Probability:

Experiment:

Any process that produces random results.

Example:

Flip a coin twice and record the results.

Heads/Heads, Heads/Tails, Tails/Heads, Tails/Tails

Sample Space:

The set of all possible outcomes of an experiment. It's abbreviated with the letter *S*, and it's like the universal set for an experiment.

Example:

From the previous experiment

$$S = \{HH, HT, TH, TT\}$$

<u>Event:</u>

Any subset of the sample space

Example:

From the previous experiment

$$S = \{HH, HT, TH, TT\}$$

E **is the event that heads occurs.** $E = \{HH, HT, TH\}$

F **is the event that tails occurs.** $F = \{HT, TH, TT\}$

G **is the event of getting the same result on both flips.** $G = \{HH, TT\}$

J **is the event of getting different results on the two flips.** $J = \{HT, TH\}$

A probability is a number between 0 and 1(inclusive) that indicates the likelihood that an event will occur.

A probability of 1 means the event must occur.

A probability of 0 means the event won't occur.

A probability of ½ means the event is just as likely to occur as not to occur.

The closer the probability value is to 1, the more likely the event will occur, and the closer the probability value is to 0, the less likely the event will occur.

Theoretical Probability and the Equally Likely assumption:

In many experiments, the outcomes in the sample space all have the same probability of occurring. Certain conditions in the experiments will allow you to make this equally likely assumption.

When you can assume equally likely outcomes, the probability of an event is determined using counting.

$$P(E)=\frac{n(E)}{n(S)}=\frac{\text{\# of outcomes in } E}{\text{\# of outcomes in } S}$$

Examples:

1. A *fair* die is rolled. $S=\{1,2,3,4,5,6\}$

The word *fair* indicates that we may make the equally likely assumption.

a) $P(\text{rolling a 1})$

$\frac{1}{6}$

b) $P(\text{rolling a 2 or a 3})$

$\frac{2}{6}=\frac{1}{3}$

c) $P(\text{rolling an odd number})$

$\frac{3}{6}=\frac{1}{2}$

d) $P(\text{rolling a 7})$

$\frac{0}{6}=0$

2. A card is *randomly selected* from a standard 52-card deck.

The phrase *randomly selected* indicates that we may make the equally likely assumption.

a) $P(\text{selecting an ace})$

$$\frac{4}{52}=\frac{1}{13}$$

b) $P(\text{selecting a red card})$

$$\frac{26}{52}=\frac{1}{2}$$

c) $P(\text{selecting a club})$

$$\frac{13}{52}=\frac{1}{4}$$

d) $P(\text{selecting a face card})$

J Q K ♦ ♠ ♥ ♣

$$\frac{12}{52}=\frac{3}{13}$$

e) $P(\text{selecting an ace or a diamond})$

A♥ **A**♣ **A**♠ **A**♦

2♦ **3**♦ **4**♦ **5**♦ **6**♦ **7**♦ **8**♦ **9**♦ **10**♦ **J**♦ **Q**♦ **K**♦

$$\frac{16}{52}=\frac{4}{13}$$

3. A *fair coin* is flipped twice. $S = \{HH, HT, TH, TT\}$

a) $P(\text{heads occurs})$

$\frac{3}{4}$

b) $P(\text{tails occurs})$

$\frac{3}{4}$

c) $P(\text{same result on both flips})$

$\frac{2}{4} = \frac{1}{2}$

d) $P(\text{different result on the two flips})$

$\frac{2}{4} = \frac{1}{2}$

e) $P(\text{two heads occur})$

$\frac{1}{4}$

f) $P(\text{three heads occur})$

$\frac{0}{4} = 0$

4. A survey of North Harris students had the following results.

	Pepperoni	Sausage	Mushroom	Total
Freshman	25	15	5	45
Sophomore	30	20	5	55
Total	55	35	10	100

A student from the survey is *selected at random*.

a) $P(\text{freshman})$

$$\frac{45}{100}=\frac{9}{20}$$

b) $P(\text{sausage})$

$$\frac{35}{100}=\frac{7}{20}$$

c) $P(\text{freshman and pepperoni})$

$$\frac{25}{100}=\frac{1}{4}$$

d) $P(\text{sausage or mushroom})$

$$\frac{45}{100}=\frac{9}{20}$$

e) $P(\text{freshman or pepperoni})$

$$\frac{75}{100}=\frac{3}{4}$$

<u>Empirical or Experimental Probability:</u>

The experiment is performed a bunch of times, *n*, and the results are recorded.

$$P(E) = \frac{\text{\# of times } E \text{ occurs}}{n}$$

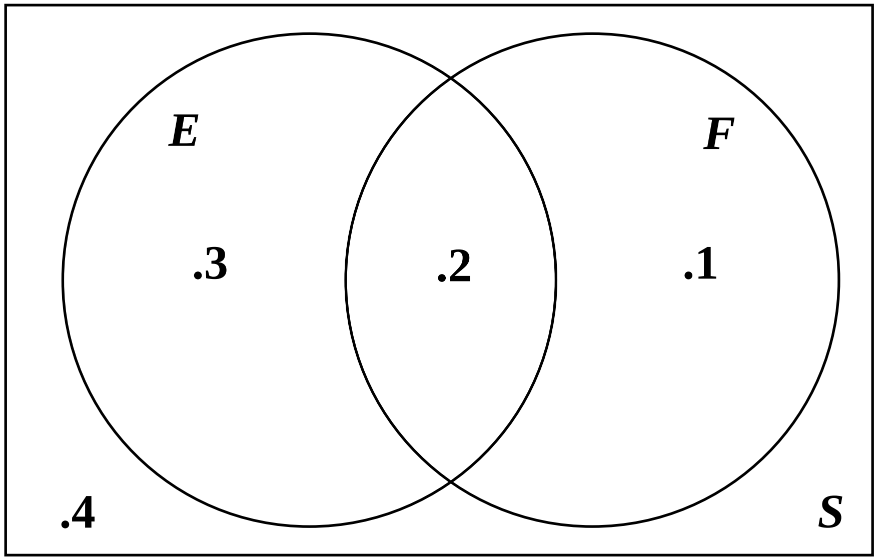
Example:

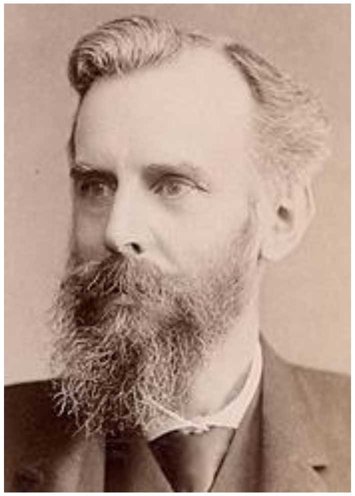
A coin is flipped 1,000 times with the following results: 450 heads and 550 tails. Find the empirical/experimental probability of flipping the coin and getting tails.

$$P(tails) = \frac{550}{1000} = \frac{11}{20}$$

Probability Diagrams and Probability Formulas:

A probability diagram is like a specialized Venn Diagram in which the probabilities of different events in the sample space are labelled.



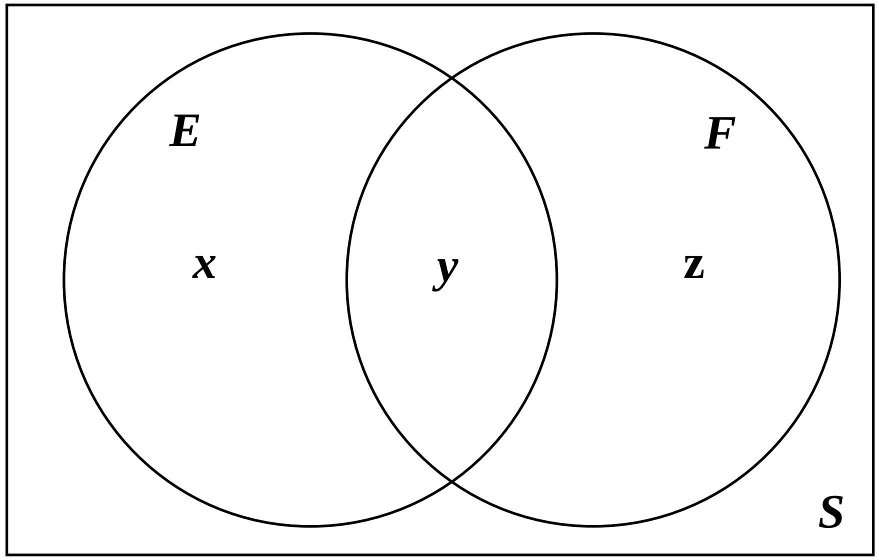


The sum of all the probabilities that make up all the disjoint regions of *S* must be 1.

$P(E) = .3 + .2 = \boxed{.5}$ $\quad P(F) = .2 + .1 = \boxed{.3}$ $\quad P(E \text{ and } F) = P(E \cap F) = .2$

$P(E \text{ or } F) = P(E \cup F) = .3 + .2 + .1 = \boxed{.6}$ $\quad P(\text{not } E) = P(\bar{E}) = .1 + .4 = \boxed{.5}$ $\quad P(\overline{E \cup F}) = .4$

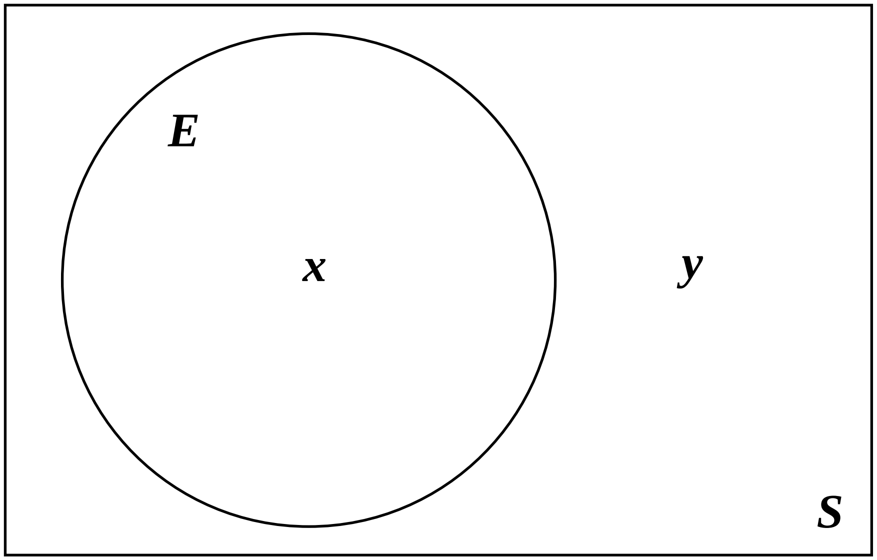
A formula for $P(E \cup F)$:



$$P(E \cup F) = x + y + z = x + y + y + z - y$$
$$= (x + y) + (y + z) - y$$
$$= P(E) + P(F) - P(E \cap F)$$

If $E \cap F = \phi$, then it's impossible for both events to occur, and they are called mutually exclusive events. In this case, $P(E \cup F) = P(E) + P(F)$

Formulas involving $P(\bar{E})$:



$$1 = P(S) = x + y = P(E) + P(\bar{E})$$

So

$$P(E) = 1 - P(\bar{E})$$

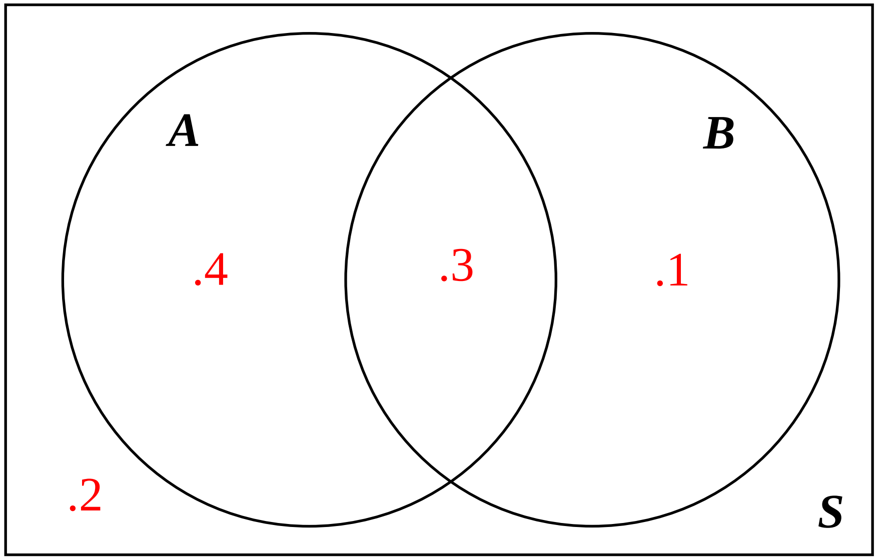
$$P(\bar{E}) = 1 - P(E)$$

Out of context example:

Suppose $P(A)=.7$**,** $P(B)=.4$**, and** $P(A\cap B)=.3$**.** Start with the probability at the end, .3, and work your way back to the beginning probability, .7.

Complete the probability diagram:

The total in *A* must be .7, the total in *B* must be .4, and the total for the diagram must be 1.



Find

$P(A\cup B)$

$.4+.3+.1=.8$

or

$P(A)+P(B)-P(A\cap B)=.8$

$P(\bar{A})$

$.1+.2=.3$

or

$1-P(A)=.3$

$P(\overline{A\cup B})$

$.2$

or

$1-P(A\cup B)=.2$

$P(A \cap \bar{B})$ $\quad$ $P(B \cap \bar{A})$ $\quad$ $P(\overline{A \cap B})$

$.4$ $\quad$ $.1$ $\quad$ $.4 + .1 + .2 = .7$ *or* $1 - P(A \cap B) = .7$

In context examples:

1. A card is randomly selected from a standard 52-card deck.

$P(\text{ace or a heart})$

$P(\text{ace or a heart})$

$= P(\text{ace}) + P(\text{heart}) - P(\text{ace and heart})$

$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \boxed{\frac{4}{13}}$

$P(\text{ace or king})$

$P(\text{ace or king})$

$= P(\text{ace}) + P(\text{king})$

$= \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \boxed{\frac{2}{13}}$

$P(\text{face card or a club})$

$P(\text{face card or a club})$

$= P(\text{face card}) + P(\text{club}) - P(\text{face card and club})$

$= \frac{12}{52} + \frac{13}{52} - \frac{3}{52} = \frac{22}{52} = \boxed{\frac{11}{26}}$

2. A survey of North Harris students had the following results.

	Pepperoni	Sausage	Mushroom	Total
Freshman	**25**	**15**	**5**	**45**
Sophomore	**30**	**20**	**5**	**55**
Total	**55**	**35**	**10**	**100**

A student from the survey is *selected at random*.

a) $P(\text{sausage or mushroom})$

$$P(\text{sausage or mushroom}) = P(\text{sausage}) + P(\text{mushroom})$$

$$= \frac{35}{100} + \frac{10}{100} = \frac{45}{100} = \boxed{\frac{9}{20}}$$

b) $P(\text{freshman or pepperoni})$

$$P(\text{freshman or pepperoni}) = P(\text{freshman}) + P(\text{pepperoni}) - P(\text{freshman and pepperoni})$$

$$= \frac{45}{100} + \frac{55}{100} - \frac{25}{100} = \frac{75}{100} = \boxed{\frac{3}{4}}$$