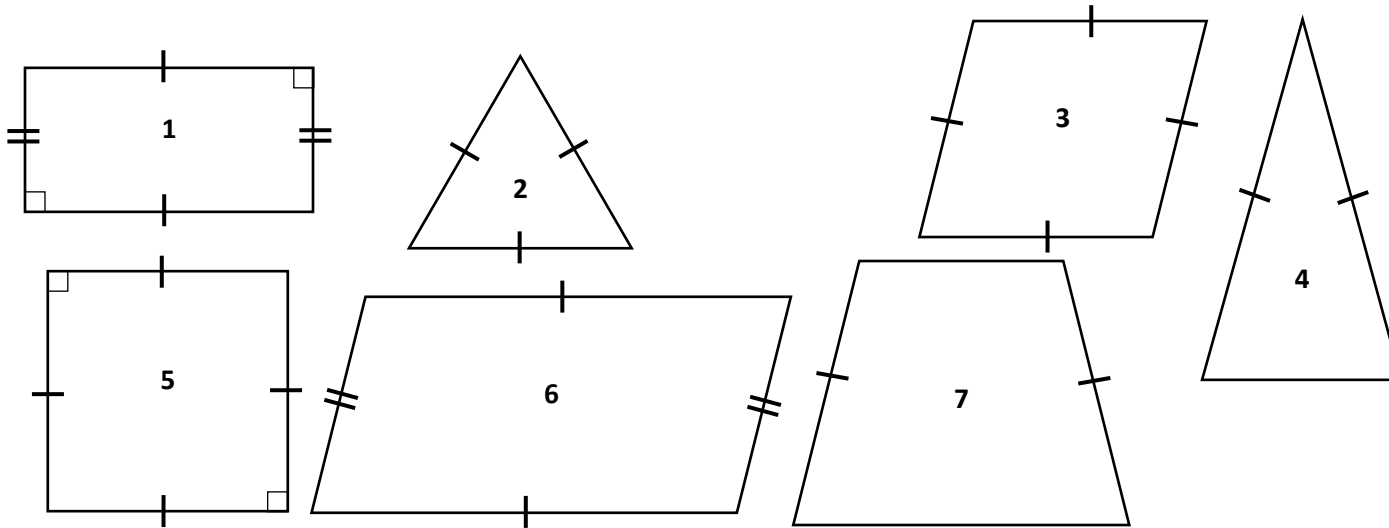
1) **Which shapes are quadrilaterals?**
   1,3,5,6,7

2) **Which shapes are triangles?**
   2,4

3) **Which shapes are rectangles?**
   1,5

4) **Which shapes are parallelograms?**
   1,3,5,6

5) **Which shapes are squares?**
   5

6) **Which shapes are rhombi?**
   3,5

7) **Which shapes are isosceles triangles?**
   2,4

8) **Which shapes are equilateral triangles?** 2

How many squares are in the following figure consisting of 8 identical squares?
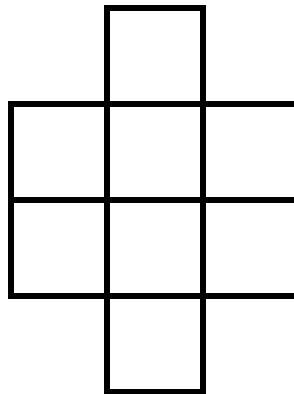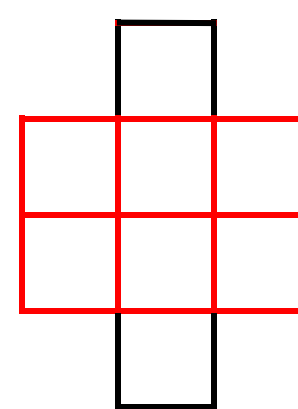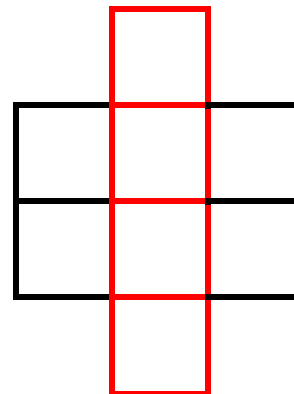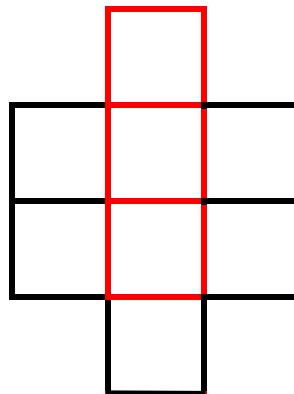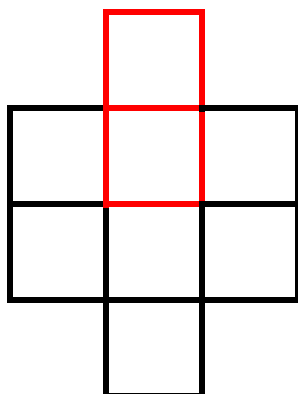
Systematic Counting:

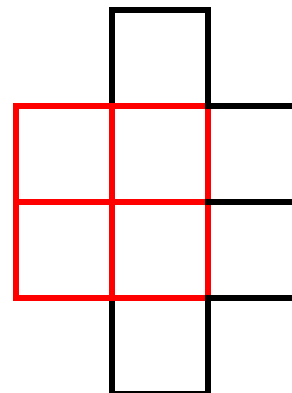| 1 small square	4 small squares	Total
8	2	10
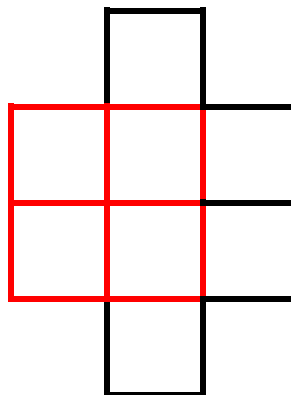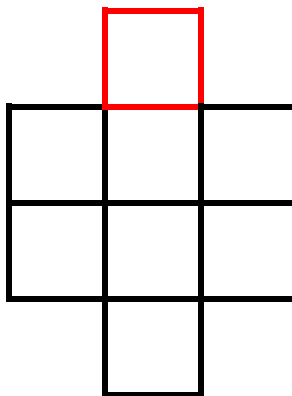
How many rectangles?
Systematic Counting:

| 1 small square	2 small squares	3 small squares	4 small squares	6 small squares
8	9	4	3	1
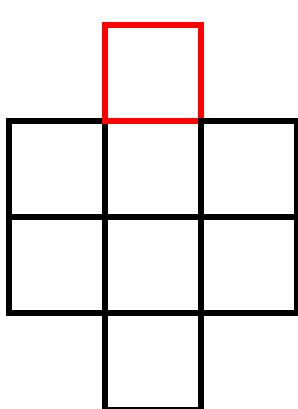
$8+9+4+3+1=\boxed{25}$

Symmetry:

Reflection Symmetry:

A figure has reflection symmetry if there is a line called the axis of symmetry along which the figure can be folded and have the portion of the figure on one side of the line exactly match up with the portion of the figure on the other side of the line.
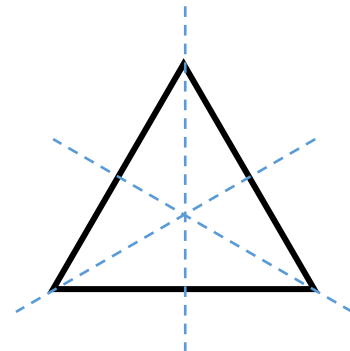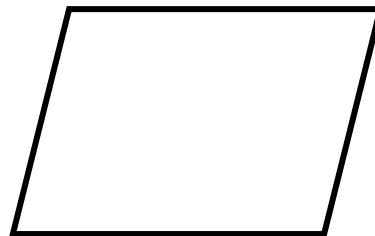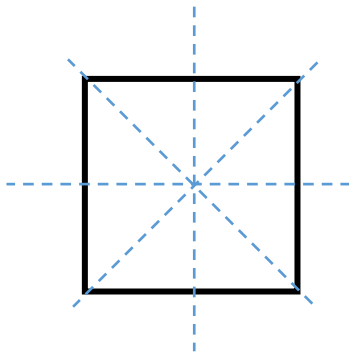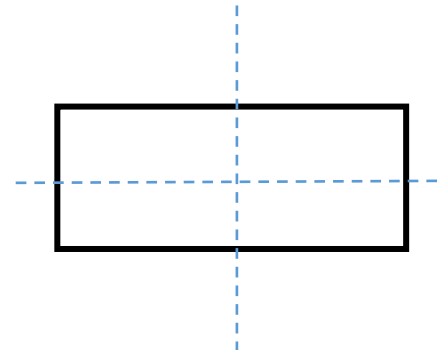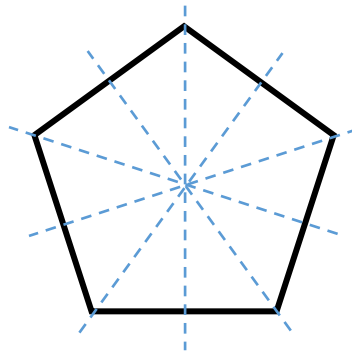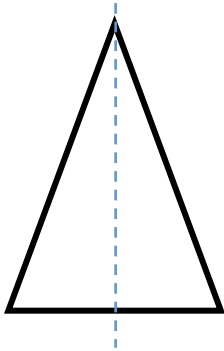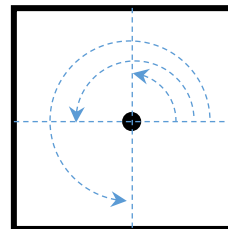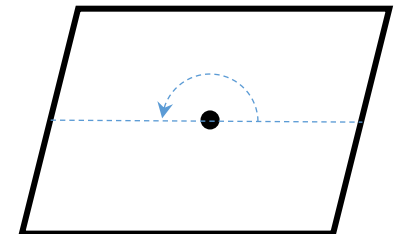
Rotation Symmetry:

A figure has rotation symmetry if there is a point called the center of rotation around which a copy of the figure can be rotated less than a full turn and exactly match up with the original figure. The amount of rotation that leads to an exact matchup is called an angle of rotation symmetry.
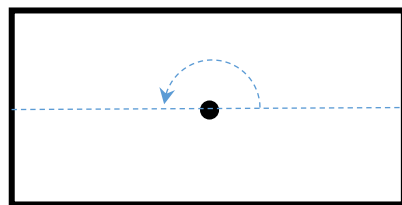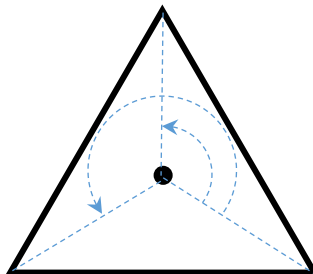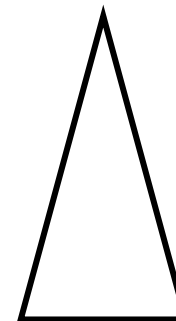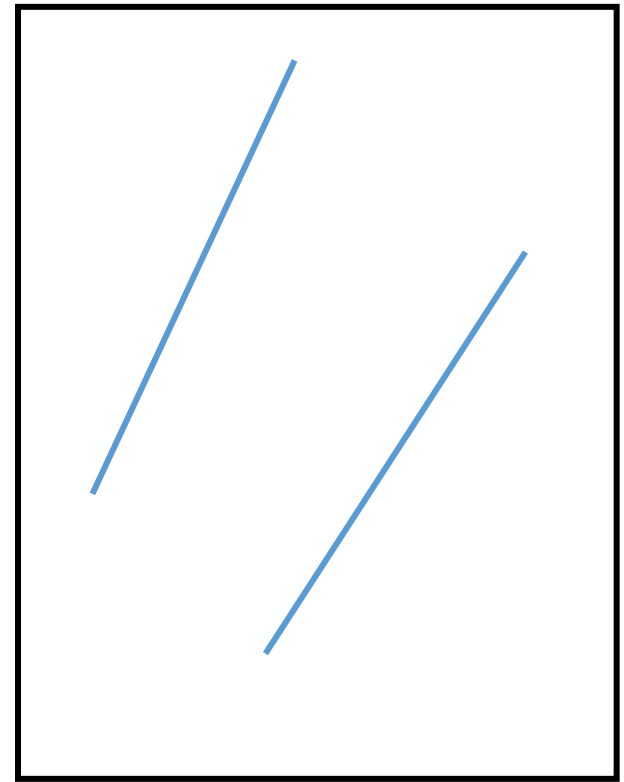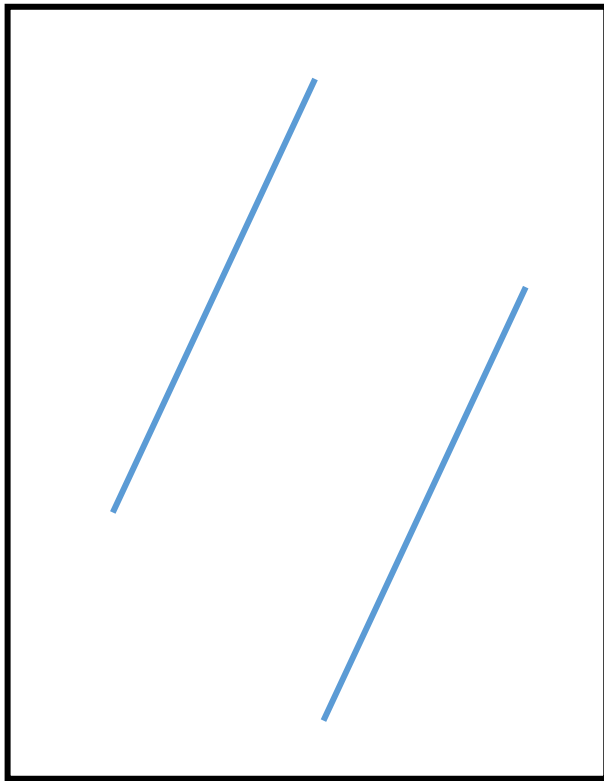
180°

90°,180°,270°

180°

120°,240°

Paper Folding Tests:

Parallel Line Segments Test:

Line segments, *l* and *m* are parallel if when you fold line segment *l* onto itself, line segment *m* is folded onto itself or an extension of itself.

Perpendicular Line segments Test:

The intersecting line segments *l* and *m* are perpendicular if when you fold the line segment *l* onto itself at the point of intersection, the line segment *m* lies along the fold line.