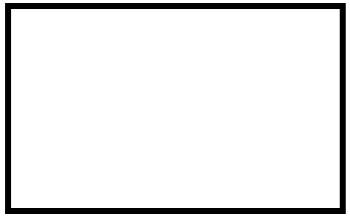
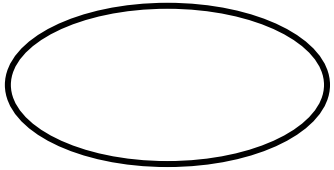
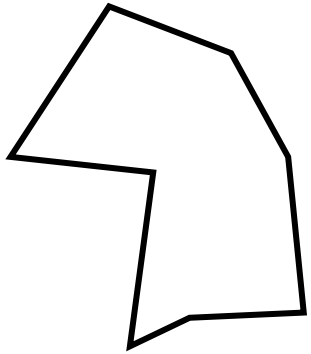
Simple Closed Curve:

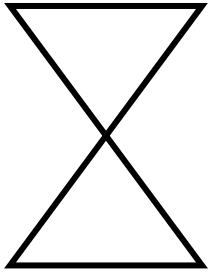
It's a curve in the plane that can be traced with the same starting and stopping point without crossing or retracing any part of the curve.

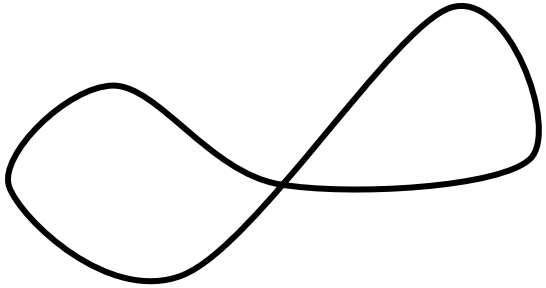


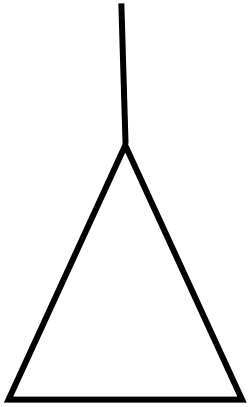




The above are simple closed curves.



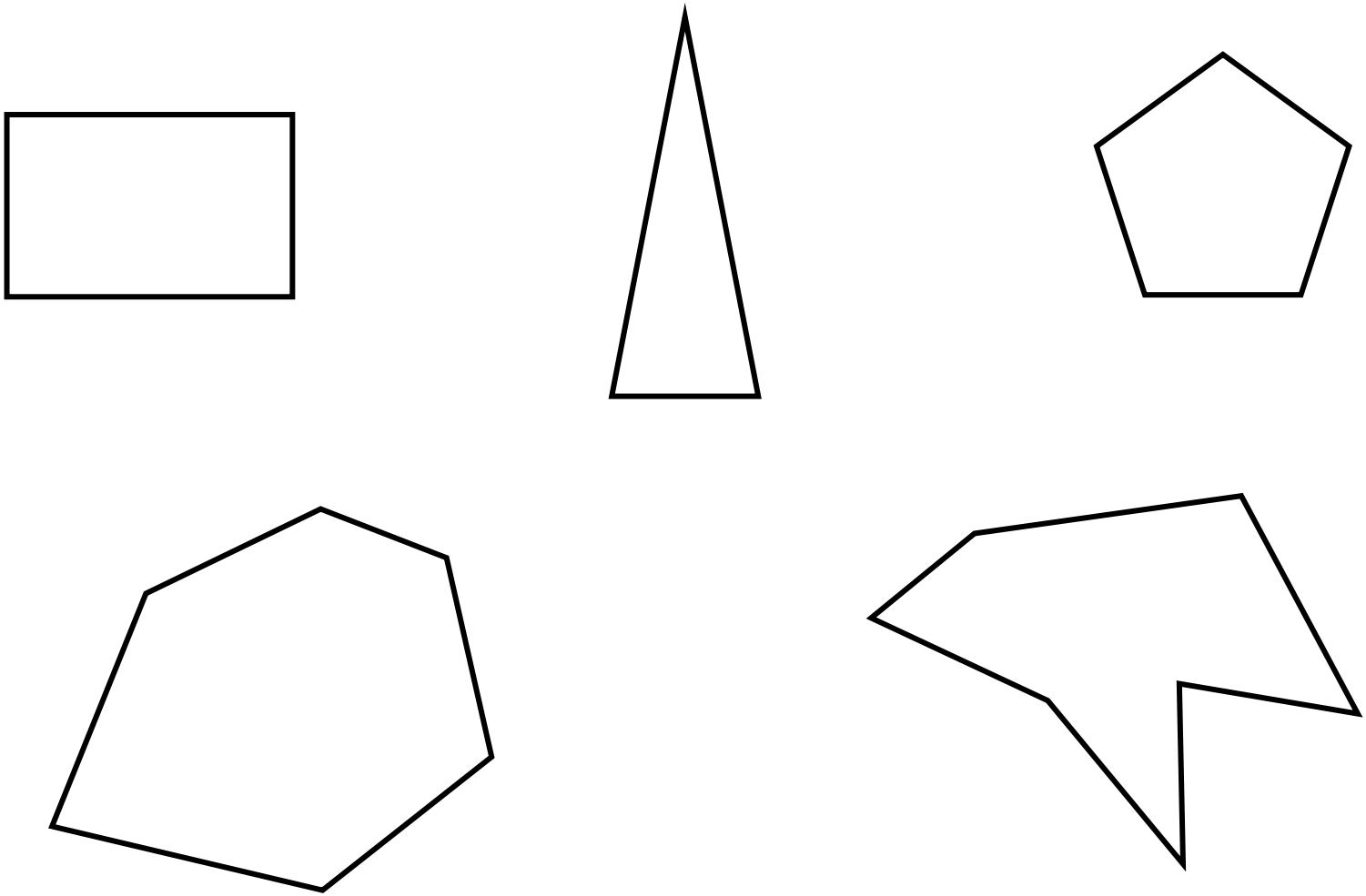




The above are not simple closed curves.

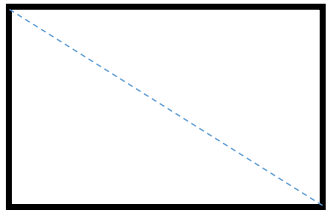
Polygon:

It's a simple closed curve made up of line segments.

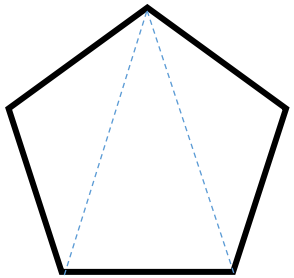


The Angle Sum of a Polygon:

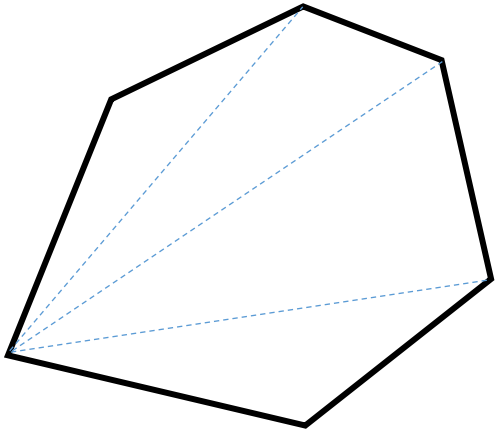
The angle sum of a polygon can be determined by dissecting it into triangles and using the angle sum of a triangle.



2 triangles = 360°

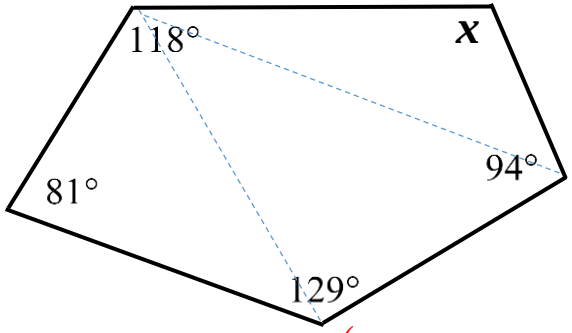


3 triangles = 540°

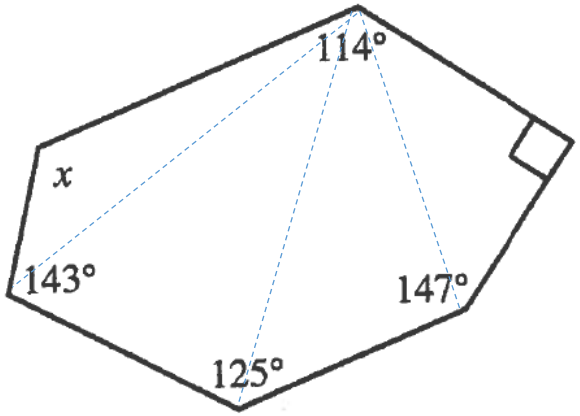


4 triangles = 720°

Find the missing angle measure in the following polygons.



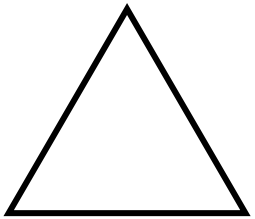
3 triangles = $540°$, so the missing angle is $540° - (118° + 81° + 129° + 94°) = 118°$.



4 triangles = $720°$, so the missing angle is $720° - (143° + 125° + 147° + 90° + 114°) = 101°$.

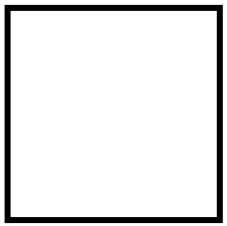
Regular Polygon/Regular *n*-gon:

It's a polygon with all sides of the same length and all vertex angles of the same measure. The *n* refers to the number of sides, and all of them enclose a convex region.



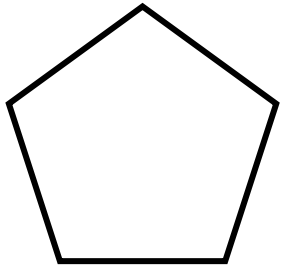
3-gon

equilateral triangle



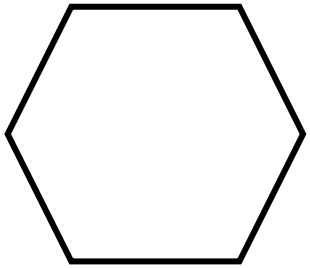
4-gon

square



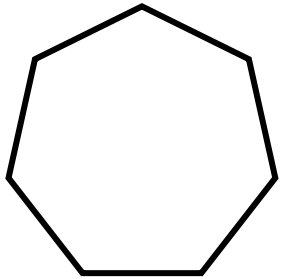
5-gon

regular pentagon



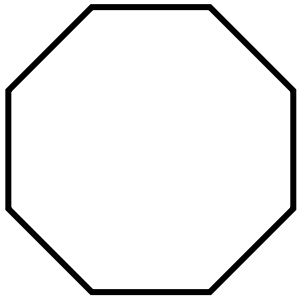
6-gon

regular hexagon



7-gon

regular heptagon

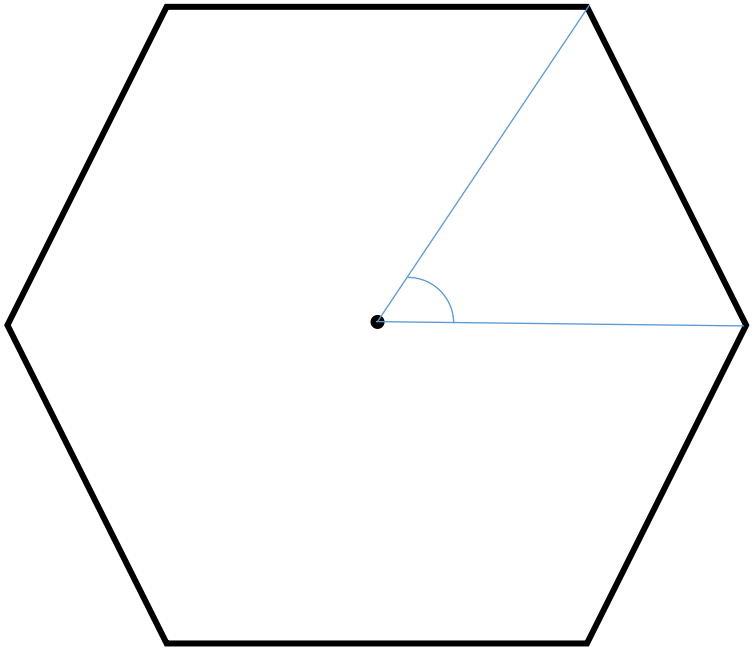


8-gon

regular octagon

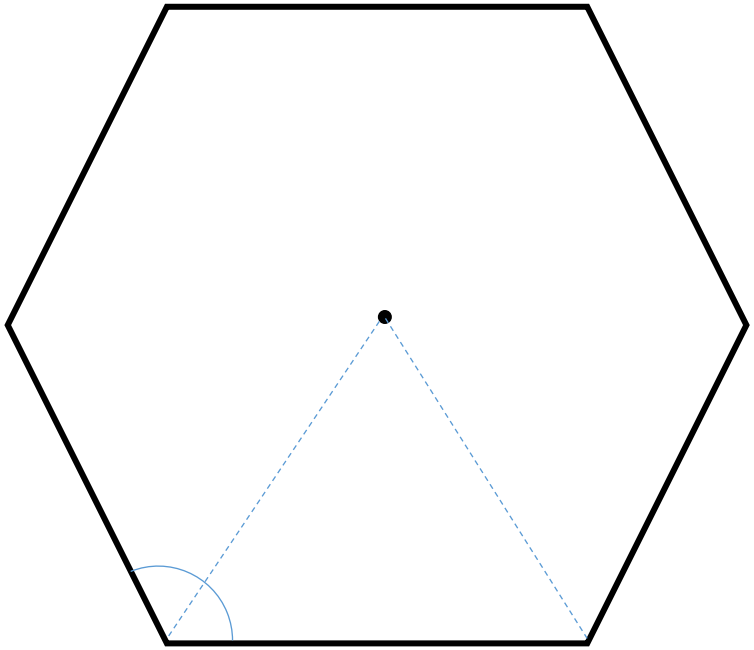
The Angles of a Regular n-gon:

Central Angle:



Since it takes *n* central angles to get $360°$**,** $\text{central angle} = \frac{360°}{n}$**.**

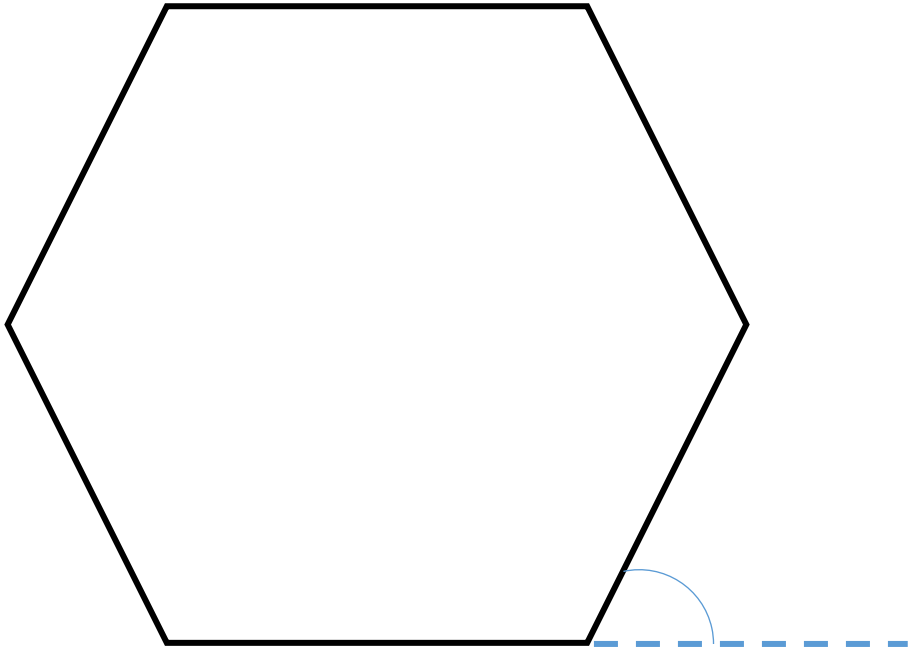
Vertex Angle:



Since a central angle and a vertex angle give $180°$**,** $\text{vertex angle} = 180° - \frac{360°}{n}$**, or**

$$\text{vertex angle} = \frac{180°n - 360°}{n} = \frac{(n-2)180°}{n}.$$

Exterior Angle:



Since vertex angle and exterior angle give $180°$**, and vertex angle and central angle give** $180°$**, then exterior angle equals central angle.**

$$\text{exterior angle} = \frac{360°}{n}$$

Angle Sum of a Regular *n*-gon:

Since the measure of a vertex angle of a regular *n*-gon is $\frac{(n-2)180°}{n}$, and there are *n* of them in the regular *n*-gon, the angle sum must be $n \cdot \frac{(n-2)180°}{n} = (n-2)180°$.

n	**central angle** $\frac{360°}{n}$	**vertex angle** $180° - \frac{360°}{n}$	**exterior angle** $\frac{360°}{n}$	**angle sum** $(n-2)180°$
3	120°	60°	120°	180°
4	90°	90°	90°	360°
5	72°	108°	72°	540°
6	60°	120°	60°	720°

If a vertex angle of a regular *n*-gon is $162°$, then what's the value of *n*?

$$\text{vertex angle} = 180° - \frac{360°}{n} = 162°\text{, so } \frac{360°}{n} = 18°\text{, so } n = 20.$$

If an exterior angle of a regular *n*-gon is $45°$, then what's the value of *n*?

$$\text{exterior angle} = \frac{360°}{n} = 45°\text{, so } n = 8.$$

If a vertex angle of a regular *n*-gon is $150°$, then what's the measure of a central angle?

vertex angle = $180°$ **– central angle, so central angle =** $30°$**.**