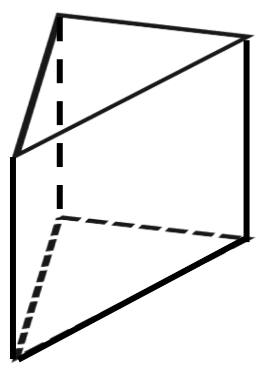
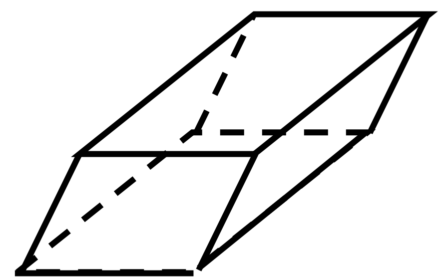
Volume of 3-d Objects:

Prisms:

A prism is a Polyhedron with two opposite faces that are identical polygonal regions called the bases. The vertices of the bases are joined with segments to form the lateral faces which are parallelograms. If the lateral faces are rectangles, the prism is called a right prism, otherwise, it's called an oblique prism.

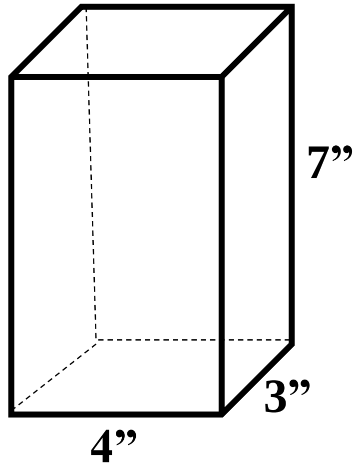


right prism



oblique prism

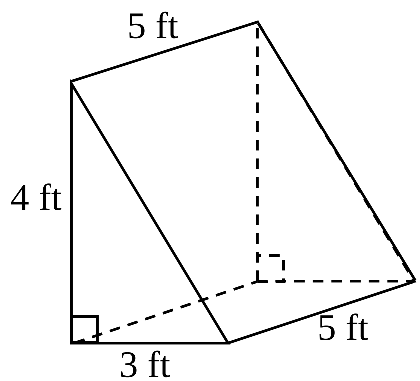
Right rectangular prism: The volume is just the product of the three dimensions.



$$V = (3)(4)(7) = \boxed{84\, in^3}$$

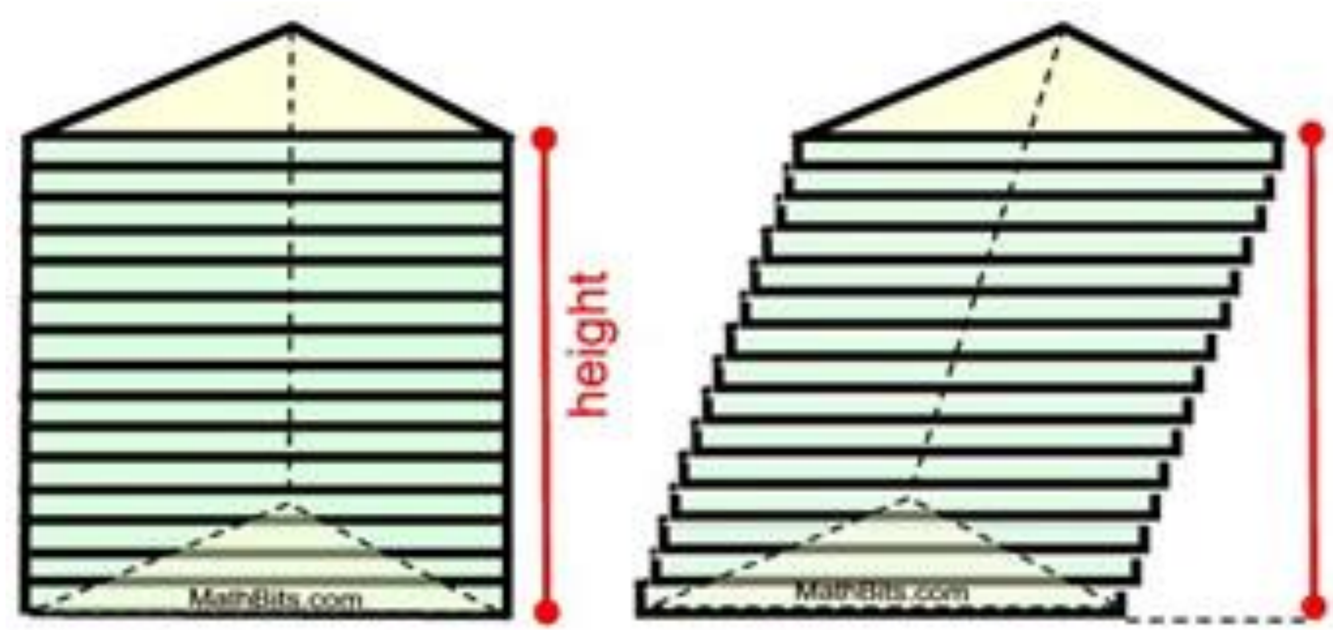
In general, for right prisms, the volume is the area of one base times the height.

Right triangular prism:



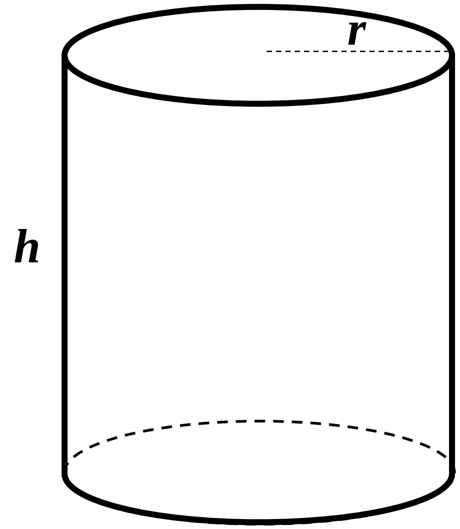
$$V = \underbrace{\frac{1}{2}(3)(4)}_{\text{area of base}}(5) = \boxed{30\ ft^3}$$

For oblique prisms, we can use Cavalieri's Principle: If two solids have the same cross-sections, then they have the same volume.

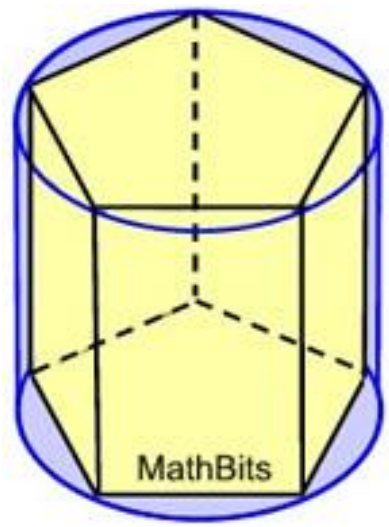


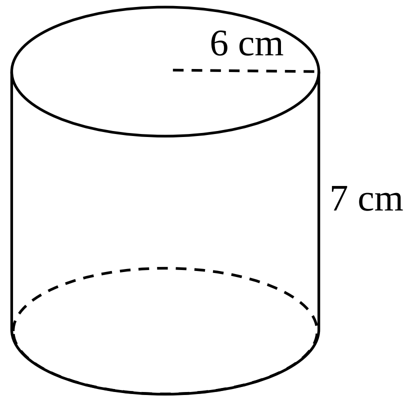
So the volume of an oblique prism is the product of the area of a base and the height.

Right Circular Cylinders:



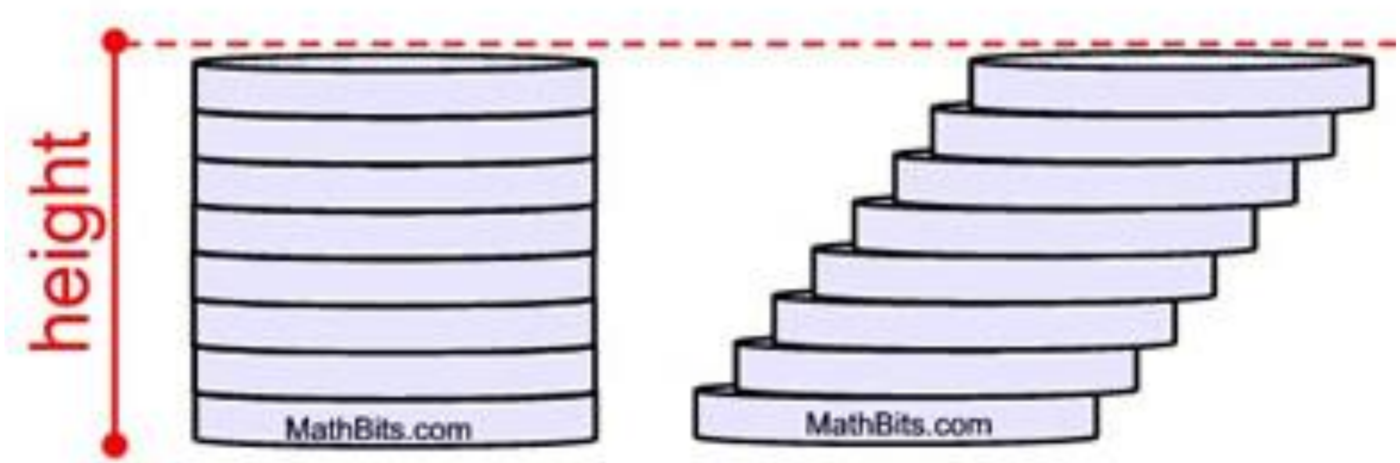
A right circular cylinder can be arbitrarily approximated by a right prism, so it's not hard to believe that the volume of a right circular cylinder is the area of a base times the height. $V = \underbrace{\pi r^2}_{\text{area of base}} \cdot h$





$$V = \underbrace{\pi(6)^2}_{\text{area of base}}(7) = \boxed{252\pi\, cm^3}$$

Again, from Cavalieri's Principle,

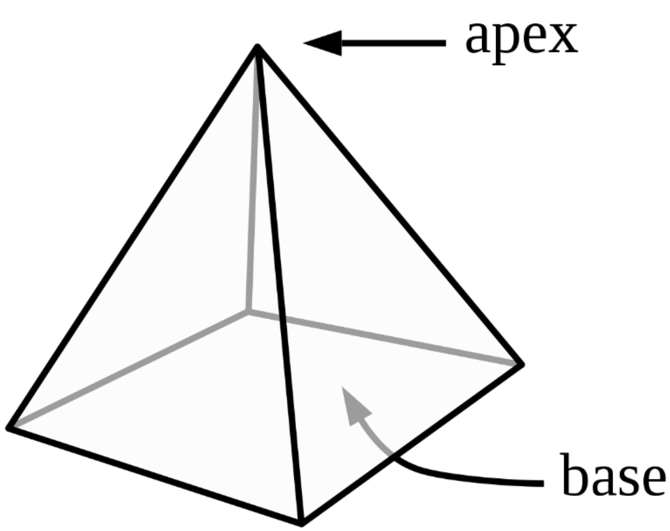


the volume of an oblique circular cylinder is the product of the area of a base and the height.

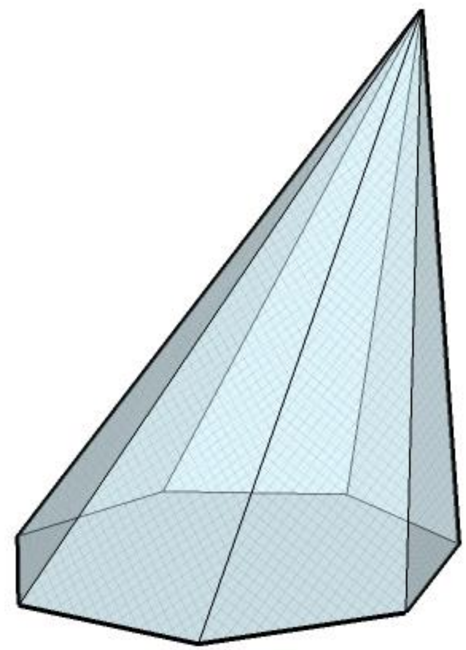
<u>Pyramids:</u>

A pyramid is a polyhedron formed by connecting the vertices of a polygonal region called the base to another point not in the plane of the base called the apex using segments. Pyramids whose bases are regular polygonal regions are classified into two groups:

If the lateral faces are isosceles triangles, then it's a right regular pyramid. Otherwise, it's an oblique regular pyramid.



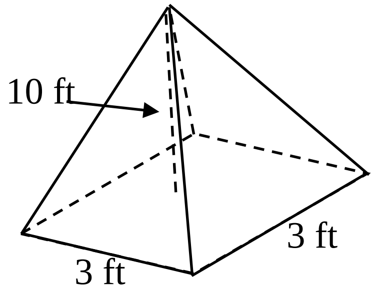
right regular pyramid



oblique regular pyramid

The textbook provides an argument that the volume of a right pyramid is one-third the volume of the prism with the same base, i.e. $\frac{1}{3}Ah$.

Right square pyramid:



$$V = \frac{1}{3} \cdot \underbrace{(3)^2}_{\text{area of base}} (10) = \boxed{30\ ft^3}$$

Cavalieri's Principle implies that the volume of an oblique pyramid is also $\frac{1}{3}Ah$.

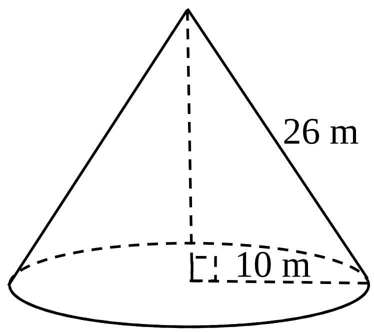
A pyramid has a height of *h* and a square base with a side length of *s*. If the base stays the same, but the height is doubled, how does the volume of the new pyramid compare to the volume of the original pyramid?

The original volume is $\frac{1}{3}s^2h$. The new volume is $\frac{1}{3}s^2(2h) = 2\cdot\frac{1}{3}s^2h$, so the volume of the new pyramid is twice the volume of the original pyramid.

A pyramid has a height of *h* and a square base with a side length of *s*. If the height stays the same, but the side length of the base is doubled, how does the volume of the new pyramid compare to the volume of the original pyramid?

The original volume is $\frac{1}{3}s^2h$. The new volume is $\frac{1}{3}(2s)^2 h = 4\cdot\frac{1}{3}s^2h$, so the volume of the new pyramid is four times the volume of the original pyramid.

Cones can be arbitrarily approximated by pyramids, so it should seem reasonable that the volume of a cone is also $\frac{1}{3}Ah$.



From the Pythagorean theorem, the height of the cone is $\sqrt{26^2 - 10^2} = 24m$. The volume is $\frac{1}{3}\pi(10)^2(24) = \boxed{800\pi\, m^3}$.