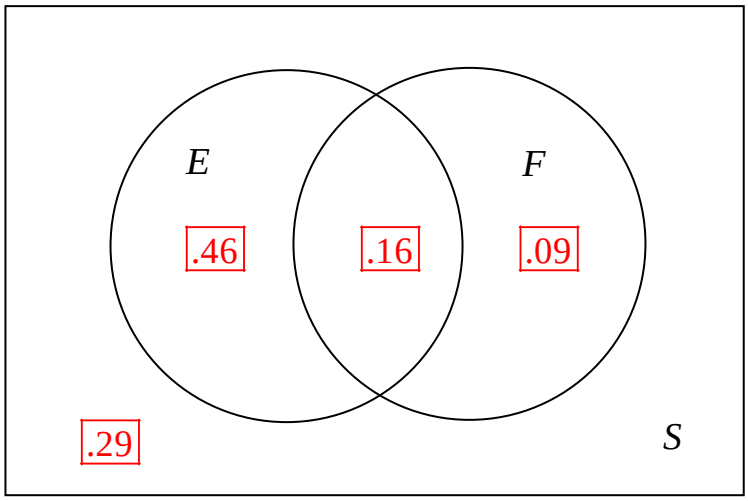
**Math 1351 Review #2(answers)**

**1.** *Suppose that* $P(E)=.62$, $P(F)=.25$, *and* $P(E\cap F)=.16$.

**a)** Complete the following probability diagram:




**b)** Find $P(E\cup F)$

$.46+.16+.09=\boxed{.71}$

**c)** Find $P(E\cap \bar{F})$

$\boxed{.46}$

**d)** Find $P(\bar{E}\cap \bar{F})$

$\boxed{.29}$

**e)** Find $P(E\,|\,F)$

$$\frac{P(E\cap F)}{P(F)}=\frac{.16}{.25}=\boxed{.64}$$

**f)** Find $P(F\,|\,E)$

$$\frac{P(E\cap F)}{P(E)}=\frac{.16}{.62}=\boxed{\frac{8}{31}}$$

**g)** Find the odds in favor of event *F*.

$P(F):P(\bar{F})$

$.25:.75$

$\boxed{1:3}$

**2.** At a company, ID numbers consist of two letters(A-Z) followed by 5 digits(0-9) with no repeating letters or digits.

**a)** How many different ID numbers are possible?

| | | |
|---|---|---|
| ${}_{26}P_2=650$ | ${}_{10}P_5=30,240$ | $\boxed{19{,}656{,}000}$ |
| # of two letter permutations | # of 5 digit permutations | Total |

OR

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 26 | 25 | 10 | 9 | 8 | 7 | 6 | $\boxed{19{,}656{,}000}$ |
| 1st letter | 2nd letter | 1st digit | 2nd digit | 3rd digit | 4th digit | 5th digit | Total |

**b)** What is the probability that a randomly assigned ID number would have the letters B and P on it?

| B | P | 10 | 9 | 8 | 7 | 6 | 30,240 |
|---|---|---|---|---|---|---|---|
| 1st letter | 2nd letter | 1st digit | 2nd digit | 3rd digit | 4th digit | 5th digit | Total |

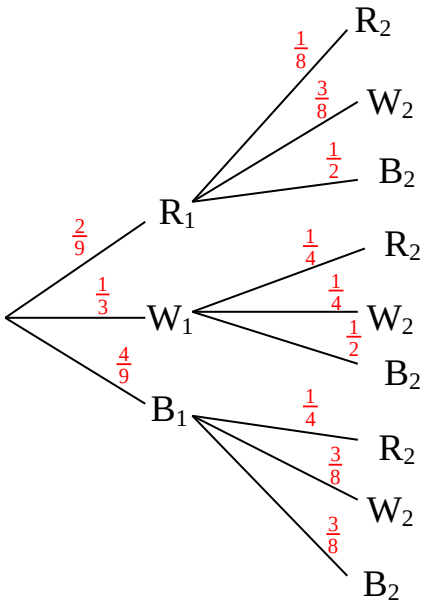
OR

| P | B | 10 | 9 | 8 | 7 | 6 | 30,240 |
|---|---|---|---|---|---|---|---|
| 1st letter | 2nd letter | 1st digit | 2nd digit | 3rd digit | 4th digit | 5th digit | Total |

$$\frac{\#\text{ of ID's with B and P}}{\#\text{ of ID's}} = \frac{60,480}{19,656,000} = \boxed{\frac{1}{325}}$$

**3.** A box contains two red, three white, and four blue tickets. Two tickets will be randomly drawn without replacement.

**a)** Complete the probability tree for this experiment.




**b)** Find the probability of drawing a red ticket and a blue ticket in either order.

$$P(\text{red and blue}) = P(R_1 \cap B_2) + P(B_1 \cap R_2) = \frac{2}{9}\cdot\frac{1}{2} + \frac{4}{9}\cdot\frac{1}{4} = \boxed{\frac{2}{9}}$$

**c)** Find the probability of drawing a red ticket on the second draw.

$$P(R_2) = \frac{2}{9}\cdot\frac{1}{8} + \frac{1}{3}\cdot\frac{1}{4} + \frac{4}{9}\cdot\frac{1}{4} = \frac{1}{36} + \frac{1}{12} + \frac{1}{9} = \boxed{\frac{2}{9}}$$

**d)** Find the probability of drawing a blue ticket on the second draw.

$$P(B_2) = \frac{2}{9}\cdot\frac{1}{2} + \frac{1}{3}\cdot\frac{1}{2} + \frac{4}{9}\cdot\frac{3}{8} = \frac{1}{9} + \frac{1}{6} + \frac{1}{6} = \boxed{\frac{4}{9}}$$

**e)** Find the probability of drawing a white ticket on the second draw.

$$P(W_2) = 1 - P(R_2) - P(B_2) = 1 - \frac{2}{9} - \frac{4}{9} = \boxed{\frac{1}{3}}$$

**f)** Find the probability that both tickets drawn will have the same color.

$$P(\text{same color}) = P(R_1 \cap R_2) + P(W_1 \cap W_2) + P(B_1 \cap B_2) = \frac{2}{9} \cdot \frac{1}{8} + \frac{1}{3} \cdot \frac{1}{4} + \frac{4}{9} \cdot \frac{3}{8}$$

$$= \frac{1}{36} + \frac{1}{12} + \frac{1}{6} = \boxed{\frac{5}{18}}$$

**g)** If a blue ticket is drawn second, what is the probability that a red ticket was drawn first?

$$P(R_1 \mid B_2) = \frac{P(R_1 \cap B_2)}{P(B_2)} = \frac{\frac{2}{9} \cdot \frac{1}{2}}{\frac{4}{9}} = \boxed{\frac{1}{4}}$$

**4.** A team of four players is to be selected from a group of eight boys and six girls.

**a)** How many different teams are possible?

$$_{14}C_4 = \boxed{1001}$$

**b)** How many different teams are possible if there must be two boys and two girls?

| $_8C_2 = 28$ | $_6C_2 = 15$ | $\boxed{420}$ |
|---|---|---|
| Which 2 boys? | Which 2 girls? | Total |

**c)** How many different teams are possible if they must all be boys?

$$_8C_4 = \boxed{70}$$

**d)** If a team is randomly assembled, what is the probability that it will have two boys and two girls?

$$\frac{420}{1001} = \boxed{\frac{60}{143}}$$

**e)** If a team is randomly assembled, what is the probability that it will have all boys?

$$\frac{70}{1001} = \boxed{\frac{10}{143}}$$

**f)** If a team is randomly assembled, what is the probability that it will have at least one girl?

$$P(\text{at least one girl}) = 1 - P(\text{no girls}) = 1 - P(\text{all boys}) = 1 - \frac{10}{143} = \boxed{\frac{133}{143}}$$

**5.** The table gives the results of a survey question which asked: "Are federal income taxes too high, about right, too low, or don't know?". If a respondent is chosen at random, determine the following:

| | Too High | About Right | Too Low | Don't Know | Total |
|---|---|---|---|---|---|
| **Male** | 289 | 192 | 6 | 10 | 497 |
| **Female** | 257 | 153 | 3 | 14 | 427 |
| **Total** | 546 | 345 | 9 | 24 | 924 |

**a)** $P(Female)$

$\frac{427}{924} = \boxed{\frac{61}{132}}$

**b)** $P(About\ Right)$

$\frac{345}{924} = \boxed{\frac{115}{308}}$

**c)** $P(Male\ or\ Too\ Low)$

$\frac{500}{924} = \boxed{\frac{125}{231}}$

**d)** $P(Female\ and\ Don't\ Know)$

$\frac{14}{924} = \boxed{\frac{1}{66}}$

**e)** $P(Female|Don't\ Know)$

$\frac{14}{24} = \boxed{\frac{7}{12}}$

**f)** $P(Don't\ Know|Male)$

$\boxed{\frac{10}{497}}$

**6.** A player rolls a fair die and receives a number of dollars equal to the number of dots showing on the face of the die.

**a)** If the game costs \$1 to play, what's the expected value of the game?

Let $X$ be the gross winnings in a single play of the game.

| $X$ | \$1 | \$2 | \$3 | \$4 | \$5 | \$6 |
|---|---|---|---|---|---|---|
| $P(X)$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |

$$E(X) = \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6} = \frac{21}{6} = \$3.50$$

If you pay \$1 per play, the expected value of the game is $\boxed{\$2.50}$.

**b)** If the game costs \$2 to play, what's the expected value of the game?

If you pay \$2 per play, the expected value of the game is $\boxed{\$1.50}$.

**c)** What is the most the player should be willing to pay to play the game and not lose money in the long run?

$\boxed{\$3.50}$

**7.** A die is rolled 100 times with the following results:

| **Outcome** | **1** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|---|
| **Frequency** | 17 | 16 | 17 | 19 | 13 | 18 |

**a)** Find the experimental probability of rolling a 4.

$$\frac{19}{100}$$

**b)** Find the experimental probability of rolling an odd number.

$$\frac{47}{100}$$