

**Counting Methods:**

**Example:**

**A pen has tip options of regular tip, fine tip, or micro tip, and it has ink color options of red ink or green ink. How many different pens are possible?**

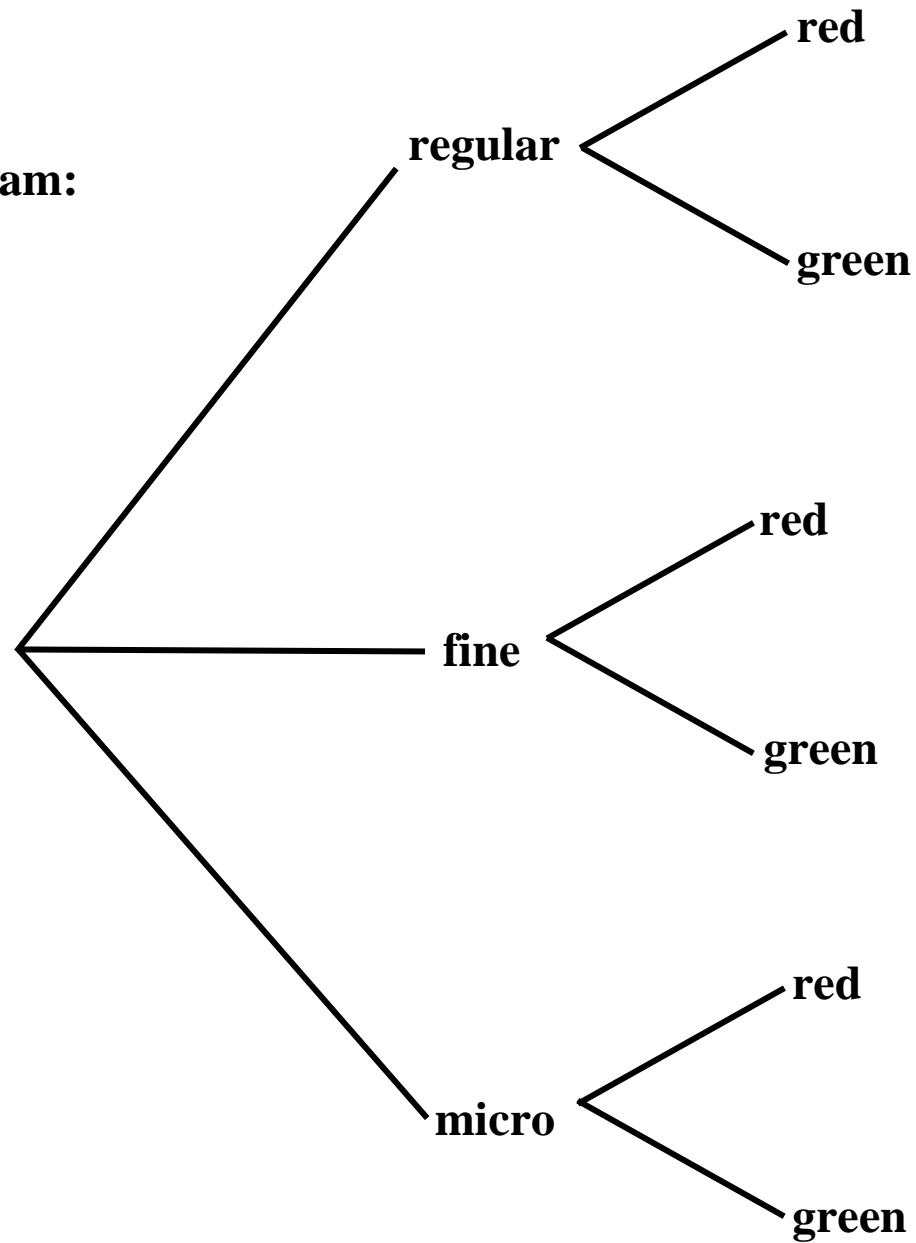
**Using a table:**

	regular	fine	micro
red			
green			



**The number of pens possible is the number of cells in the table:  $3 \times 2$ .**

**Using a Tree Diagram:**



**The number of pens possible is the number of branch tips on the right:  $3 \times 2$ .**

**The Fundamental Counting Principle/Property:**

**If a sequence of decisions is to be made, then the number of different ways of making all the decisions is the product of the number of options for each decision.**

**Examples:**

- 1. A meal consists of 1 of 8 appetizers, 1 of 10 entrees, and 1 of 5 desserts. How many different meals are possible?**



- 2. In a race with 5 horses, how many different first, second, and third place finishes are possible?**



**3. In a certain small state, license plates consist of three letters followed by two digits.**

**a) How many different plates are possible?**



**b) How many if letters can't repeat?**

**c) How many if digits can't repeat?**

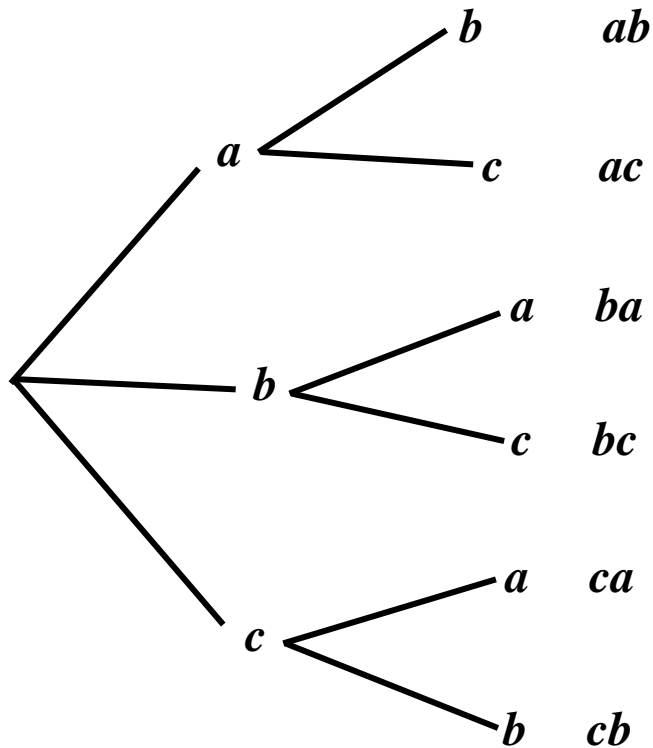
**d) How many if no repeats?**

## Permutations:

A permutation is an arrangement of objects in a particular order.

**Example:**

Find all the permutations of the objects  $\{a,b,c\}$  of size 2.



There are 6 permutations of size 2 from the 3 objects. If we just wanted to know how many, we could have used the Fundamental Counting Principle.



In general, the number of permutations of size  $r$  from  $n$  objects is abbreviated as  ${}_nP_r$ . So far, we know that  ${}_3P_2 = 6$ . There's a nice formula for the value of  ${}_nP_r$  in general, but it involves things called factorials.

**Factorials:**

$$n! = n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1 \text{ or } n! = 1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n$$

$$\text{So } 1! = 1.$$

$$2! = 2 \cdot 1 = 2$$

$$3! = 3 \cdot 2 \cdot 1 = 6$$

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 4 \cdot 3! = 4 \cdot 6 = 24$$

$$5! = 5 \cdot 4! = 5 \cdot 24 = 120$$

$$6! = 6 \cdot 5! = 6 \cdot 120 = 720$$

By special definition,  $0! = 1$ .

$${}_nP_r = \frac{n!}{(n-r)!}$$

Let's check it out for  ${}_3P_2$ , which we already know is equal to 6.

$${}_3P_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = \frac{6}{1} = 6$$

**Examples:**

1. Five solo singers are to perform their acts at a nightclub on Saturday night. How many different orders of their appearances are possible?

$${}_5P_5$$

Or

**Fundamental Counting Principle**



**2. From a group of 6 people, a president, vice-president, and secretary will be selected, how many different selections are possible?**

$${}_6P_3$$

**Or**

**Fundamental Counting Principle**



**3. In a race with 8 horses, how many different first, second, and third place finishes are possible?**

$${}_8P_3$$

**Or**

**Fundamental Counting Principle**



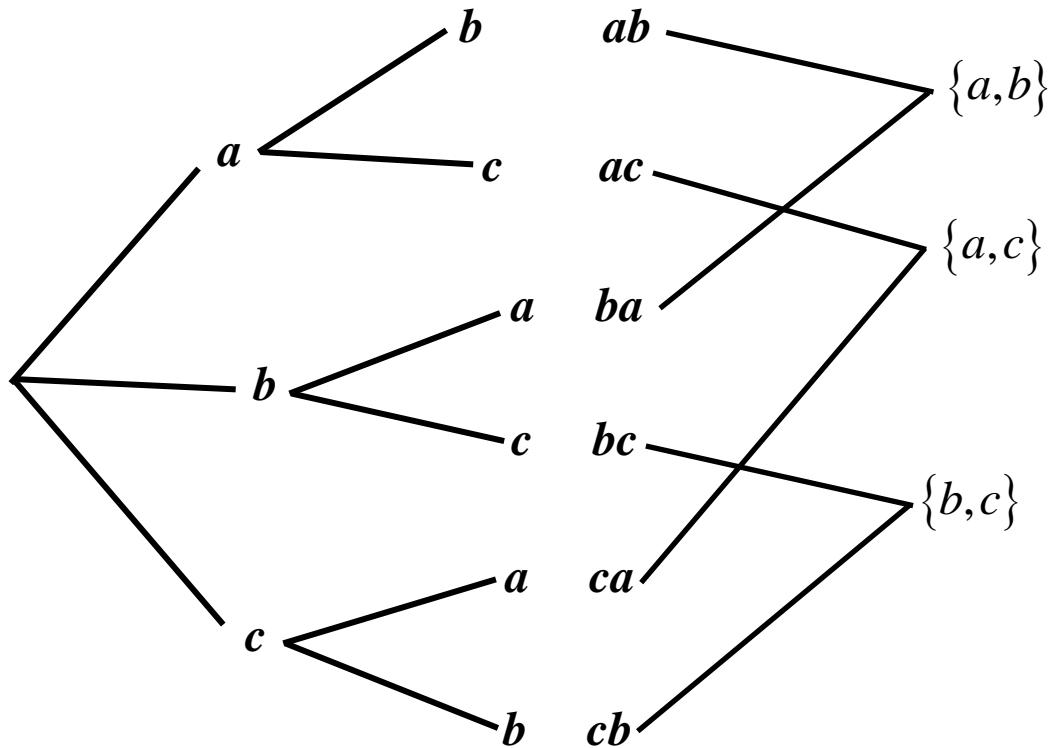
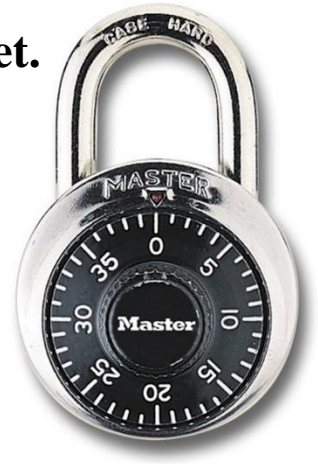


### Combinations:

A combination is a selection of objects without regard to order, i.e. a subset.

### Example:

Find all the combinations of the objects  $\{a,b,c\}$  of size 2.



There are 6 permutations of size 2 from the 3 objects, but only 3 combinations of size 2 from the 3 objects.

In general, the number of combinations of size  $r$  from  $n$  objects is abbreviated as

${}_nC_r$ . So far, we know that  ${}_3C_2 = 3$ , and  ${}_3C_2 = 3 = \frac{6}{2} = \frac{{}_3P_2}{2!}$ . This is true in general, and

leads to a nice formula for  ${}_nC_r$ .  ${}_nC_r = \frac{n!}{r!(n-r)!}$

**Examples:**

1. A three-person committee is to be selected from a group of 10 people. How many different committees are possible?



2. In a certain lottery, you must select 6 numbers from the numbers 1-50. How many different lottery selections are possible?



**3. A group consists of 7 men and 8 women. A committee of 4 people will be selected.**

**a) How many different 4-person committees are possible?**



**b) How many different 4-person committees consisting of 4 women are possible?**

**c) How many different 4-person committees consisting of 3 women and 1 man are possible?**

**d) How many different 4-person committees consisting of 2 women and 2 men are possible?**

**e) How many different 4-person committees have at least 1 man?**

**Finding Probabilities Using Counting Techniques:**

In the case of the equally likely assumption,  $P(E) = \frac{n(E)}{n(S)}$ .

**Examples:**

**1. Allen, Bob, Carl, and David will be seated in a row of 4 chairs at random.**

**a) How many different ways can they be seated?**



**b) How many different ways can Carl be seated in the first chair and Allen in the fourth chair?**

**c) What's the probability that Carl will be seated in the first chair and Allen in the fourth chair?**

**2. A group consists of 4 men and 5 women. Three people will be selected at random to attend a conference.**

**a) How many different selections of 3 people from the group are possible?**

**b) What's the probability that the 3 people selected are all women?**



**c) What's the probability that the 3 people selected are all men?**

**d) What's the probability that the 3 people selected will consist of 2 women and 1 man?**

**3. Three cards will be randomly selected from a 52-card deck without replacement.**

**a) What's the probability that it will consist of all hearts?**



**b) What's the probability that it will consist of exactly 2 aces?**

**c) What's the probability that it will consist of 2 aces and a king?**