Bird Watching And Triangles?

1. In the world of triangles with sides whose lengths are whole numbers, the Heron triangle has a whole number area. Heron triangles are uncommon but not as rare as the Pythagorean triangles. Heron triangles are named after Heron of Alexandria, a Greek scientist who lived during the first century A.D. and is credited with the ingenious formula relating the area for a triangle to the lengths of its sides:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Here *a*, *b*, and *c* stand for the lengths of the sides, and $s = \frac{a+b+c}{2}$, called the semiperimeter. As with Pythagorean triangles, we will distinguish between primitive Heron triangles and nonprimitive ones. Primitive Heron triangles have side measurements whose greatest common factor is 1.

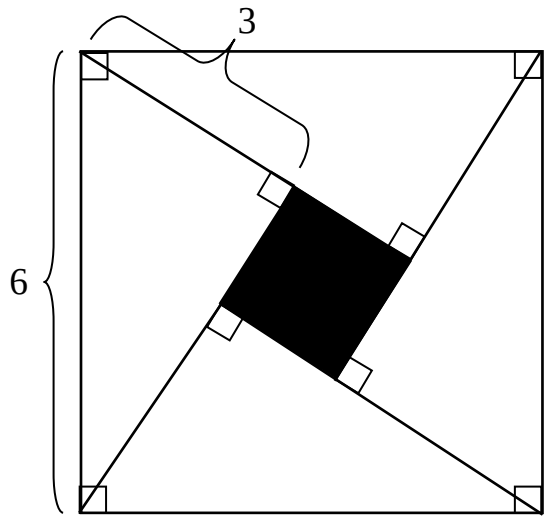
a) Find all the different primitive Heron triangles with area less than or equal to 100.

b) How many different primitive Heron triangles are there with sides less than or equal to 25?

Getting started: The 3-4-5 right triangle is a Heron triangle since its area is 6. The triangle with sides of 13, 14, and 15 is a Heron triangle because its area is a whole number.

Who Said Holes Have To Be Round?

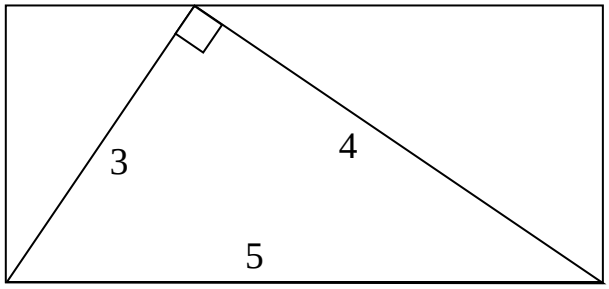
2. Find the exact area of the square hole in the middle of the square.



{Hint: Find the areas of the four right triangles, and subtract it from the area of the large square. Or use the Pythagorean Theorem to find the side measurement of the square. Express your answer using square-roots.}

Trigonometry Is Overrated.

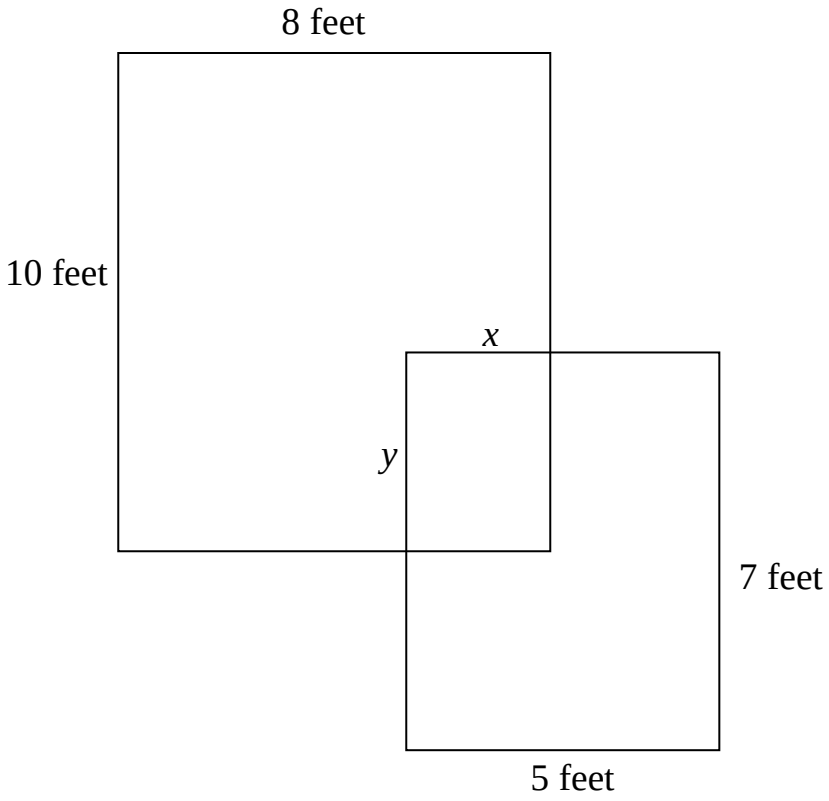
3. What is the height of this rectangle containing a 3-4-5 right triangle?



{Hint: The area of the rectangle is twice the area of the right triangle.}

What's The Difference?

4. Consider the two overlapping rectangles below:

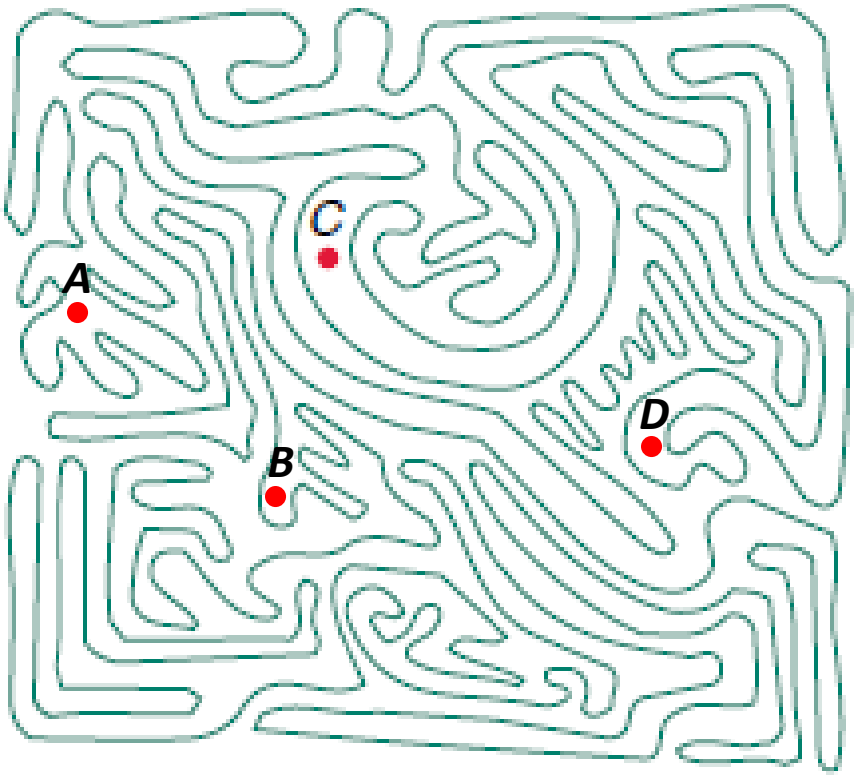


What is the difference between the areas of the non-overlapping regions of the rectangles?

{Hint: The area of the first non-overlapping region is $80 - xy$*. Find the area of the second non-overlapping region, and subtract it from the first.}*

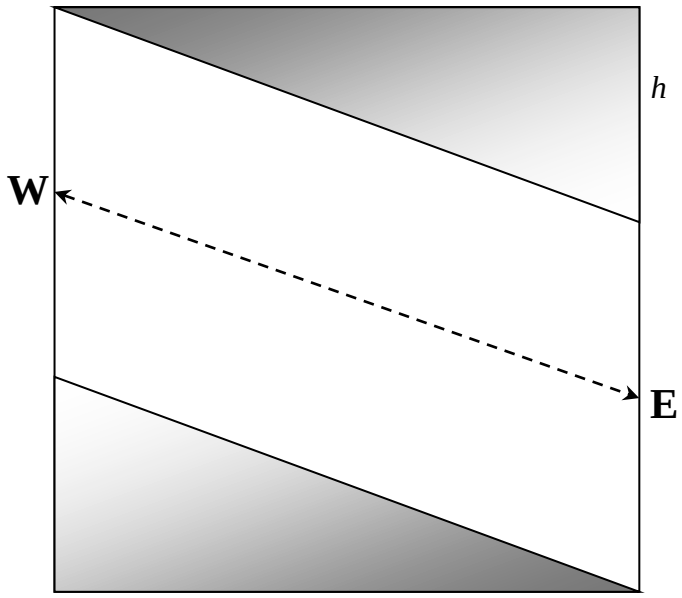
Inside-out, Outside-in.

5. Decide if the indicated points are inside or outside the given closed curve.



Communists In The Garden Of Eden.

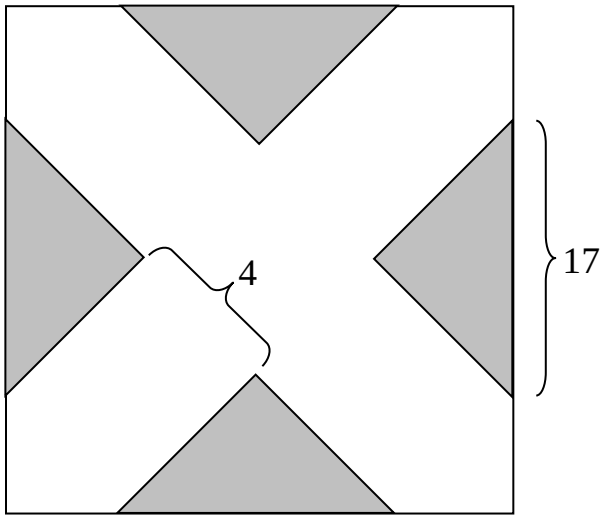
6. There has been a lot of criticism by the taxpayers of the new housing development Much Spending. Its site is a perfect square, each side being ¾ of a mile. Houses are only to be built on the shaded triangular areas; these two triangles are identical. The intervening space down the center from west to east is to be a communal garden. It is the devotion of so much space to this garden that has provoked adverse criticism. It occupies $\frac{7}{12}$ of the area of the development. What is the length of the communal garden area?



{Hint: The area of both triangles is the area of the rectangle formed by putting both of them together, $\frac{3}{4}h$. This must be $\frac{5}{12}$ of the total area which is $\left(\frac{3}{4}\right)^2$. The length of the communal garden area is the same as the hypotenuse of either right triangle.}

She Said Meet Me In The Middle In A Gravelly Voice.

7. Four-foot-wide gravel paths intersect within a grass-covered square region as shown.

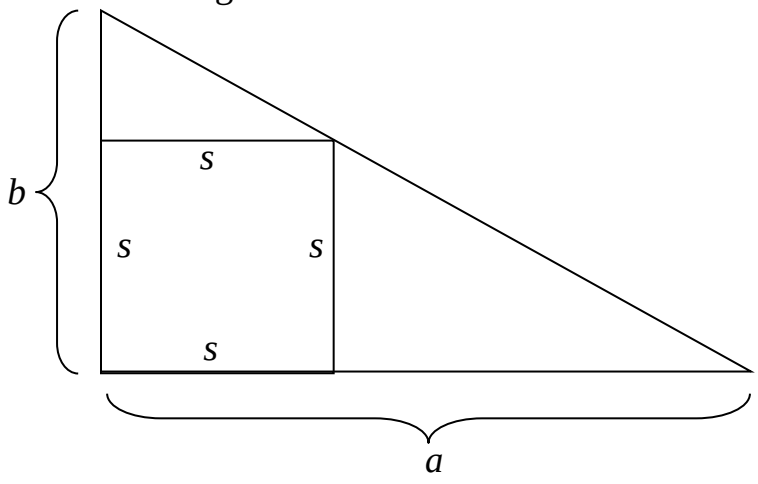


If the four triangles are right triangles, what is the total area covered by grass?

{Hint: Find the area of one of the right triangles using the Pythagorean Theorem, and then multiply it by 4, or just assemble them into a square.}

I Don't Give A Square's S!

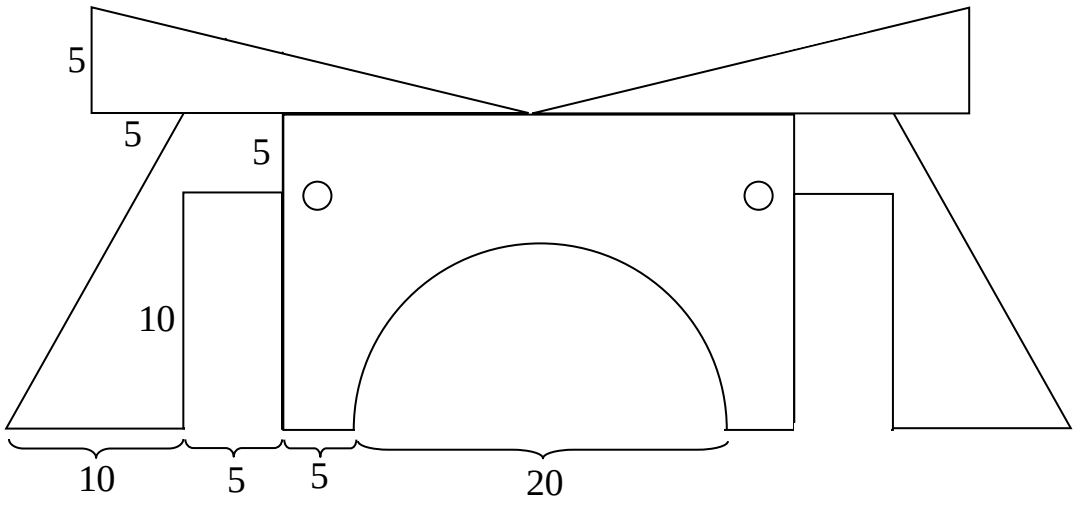
8. A right triangle with leg measurements of *a* and *b* has an inscribed square with side measurement *s* as shown in the figure. Find the value of *s*.



{Hint: The areas of the square and two little right triangles must equal the area of the big right triangle.}

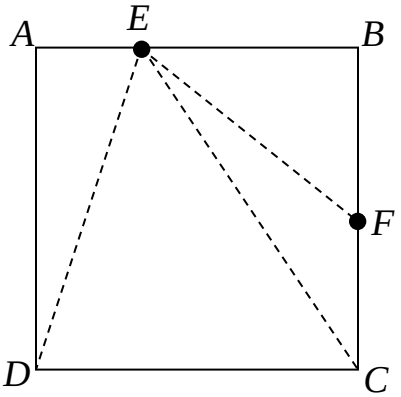
Mad Architect's Disease.

9. A mad architect has designed the symmetric building front shown in the figure. Find the area of the front of the building, not including the two circular windows of radius 2, the semicircular entrance way, and the two rectangular doorways.

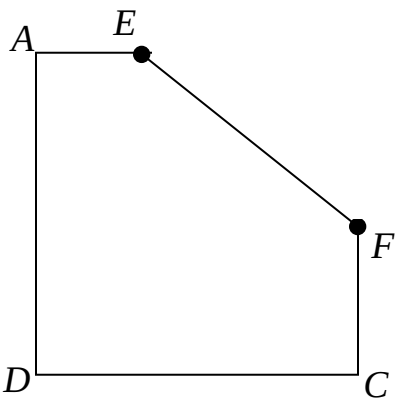


That's Sum Angle.

10. In square *ABCD*, point *E* is between *A* and *B*, and point *F* is between *B* and *C*. Find the sum of the measures of $\angle AEF$ and $\angle EFC$.



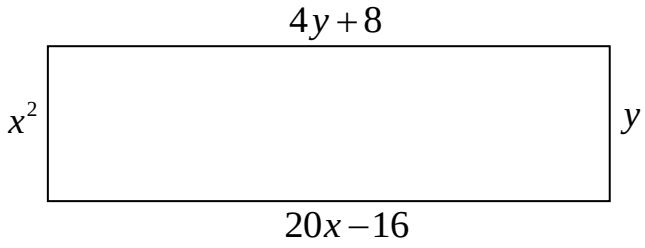
{Hint: Consider



.}

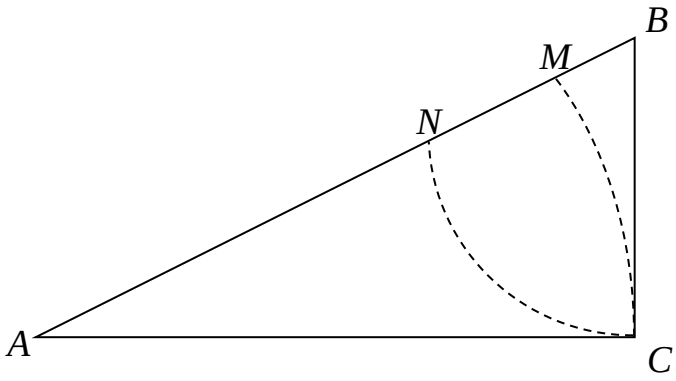
Mission: Impossible/Possible Perimeters.

11. A rectangle has the following sides. Find all possible numerical values of the perimeter of the rectangle.



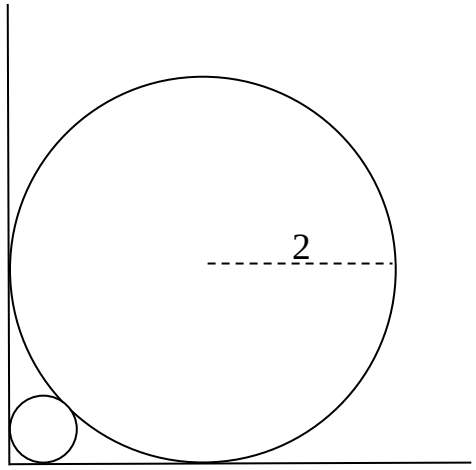
The Arc Of Triangle.

12. In right triangle ABC with legs of 5 and 12, arcs of circles are drawn, one with center A and radius 12, the other with center B and radius 5. What is the length of MN?

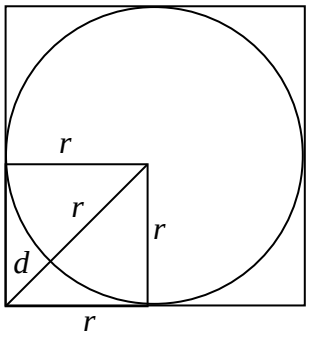


Between A Circle And A Hard Place.

13. Let *C* be a circle of radius 2 that is tangent to two line segments that form a right angle. Another circle, *S*, is tangent to the same two line segments and circle *C*. What is the radius of *S*?



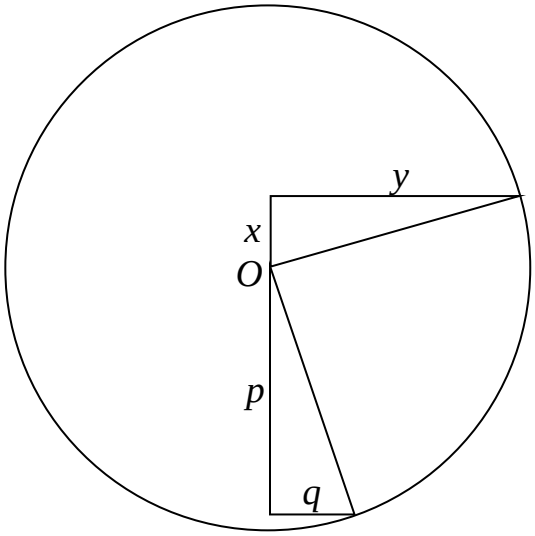
{Hint:



$$d = \sqrt{r^2 + r^2} - r\,.\}$$

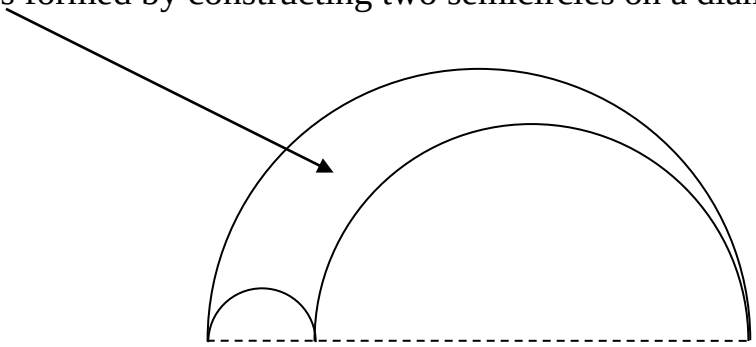
Squaring The Circle?

14. The two triangles are right triangles. The shared point, *O*, is the center of the circle, and the indicated vertices of the two triangles are on the circle. Given that $x^2 + y^2 + p^2 + q^2 = 72$, find the circumference of the circle.



The Shoemaker's Paradox.

15. An arbelos is formed by constructing two semicircles on a diameter of a larger semicircle.

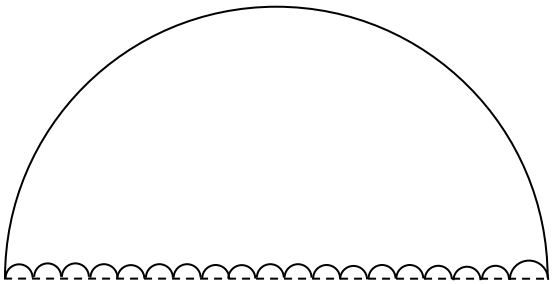


a) How does the circumference of the two smaller semicircles compare to the circumference of the larger semicircle?

b) Answer the same question except that this time there are three smaller semicircles.

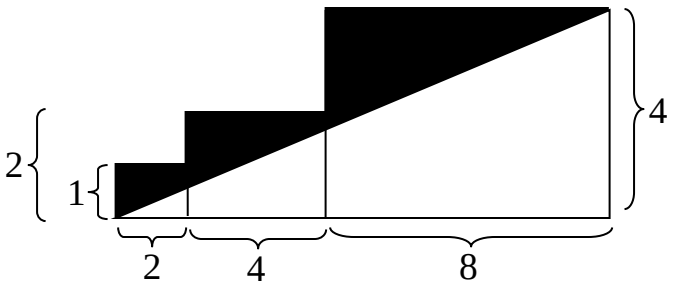
c) Answer the same question except that this time there is any number of smaller semicircles.

d) Does your answer to the part c) seem strange?



A Very Shady Question.

16. Three rectangles are connected as in the figure. The first rectangle is 2 by 1; the second rectangle is 4 by 2; the third rectangle is 8 by 4. A line is drawn from a vertex of the smallest rectangle to a vertex of the largest rectangle. Find the area of the shaded region.

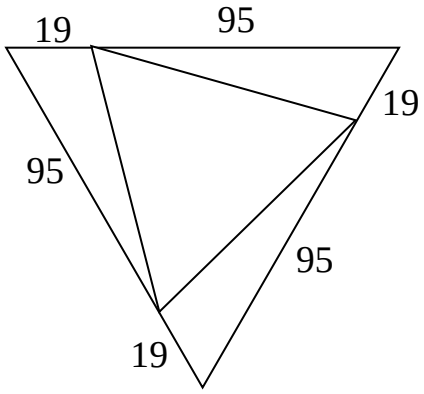


The Semicircle Of The Hypotenuse Equals The Sum Of The Semicircles Of The Two Legs.

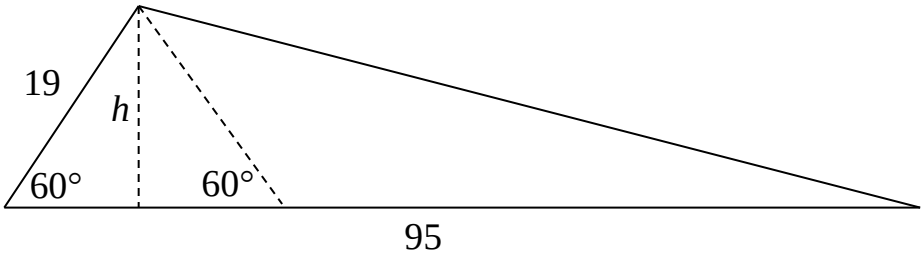
17. Semicircles drawn on each side of a triangle have areas of 9π, 16π, and 25π. Find the area of the triangle.

The Age of Equi-areas.

18. Find the ratio of the area of the smaller equilateral triangle to the area of the larger equilateral triangle.



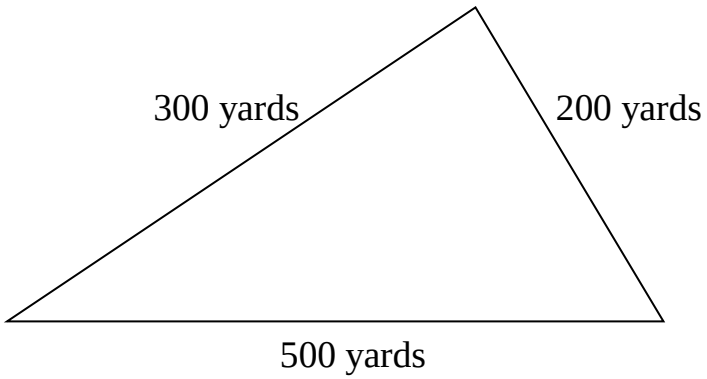
{Hint:



The area of one of the three triangles is $\frac{1}{2} \cdot 95 \cdot h$*.}*

This Land Aint Your Land, This Land Aint My Land.

19. A farmer found the following advertisement for a plot of land. Why shouldn't he buy it?

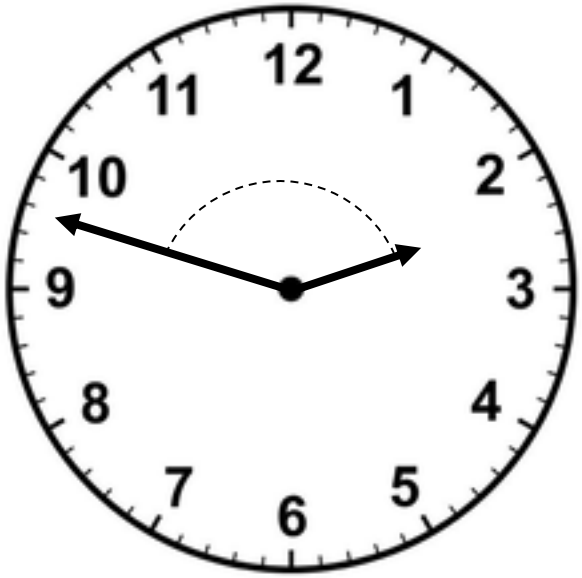


Right Triangle Or Wrong Triangle?

20. If a right triangle with integral side lengths has an area of 756 square units, what are the possible lengths of the hypotenuse of this right triangle?

Don't Tell Me No Lies, And Keep Your Hands To Yourself.

21. a) What is the measure, in degrees, of the angle formed by the hands of a clock at 2:48?

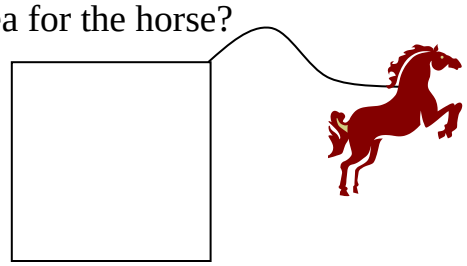


{The minute hand is $\frac{3}{5}$ of the way from 9 to 10, and the hour hand is $\frac{48}{60}$ of the way from 2 to 3.}

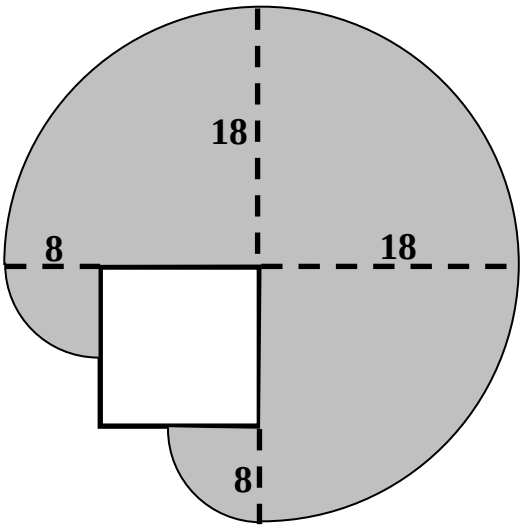
b) What is the measure, in degrees, of the angle formed by the hands of a clock at 3:36?

Grazin' In The Grass Is A Gas, Baby, Can You Dig It?

22. a) A horse is tethered by a rope to a corner on the outside of a square corral that is 10 feet on each side. The horse can graze at a distance of 18 feet from the corner of the corral where the rope is tied. What is the exact total grazing area for the horse?



{Hint:

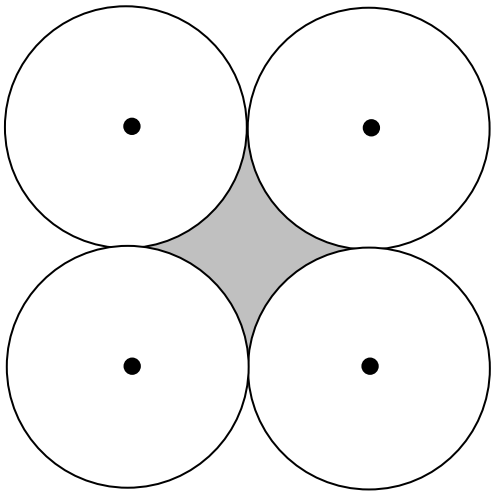


Express the answer using π .}

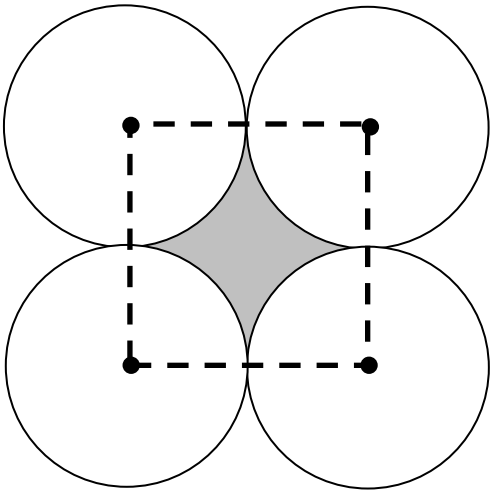
b) Do the same calculation if the horse can graze at a distance of 22 feet from the corner of the corral where the rope is tied.

How Touching!

23. Four circles, each of which has a diameter of 2 feet, touch as shown. Find the exact area of the shaded portion.



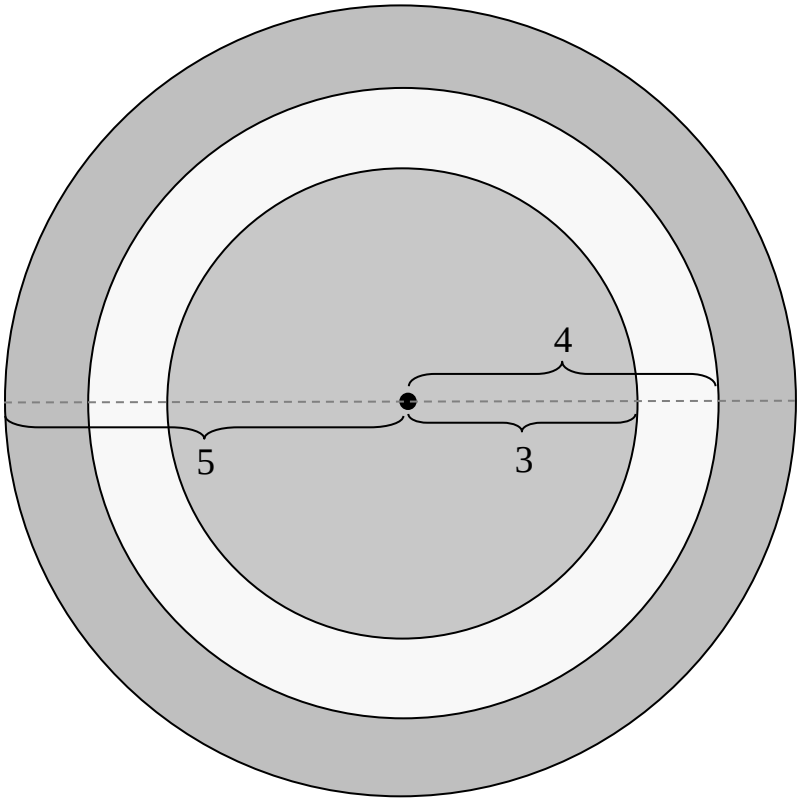
{Hint: The area of the shaded portion would be the area of the square minus the area of the four circular sectors. Express the answer using π *.*



}

On The Mark, Off The Mark, Or Bull's Eye?

24. In the following figure, the curves are concentric circles with the indicated radii. Which shaded region has the larger area, the inner circle or the outer ring?



Calculate the area of each region and check your visual estimation ability.

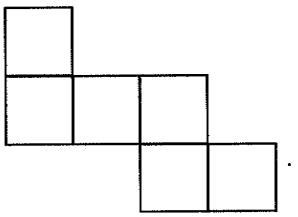
See How Everything Lines Up.

25. Given the following incomplete distance chart for 4 points in a plane, find the distance from A to B.

	A	**B**	**C**	**D**
A	**0**	**?**	**21**	**9**
B	**?**	**0**	**5**	**7**
C	**21**	**5**	**0**	**12**
D	**9**	**7**	**12**	**0**

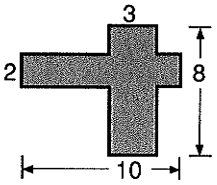
Just Go All The Way Around.

26. The following figure consists of six congruent squares, and it has an area of 294. Find the perimeter of the figure.



What An Intersecting Little Problem.

27. Find the area of the following shaded region formed by the two perpendicular intersecting rectangles.



You May Be A King Of Comedy, But Do You Know How A Clock Works?

28. On a recent episode of *Who Wants to Be a Millionaire* with Cedric the Entertainer, the following question appeared.

For which of the following times will the minute and hour hands of a clock form a right angle?

a) 4:05 **b)** 5:20

c) 3:35 **d)** 11:50

The contestant chose answer **a)** and he was told that he was correct. He wasn't correct, in fact, none of the options are correct. For t measured in minutes after midnight, $M(t)=6t$ represents the cumulative angle of the minute hand in degrees, and $H(t)=\frac{1}{2}t$ represents the cumulative angle of the hour hand in degrees. In order for the two hands to form a right angle, the difference between the cumulative angle of the minute hand and the cumulative angle of the hour hand must be an odd multiple of $90°$. So we get that

$$M(t)-H(t)=(2n-1)90;n=1,2,\ldots$$

$$6t-\tfrac{1}{2}t=(2n-1)90;n=1,2,\ldots$$

$$\tfrac{11}{2}t=(2n-1)90;n=1,2,\ldots$$

$$t=\frac{180(2n-1)}{11};n=1,2,\ldots$$

a) Use the previous formula to find the number of times from one midnight to the next that the minute and hour hands form a right angle.

{Hint: $\frac{180(2n-1)}{11} \le$ # of minutes in a 24 hour period.*}*

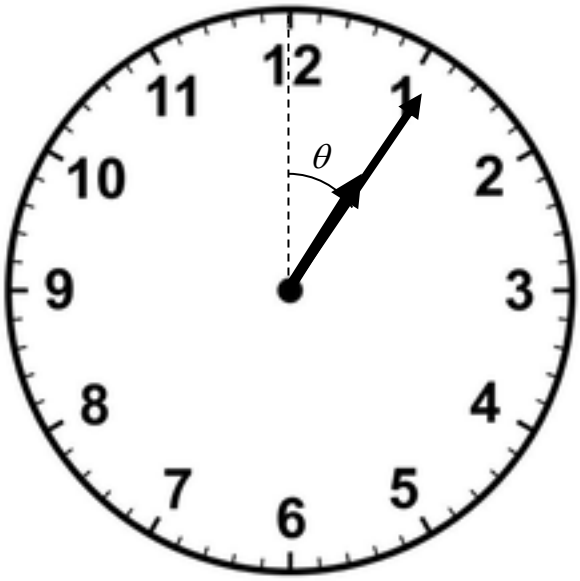
b) Use the same reasoning to find a formula for the times(in minutes after midnight) from one midnight to the next(inclusive) that the minute and hour hands point in exactly the same direction, and the number of times that it occurs.

c) Use the same reasoning to find a formula for the times(in minutes after midnight) from one midnight to the next that the minute and hour hands point in exactly opposite directions, and the number of times that it occurs.

Put Your Hands Together For This Intermission.

29. Professor I. Vadenuff stopped lecturing at noon. He announced that at noon the minute hand and hour hand are exactly lined up. He said the lecture would begin again the next time the two hands line up again. In precisely how many minutes will that be?

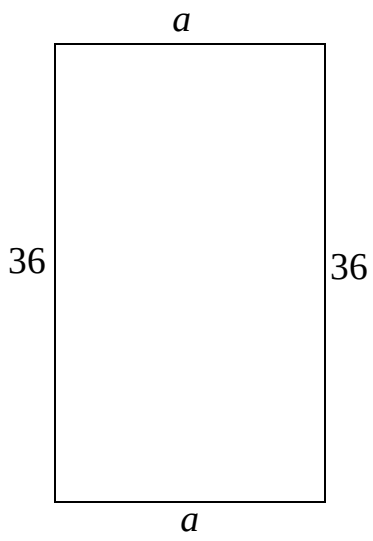
{The hands will next be together between 1:05 and 1:10.

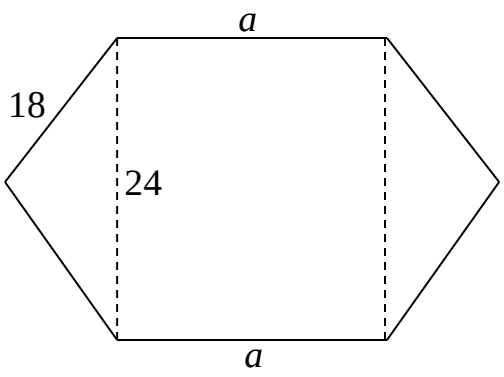


The angle θ *must have the property that* $\frac{\theta - 30}{30} = \frac{\theta}{360}$. *So the amount of time for the break is the 60 minutes from noon to 1 o'clock plus the* $\frac{\theta}{360} \cdot 60$ *minutes.}*

Can A Hexagon Be A Squashed Rectangle?

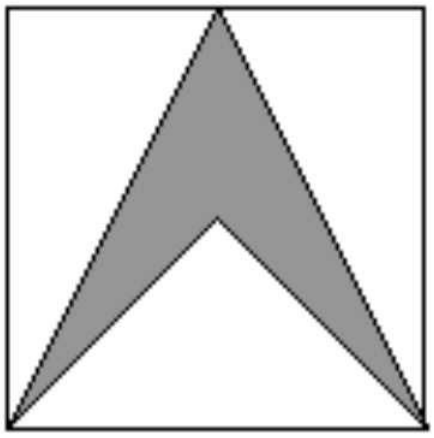
30. A rectangle has side of length a and 36. A hinge is installed at each vertex of the rectangle and the midpoints of the sides of length 36. The sides of length a can be pressed towards each other while being kept parallel to each other. This will result in a hexagon. When the distance between the sides of length a is 24, the hexagon has the same area as the original rectangle. Find the value of a^2.





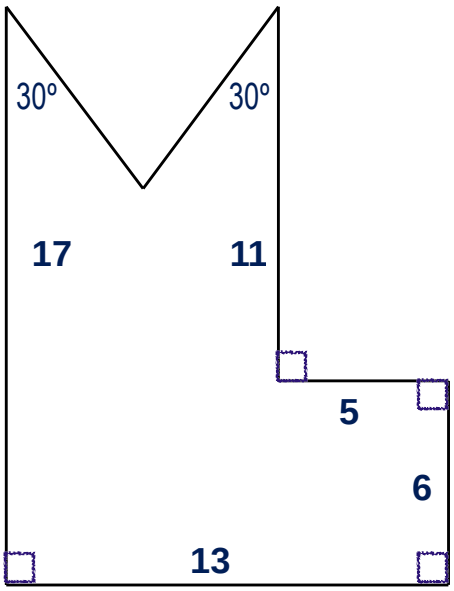
The Area Of The Arrow.

31. An arrow is formed in a square with side measurement 2 by joining the bottom vertices to the midpoint of the top edge and to the center of the square. Find the area of the arrow.



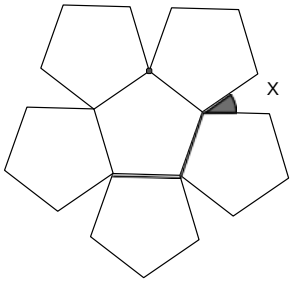
Triangulate The Perimeter.

32. Find the perimeter of the figure.



Mind The Gap.

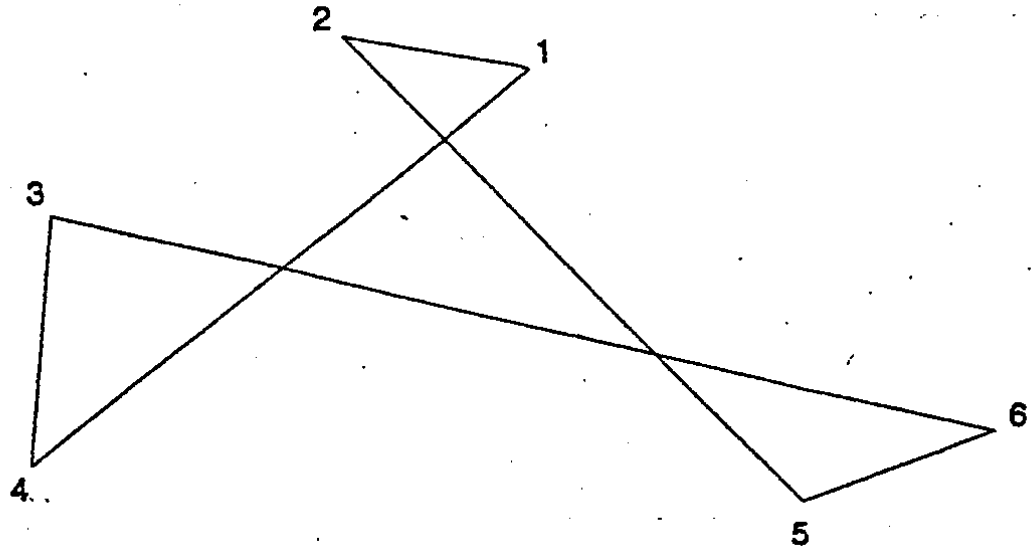
33. The six regular pentagons in the figure are congruent. At each point where three pentagons meet, they leave an angular gap, *X*. Find the measure of the gap in degrees.



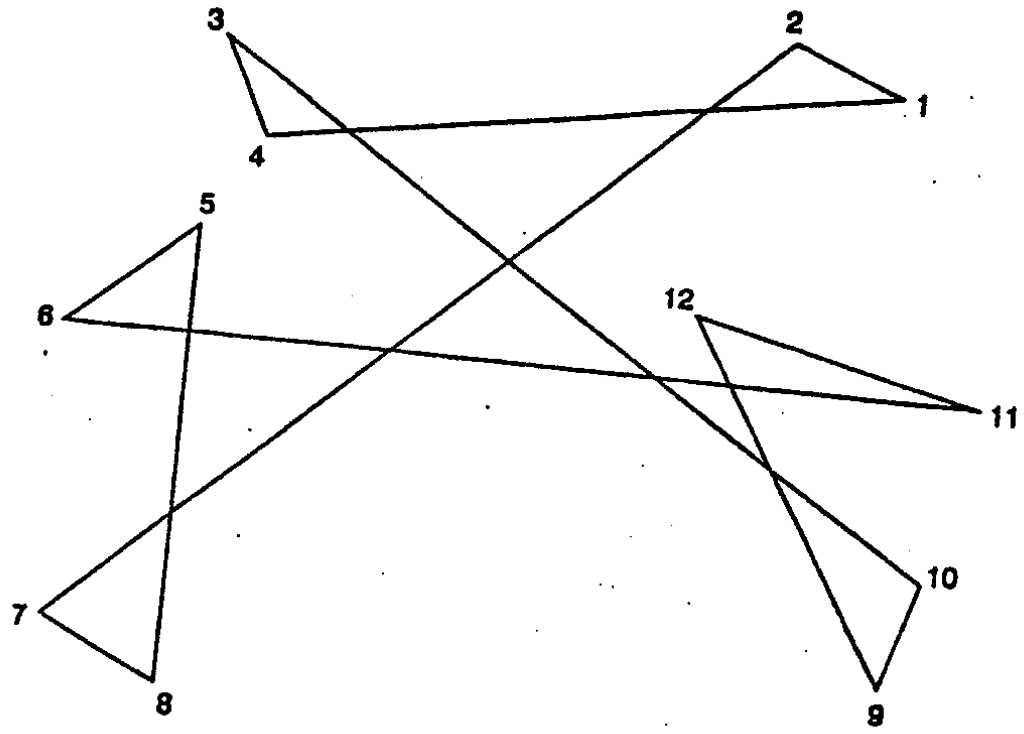
All But The Vertical Angles.

34. Determine the sum of the six angles labeled in the figures.

a)

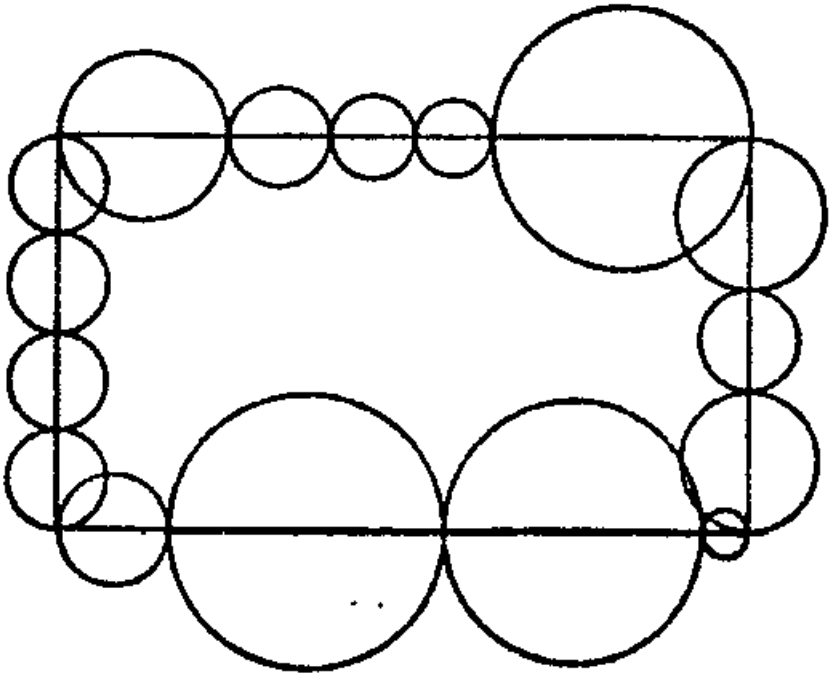


b)



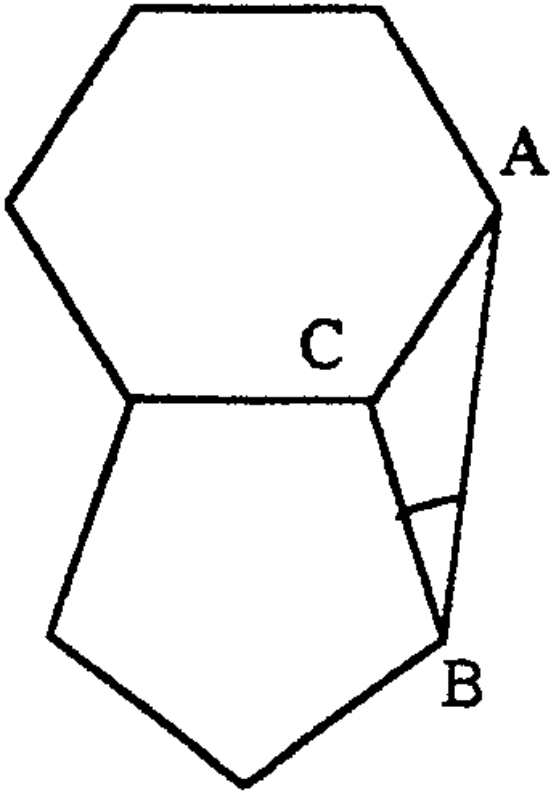
Can You Circumnavigate The Rectangle?

35. The rectangle has dimensions of 4 by 7. Determine the sum of all the circumferences of all the circles. The center of each circle lies on the rectangle, and the adjacent circles on each side are tangent to each other.



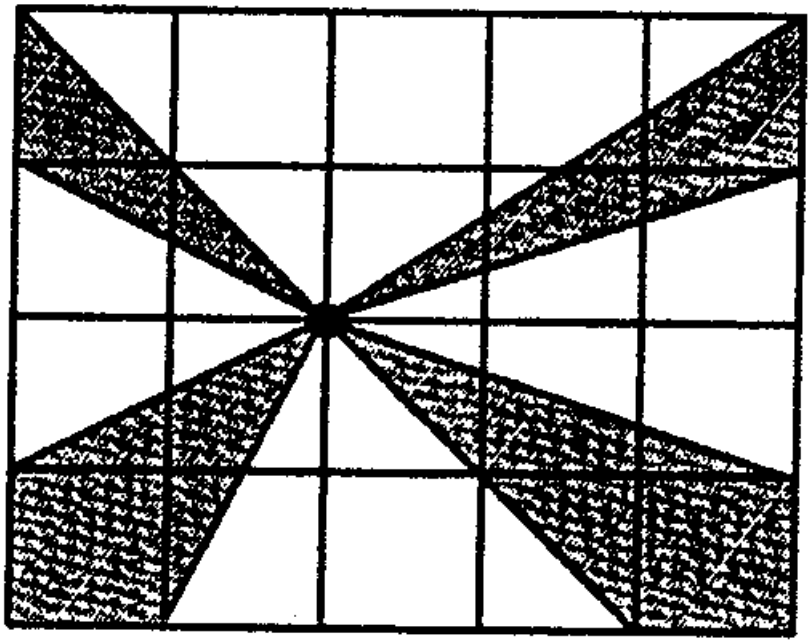
Love And Soccer.

36. The figure shows a regular hexagon adjacent to a regular pentagon. Determine the measure of angle ABC.



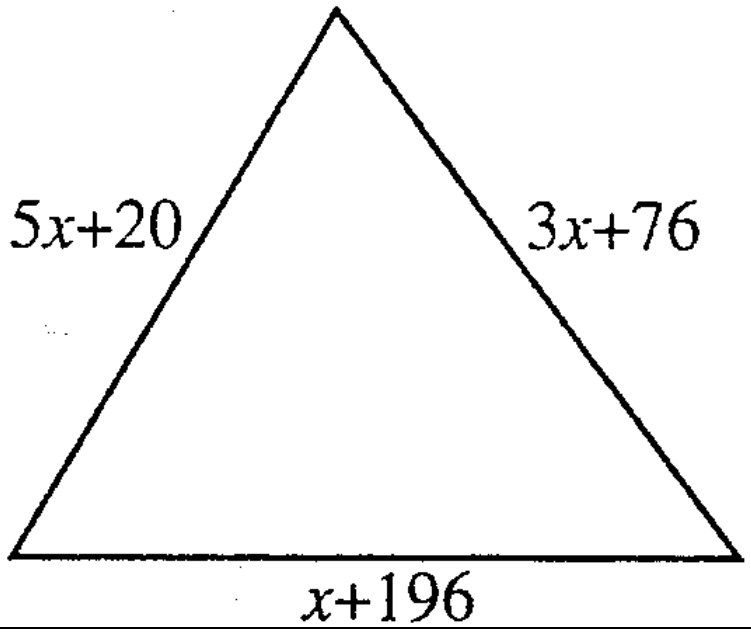
The Power Of Subtraction.

37. Determine the area of the shaded region in the 4 by 5 grid made up of 20 1 by 1 squares.



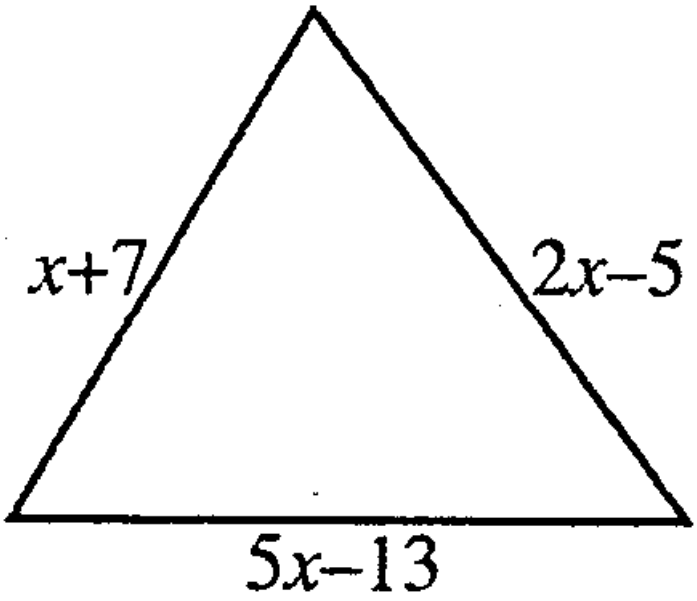
The Greatest Equality.

38. What is the greatest perimeter possible, given that the triangle is isosceles? Note: You don't know which two sides are equal!



Triangulate Your Answer.

39. Determine all the integer values of x so that $x+7$, $2x-5$, and $5x-13$ are the measures of the sides of a triangle.

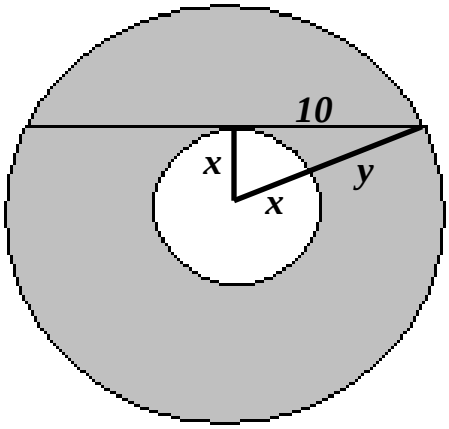


Slip Sliding Away.

40. If a 25 foot ladder is placed against a vertical wall of a building, then the foot of the ladder is 7 feet from the base of the building. If the top of the ladder slips down 4 feet, then how far will the foot of the ladder slide away from the building?

Annular Retentive?

41. a) The two circles below are concentric. The length of the chord tangent to the smaller circle is 20 inches. Find the area of the shaded annular region between the two circles.

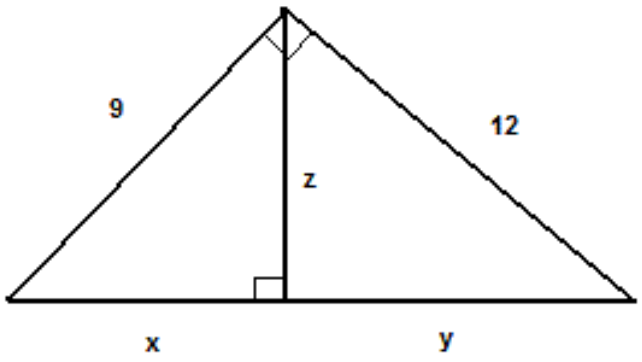


{Hint: The area of the shaded region is $\pi(x+y)^2 - \pi x^2$*, and from the Pythagorean Theorem,* $x^2 + 10^2 = (x+y)^2$*.}*

b) Calculate the area of the shaded region, if the length of the chord tangent to the smaller circle is 25 inches.

What Has 6 Legs And 3 Hypotenuses?

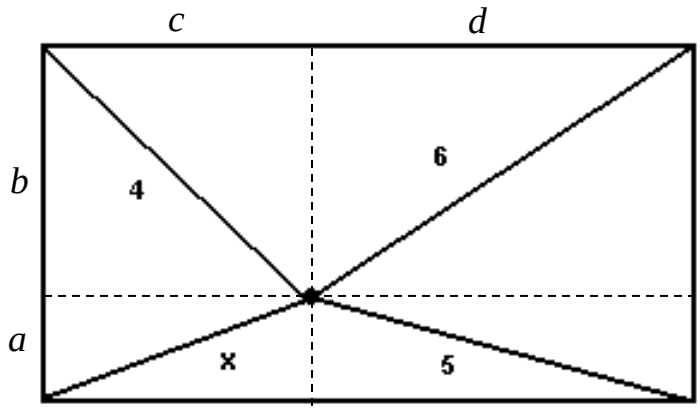
42. Find the lengths, *x*, *y*, and *z*.



{Hint: $x^2 + z^2 = 81, y^2 + z^2 = 144, (x+y)^2 = 225$*.}*

All My X's Live In Rectangles.

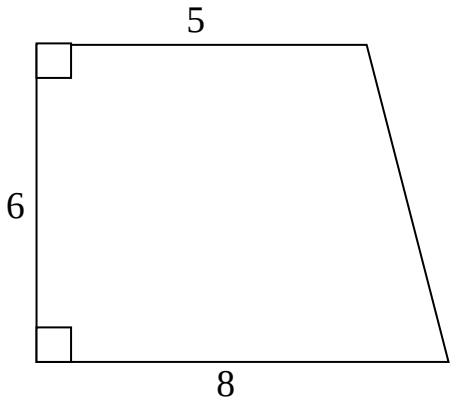
43. In the rectangle shown below, find the value of *x*.



{Hint: $a^2+c^2=x^2, b^2+c^2=16, b^2+d^2=36, a^2+d^2=25$. *Subtracting* $b^2+c^2=16$ *from* $b^2+d^2=36$ *leads to* $d^2-c^2=20$. *Subtract* $d^2-c^2=20$ *from* $a^2+d^2=25$ *to get the result.}*

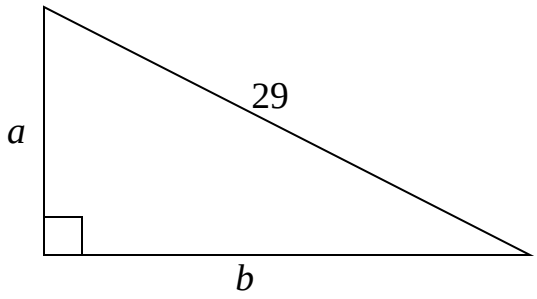
A Different Kind Of Workout For Your Quads.

44. Find the area of the following quadrilateral.



Area, Hypotenuse, Perimeter, What Does It All Mean?

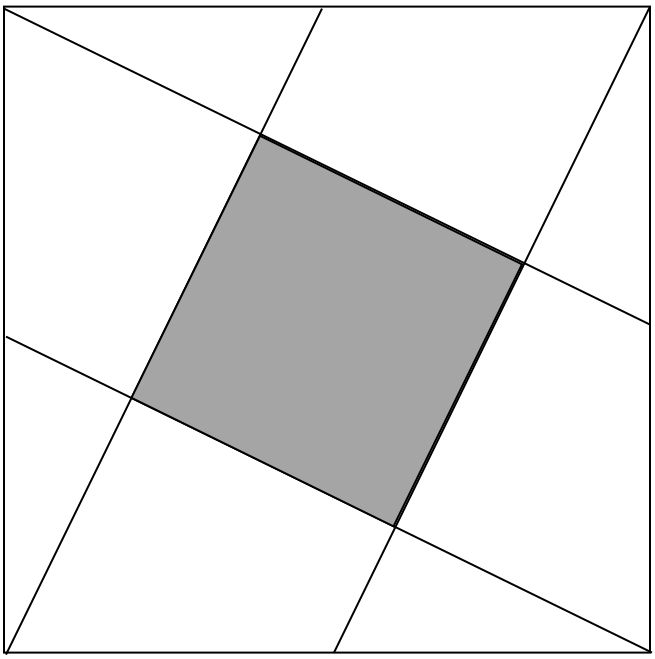
45. A right triangle has area of 210, and a hypotenuse of 29. Find the perimeter of the triangle.

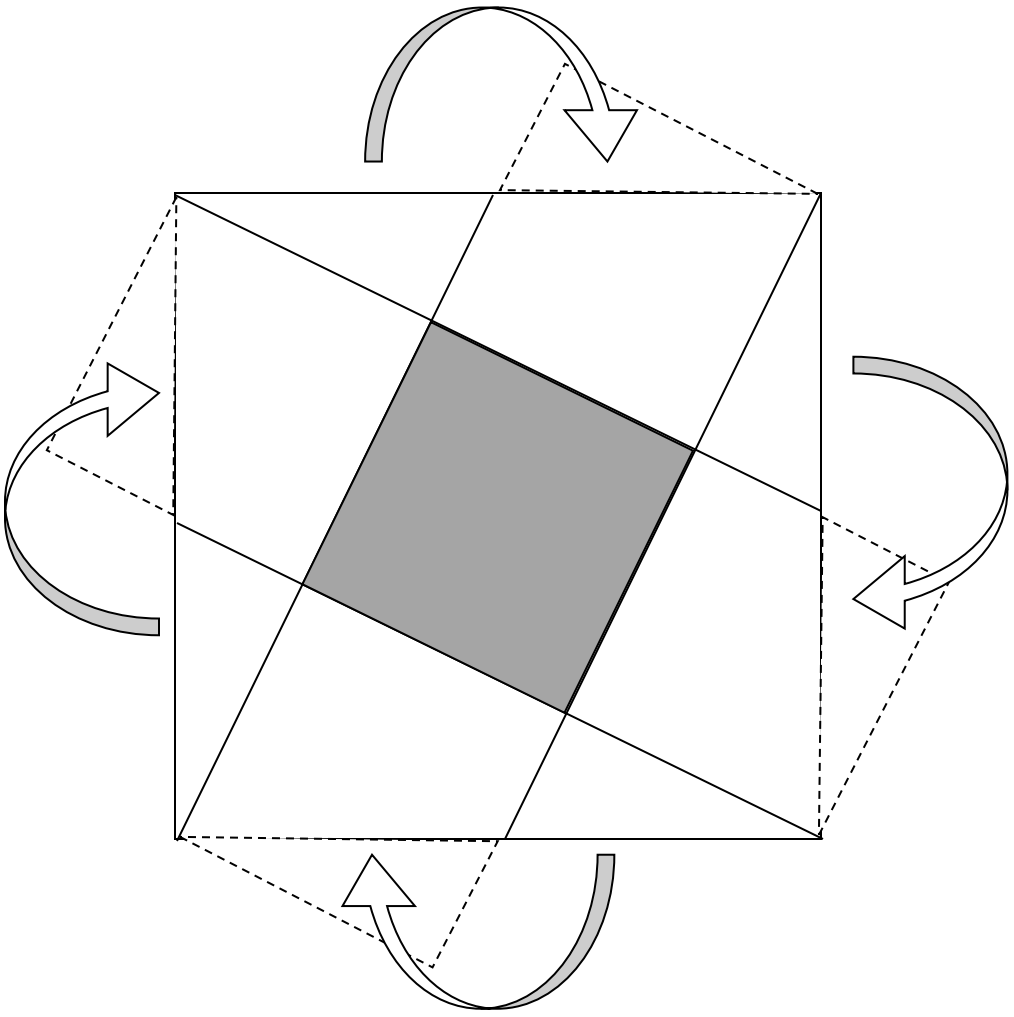


{Hint: $P = a + b + 29, \frac{1}{2}ab = 210, a^2 + b^2 = 29^2.$ $(a+b)^2 = a^2 + b^2 + 2ab = 29^2 + 4 \cdot 210.$*}*

Just Slide Your Way To An Answer.

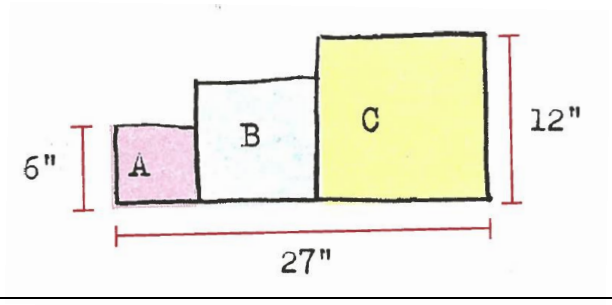
46. Consider a square of side length 1. Four line segments are drawn that connect a midpoint of one side to a vertex not on that side. Find the area of the shaded square in the center.





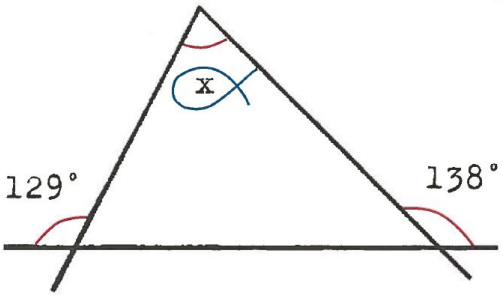
Sometimes $A^2 + B^2 \neq C^2$.

47. Three adjacent squares are given in the figure below. Determine the area of square B.



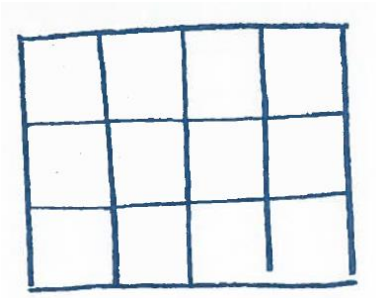
X Marks The Angle.

48. Determine the measure of angle X.



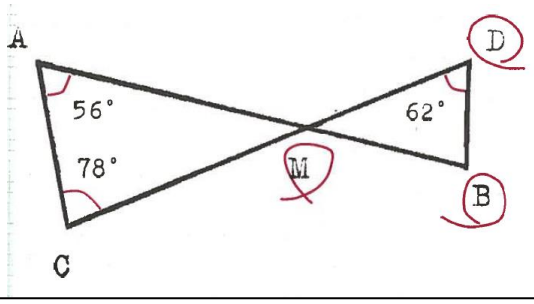
From Area To Perimeter.

49. The rectangle below is made up of 12 equal squares. The area of the rectangle is 432 square inches. What is the perimeter of the rectangle?



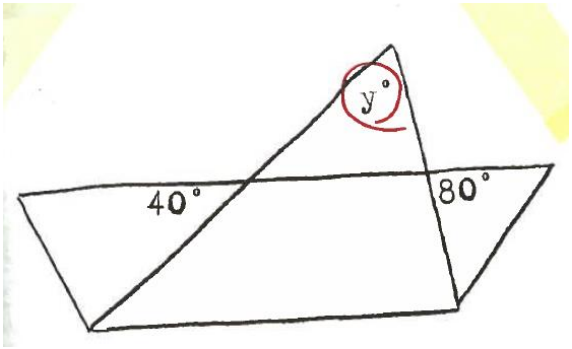
Don't Be Dumb, Find $\angle DBM$.

50. Find the measure of $\angle DBM$.



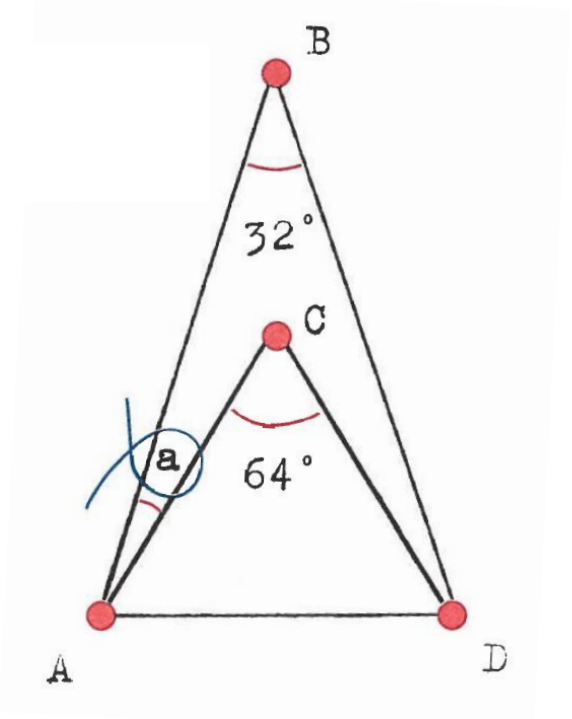
Why Ask Why?

51. What is the value of y?



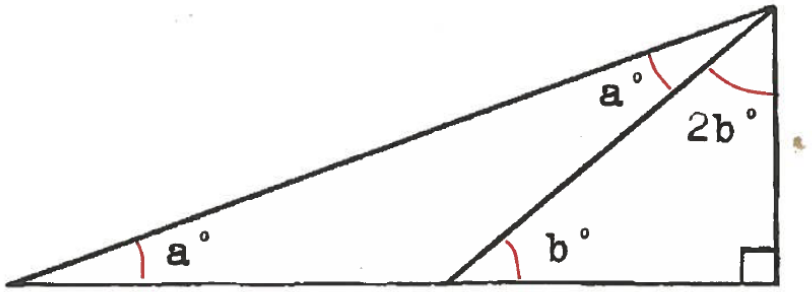
AC/DC.

52. Two isosceles triangles, ΔABD and ΔACD have the same base $\overline{AD}$. $\overline{AB} \simeq \overline{DB}$ and $\overline{AC} \simeq \overline{DC}$. Find the measure of angle a.



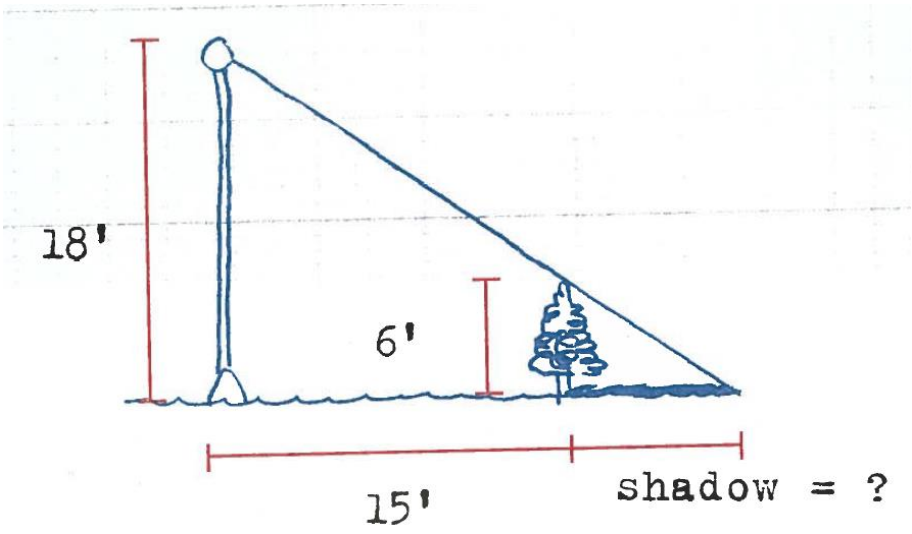
A Okay.

53. What's the value of a?



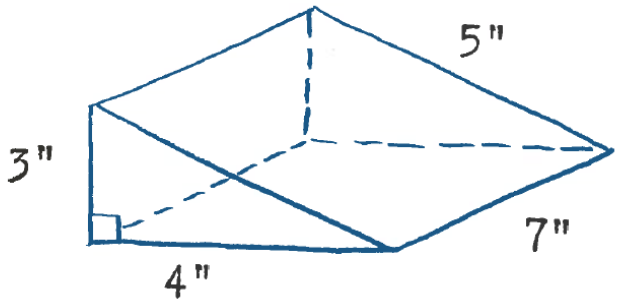
Only The Shadow Knows.

54. A tree that is 6 feet tall stands 15 feet away from a streetlamp that is 18 feet tall. What is the length of the shadow cast by the tree?



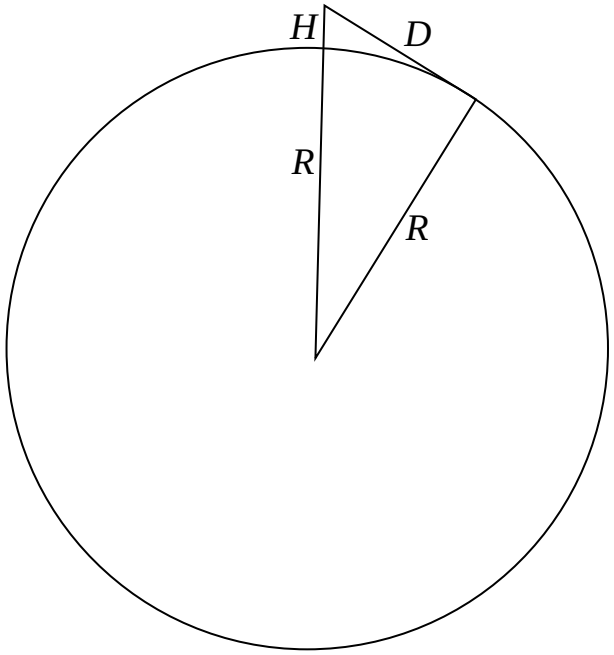
Don't Judge A Prism By Its Surface.

55. Calculate the surface area of this prism.



On A Clear Day You Can See Pretty Darn Far.

56. Assume that the Earth is smooth and spherical, and there are no intervening hills to obscure the view. If your eyes are at a height H above the ground, and you look toward the horizon, then the distance to the horizon, D is one leg of a right triangle whose other leg is the radius of the Earth, R, and whose hypotenuse is $R+H$.



From the Pythagorean Theorem, you get that $(H+R)^2 = R^2 + D^2$, and solving for D^2, leads to

$$D^2 = (H+R)^2 - R^2$$
$$= H^2 + 2HR$$

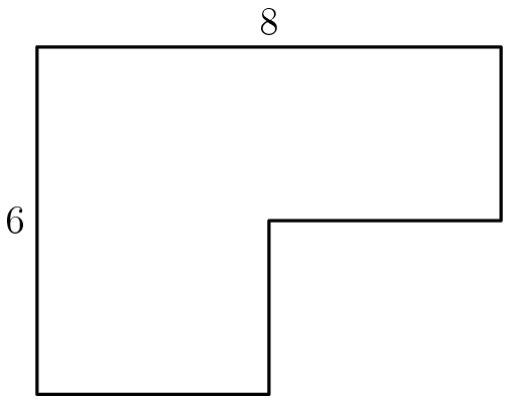
So the distance to the horizon is given by $D = \sqrt{H^2 + 2HR}$. Determine how far you can see in miles for the following heights above ground level. Assume that the radius of the Earth is 3,959 miles.

a) $H = 5\,ft.$ **b)** $H = 600\,ft.$ **c)** $H = 2,722\,ft.$ *(tallest building)*

d) $H = 29,002\,ft.$ *(Mount Everest)*

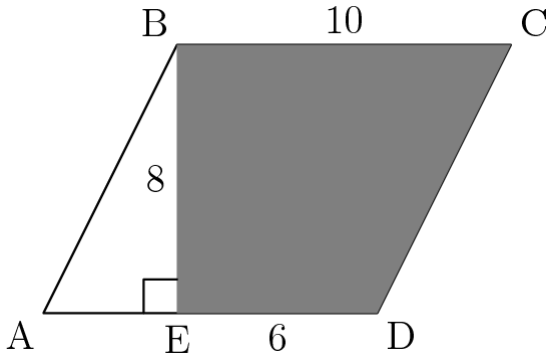
How Far Is It, All the Way Around?

57. Find the perimeter of the following figure in which all the angles are right angles.



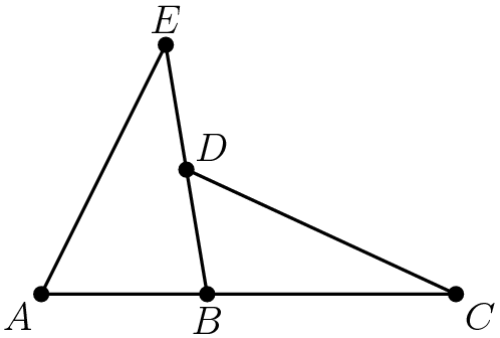
Another Shady Problem.

58. Find the area of the shaded region in the parallelogram ABCD.



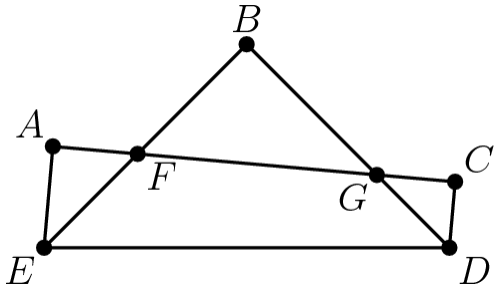
Angle On Over Here, And See if You Measure Up.

59. If $m(\angle A) = 60°$, $m(\angle E) = 40°$, and $m(\angle C) = 30°$, then what's $m(\angle BDC)$?



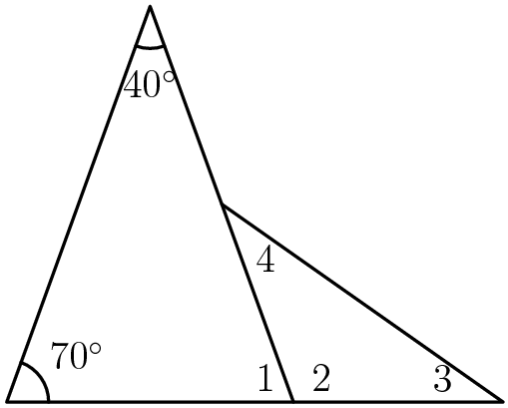
Before You Go To $\angle BED$, Check On $\angle ED(C)$.

60. $\angle A$, $\angle B$, and $\angle C$ are right angles. $m(\angle AEF) = 40°$, and $m(\angle BED) = m(\angle BDE)$. What's $m(\angle EDC)$?



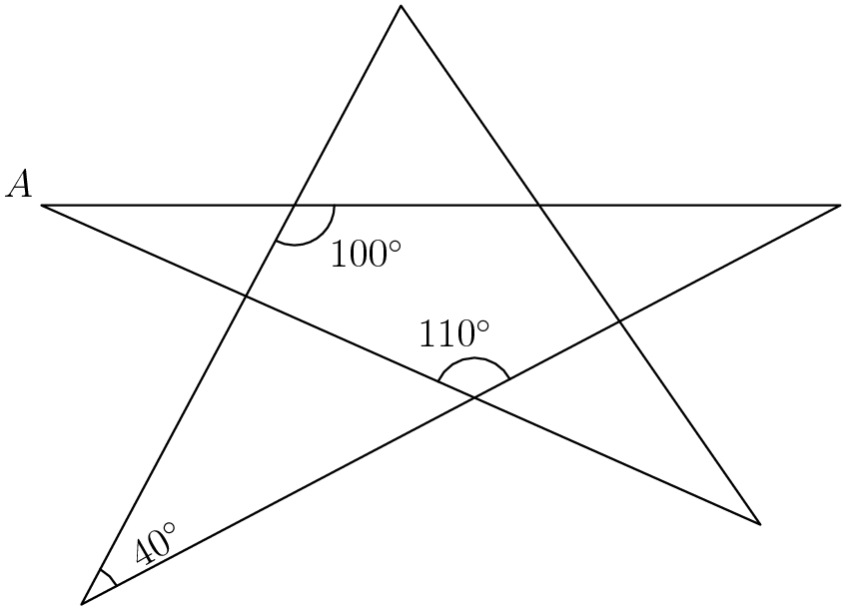
Can I Lean On You 'Cause You're (A)cute.

61. $m(\angle 3) = m(\angle 4)$ and $m(\angle 1) + m(\angle 2) = 180°$. What's $m(\angle 4)$?



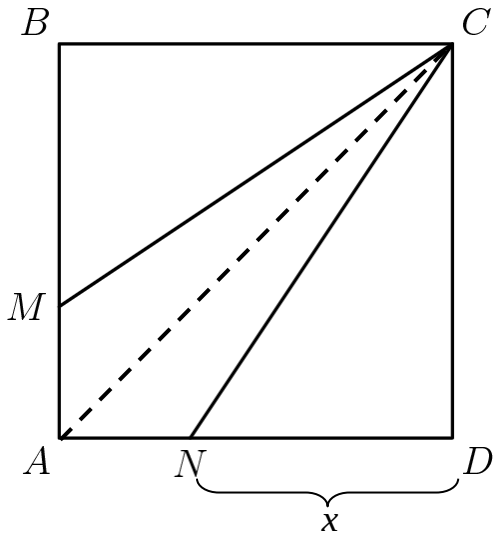
You're A Shining Star, No Matter Who You Are.

62. Find $m(\angle A)$.



If At First You Don't Succeed, Trisecting A Square.

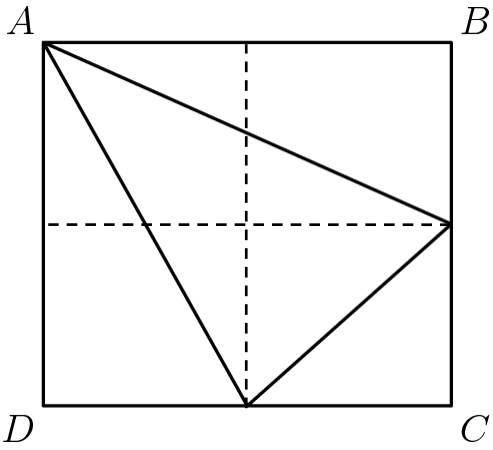
63. Square ABCD has side measurement of 3. The segments CM and CN divide the area of the square into three equal parts. How long is segment CN?



{Hint: $\frac{9}{2}-\frac{3x}{2}=\frac{9}{6}$*.}*

It's What's On The Outside That Counts.

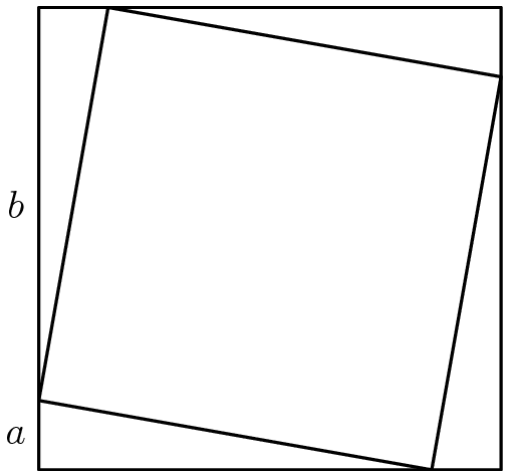
64. The area of rectangle ABCD is 72. A and the midpoints of BC and CD are joined to form a triangle. What's the area of the triangle?



{Hint: How much of the rectangle is outside of the triangle?}

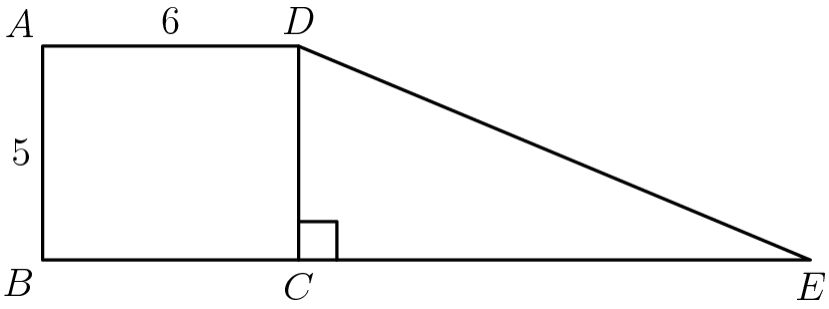
If It Looks Like Pythagoras, And Smells Like Pythagoras, It Might Not Be Pythagoras.

65. A square of area 4 is inscribed in a square of area 5 with one vertex of the smaller square on each side of the larger square. A vertex of the smaller square divides a side of the larger square into two segments, one of length *a* and the other of length *b*. What's the value of *ab*?



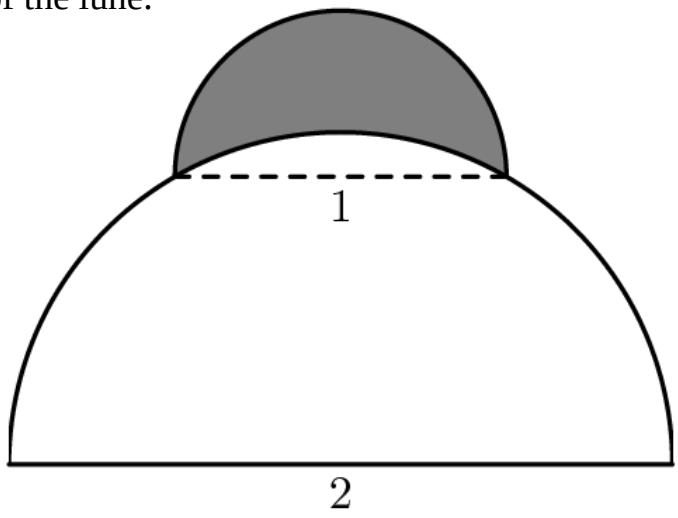
Have You Ever Felt Trapezoid In A Relationship?

66. Rectangle ABCD and triangle DCE have the same area. They are joined to form a trapezoid. What's the length of side DE?



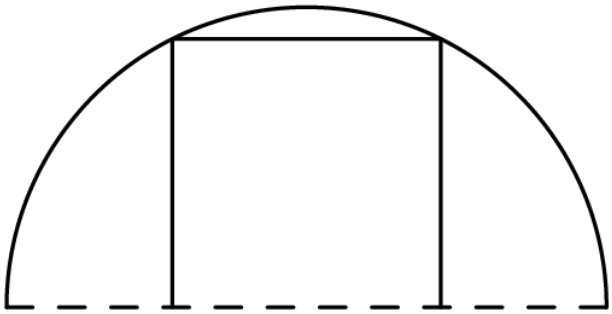
Luney Toons.

67. A semicircle of diameter 1 sits on top of a semicircle of diameter 2, as shown. The shaded region inside the smaller semicircle but outside the larger semicircle is called a lune. Determine the area of the lune.



All's Square That Ends Square.

68. A square of area 40 square units is inscribed in a semicircle, as shown. Determine the area of the semicircle.



I Put A Hex On You.

69. A regular hexagon has side length 6. Six congruent circular arcs with radius 3 are drawn centered at each of the vertices. The region inside the hexagon but outside the sectors is shaded as shown. Determine the area of the shaded region.