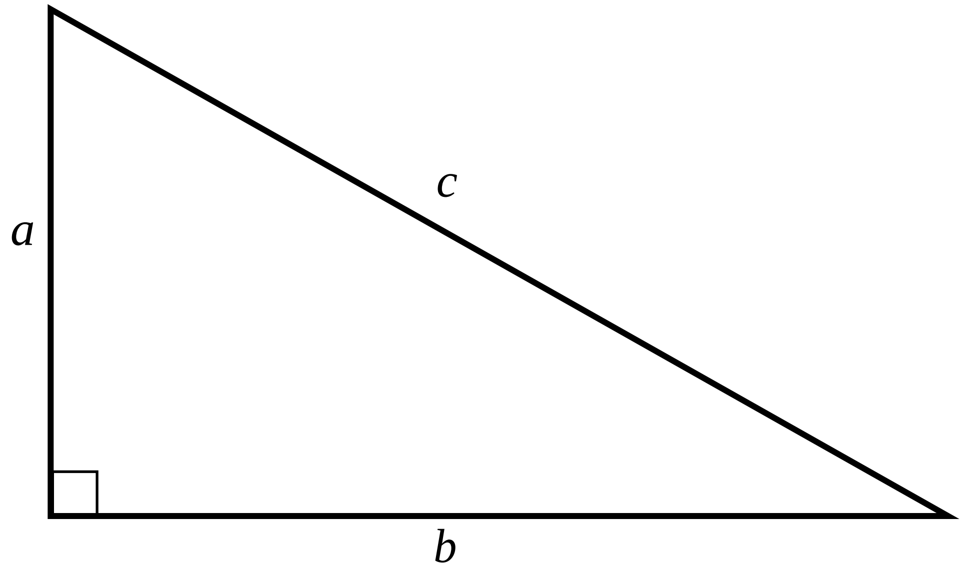
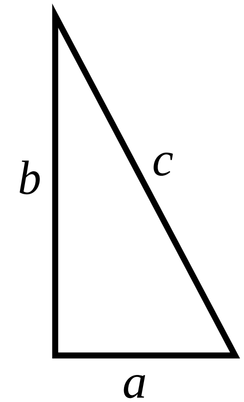
Pythagorean Theorem:

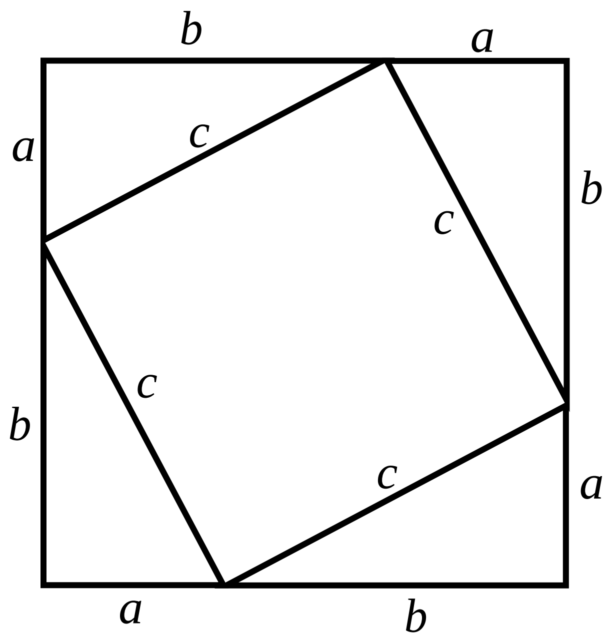
In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

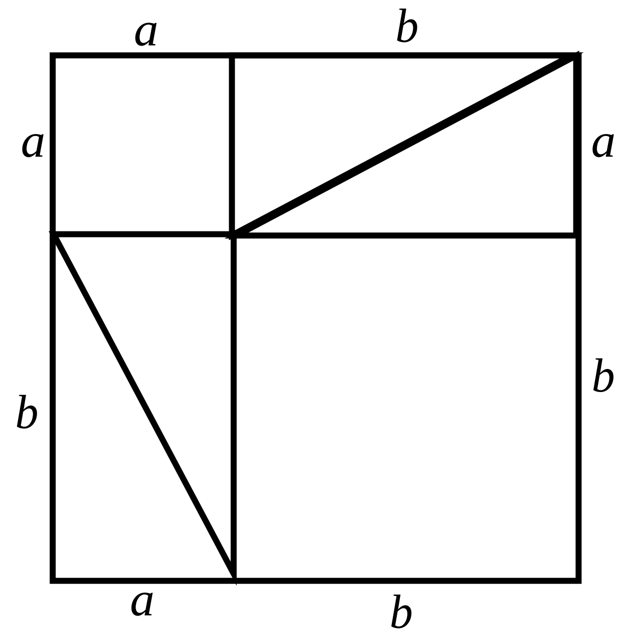


$$a^2 + b^2 = c^2$$

Proof:



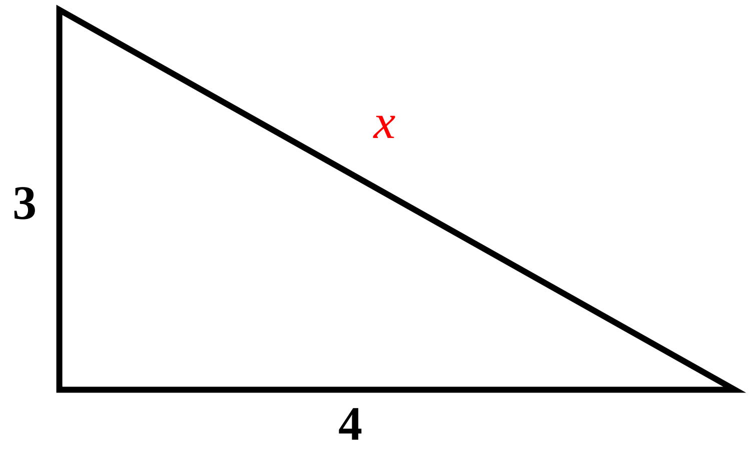




After removing the common four triangles from the two squares, you get that the area of the square of side length *c* must equal the sum of the areas of the squares of side lengths *a* and *b*. $c^2 = a^2 + b^2$**.**

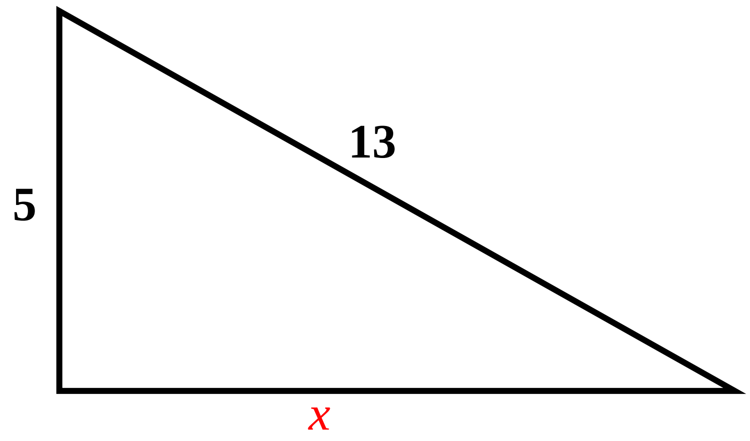
Examples:

1. Find the missing measurement in the right triangle.



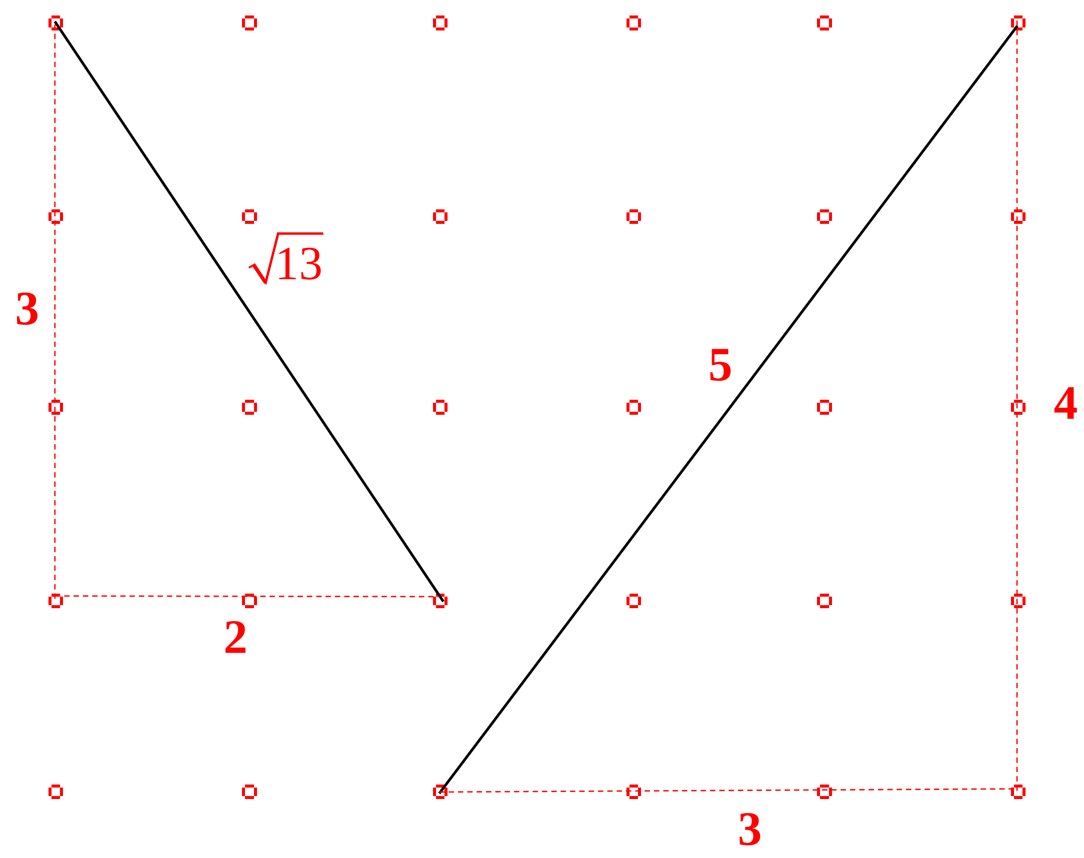
$$x^2 = 3^2 + 4^2 = 25 \Rightarrow x = \boxed{5}$$

2. Find the missing measurement in the right triangle.

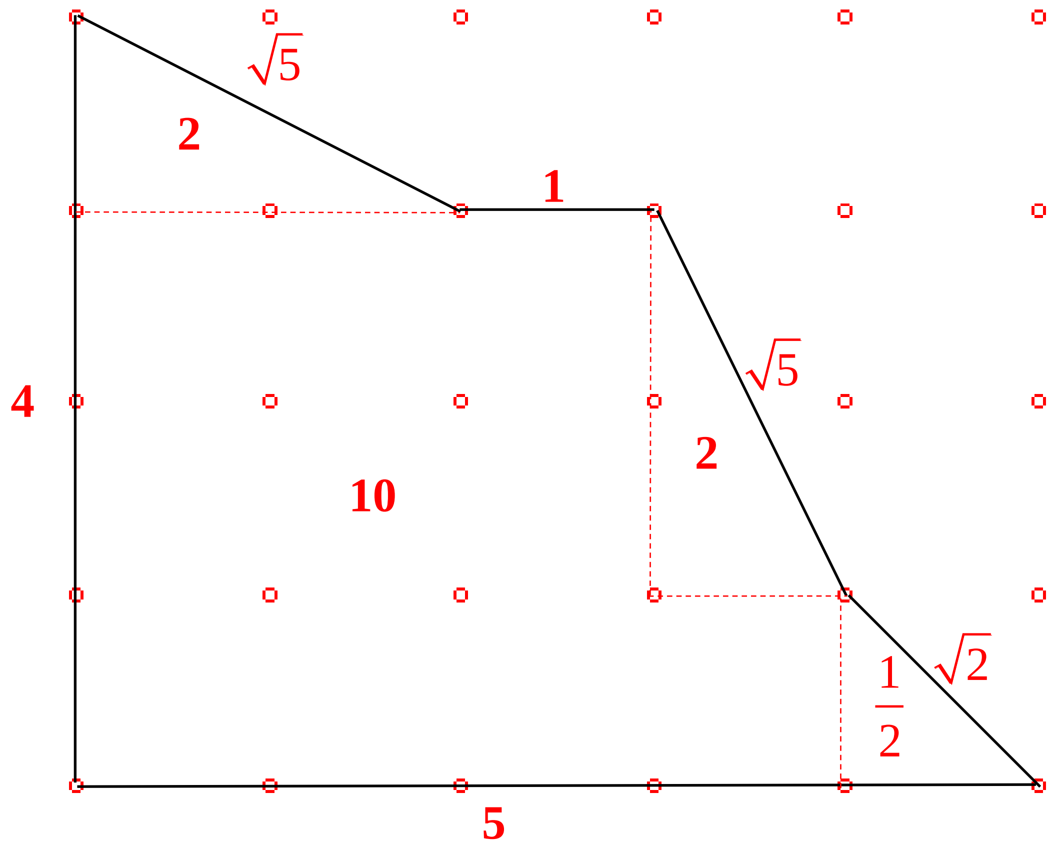


$$x^2 = 13^2 - 5^2 = 144 \Rightarrow x = \boxed{12}$$

3. **A square lattice is an equally spaced arrangement of points vertically and horizontally. Determine the lengths of the segments constructed in a square lattice.**



4. Determine the perimeter and area of the polygon constructed in a square lattice.



$$perimeter = 5 + \sqrt{2} + \sqrt{5} + 1 + \sqrt{5} + 4 = \left(10 + 2\sqrt{5} + \sqrt{2}\right) units$$

$$area = 2 + 10 + 2 + \frac{1}{2} = 14\tfrac{1}{2}\ square\ units$$

Triangle Inequality:

The length of one side of a triangle must be smaller the sum of the lengths of the other two sides and greater than the difference in the lengths of the other two sides.

Determine if the following triples of numbers could be the side measurements of a triangle:

1. 1, 1, 3

Is $3<1+1$**?**	**Is** $3>1-1$**?**	**Is** $1<3+1$**?**	**Is** $1>3-1$**?**
No	**Yes**	**Yes**	**No**

If any of the answers is no, then they can't be the side measurements of a triangle.

So 1, 1, 3 can't be the side measurements of a triangle.

2. 1, 2, 3

Is $3 < 1 + 2$**?** **Is** $1 > 3 - 2$**?** **Is** $2 > 3 - 1$**?**

No **No** **No**

So 1, 2, 3 can't be the side measurements of a triangle.

3. 2, 3, 4

Is $2 < 3 + 4$**?** **Is** $2 > 4 - 3$**?** **Is** $3 < 4 + 2$**?** **Is** $3 > 4 - 2$**?** **Is** $4 < 3 + 2$**?** **Is** $4 > 3 - 2$**?**

Yes **Yes** **Yes** **Yes** **Yes** **Yes**

So 2, 3, 4 can be the side measurements of a triangle.

The lengths of two sides of a triangle are 5 and 9. Which of the following could be the length of the third side?

The third side length must be smaller than 5 + 9 = 14, and larger than 9 – 5 = 4.

2

2 is not between 4 and 14, so no.

13

13 is not between 4 and 14, so no.

15

15 is between 4 and 14, so yes.

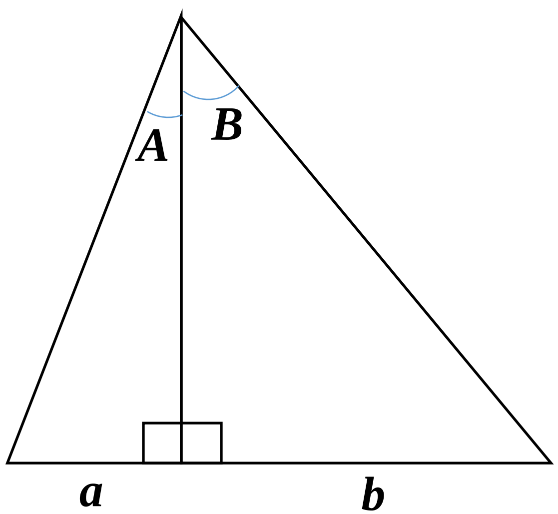
45

45 is not between 4 and 14, so no.

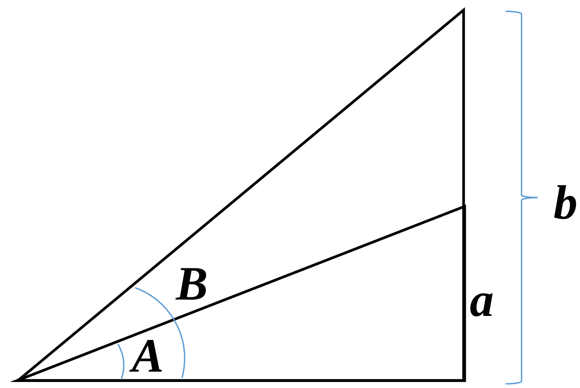
<u>Side Angle Inequality Theorem:</u> **In a triangle, the side opposite the largest angle is the longest. The side opposite the smallest angle is the shortest. If two angles have the same measure, then the opposite sides have the same length.**

Here's why:

Suppose that you have two right triangles with a common leg.



Now let's reposition them.

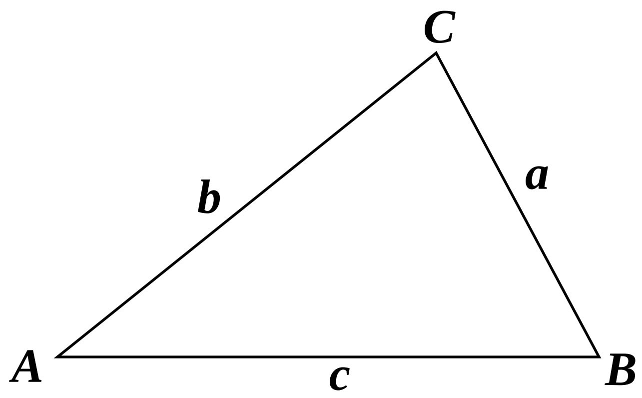


$$a < b \Leftrightarrow m(\angle A) < m(\angle B)$$

$$a > b \Leftrightarrow m(\angle A) > m(\angle B)$$

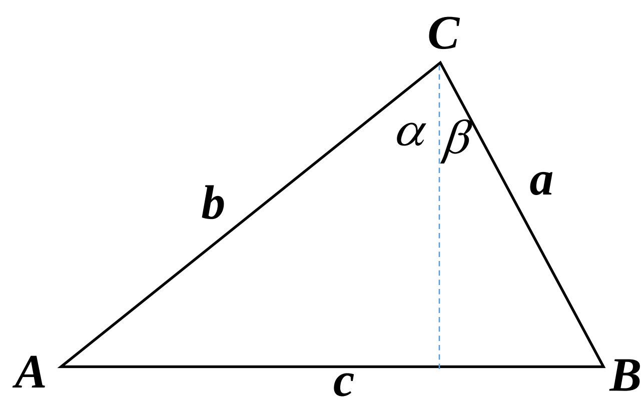
$$a = b \Leftrightarrow m(\angle A) = m(\angle B)$$

Now suppose you're given the scalene triangle



with $a < b < c$**.**

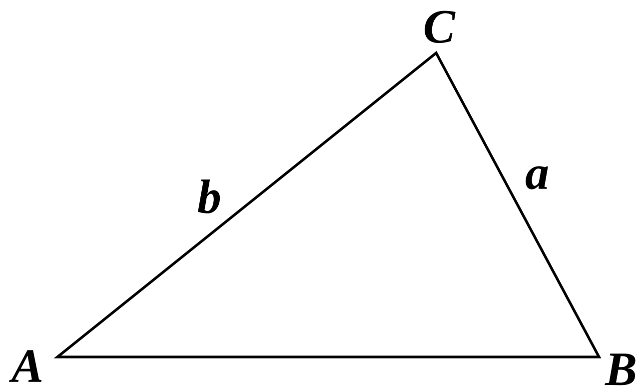
Add a perpendicular segment.



From the discussion about right triangles, $m(\angle\alpha) > m(\angle\beta) \Rightarrow m(\angle A) < m(\angle B)$**.**

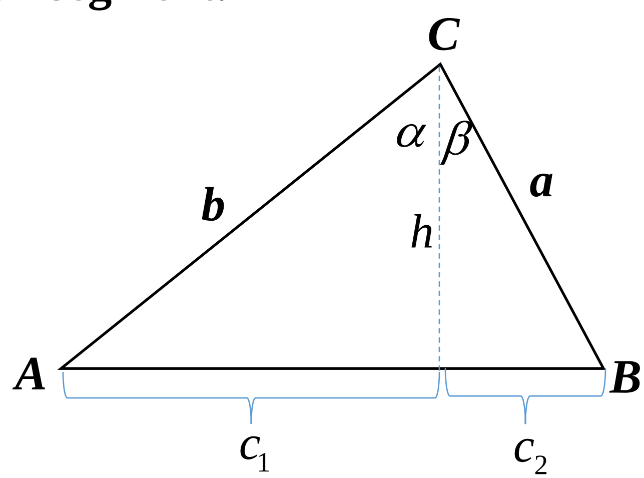
Showing that $m(\angle B) < m(\angle C)$ **can be done similarly.**

Now suppose you're given the scalene triangle



with $m(\angle A) < m(\angle B) < m(\angle C)$**.**

Again, add a perpendicular segment.

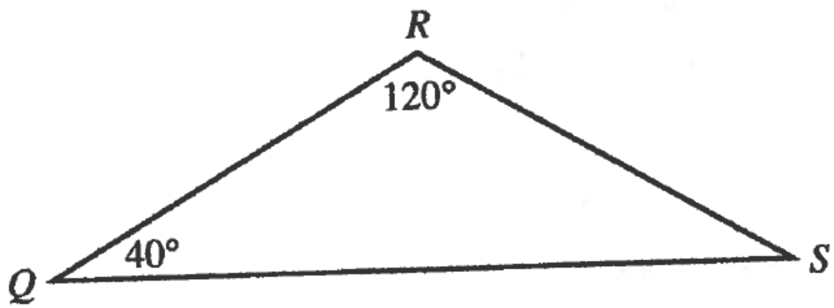


From the discussion about right triangles,

$$m(\angle A) < m(\angle B) \Rightarrow m(\angle \alpha) > m(\angle \beta) \Rightarrow c_1 > c_2$$

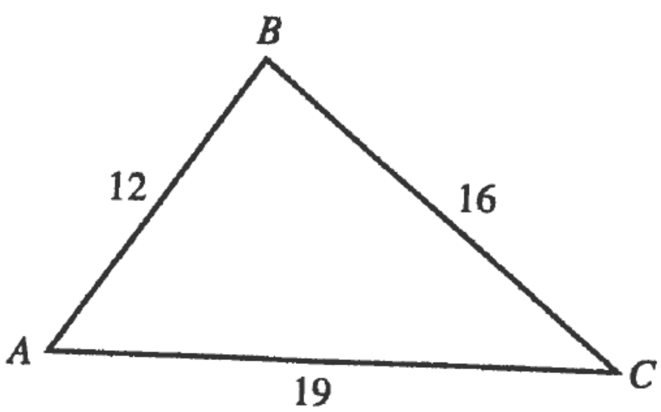
From the Pythagorean Theorem, $a = \sqrt{c_2^2 + h^2} < \sqrt{c_1^2 + h^2} = b$**.**

Showing that $b < c$ **can be done similarly.**



Which side is the shortest?

$\overline{QR}$



Which angle is the largest?

$\angle B$