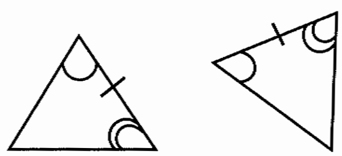
Math 1351 Activity 5(Chapter 14)(Due with Final Exam) **Group #_______**

Three Methods of Proving Triangles Congruent

Side—Side—Side (SSS)	3 sides of one Δ ≅ to 3 sides of another Δ.
Side—Angle—Side (SAS)	2 sides and the included angle of one Δ ≅ to 2 sides and the included angle of another Δ.
Angle—Side—Angle (ASA)	2 angles and the included side of one Δ ≅ to 2 angles and the included side of another Δ.

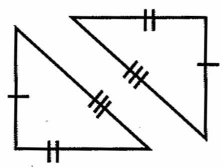
Determine which of the above methods can prove the triangles congruent. If SSS, place an X in the left letter box; if SAS, place an X in the middle letter box; if ASA, place an X in the right letter box. The remaining letters will spell the name of a U.S. president who developed a proof of the Pythagorean Theorem.

1.



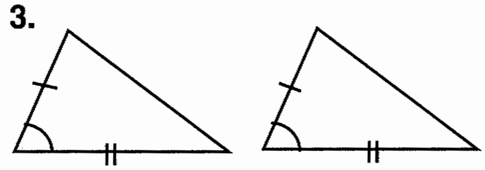
J A B

2.



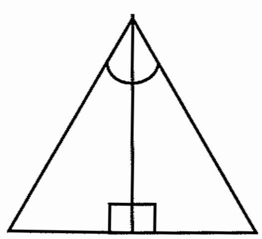
R M E

3.



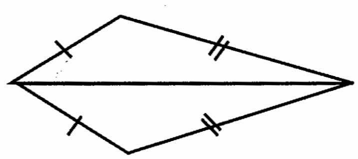
S P A

4.



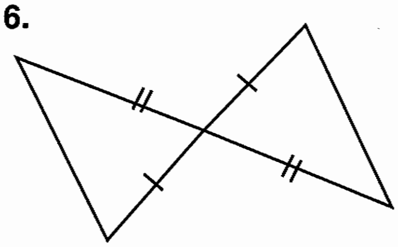
B R E

5.



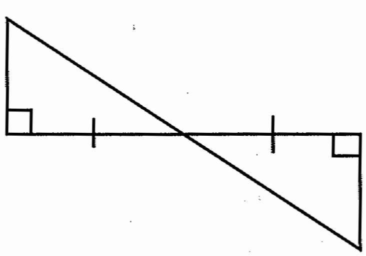
S A M

6.



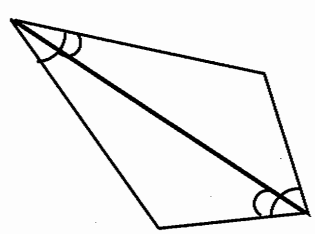
G L A

7.



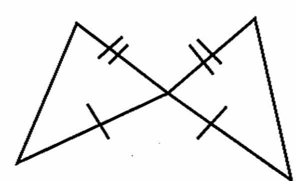
R F L

8.



I E A

9.



L T D

____ ____ ____ ____ ____ ____ ____ ____ ____ ____

____ ____ ____ ____ ____ ____ ____ ____

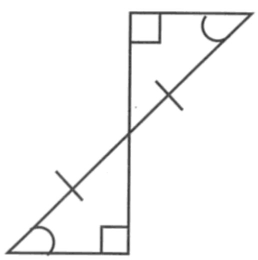
Two Additional Triangle Congruence Methods	
Angle—Angle—Side (AAS)	Two angles and a nonincluded side of one Δ ≅ to two angles and a nonincluded side of another Δ.
Hypotenuse—Leg (HL)	In a right Δ, the hypotenuse and one leg ≅ to the hypotenuse and leg of another right Δ.

Use the following box as a guideline.

SSS	SAS	ASA	AAS	HL

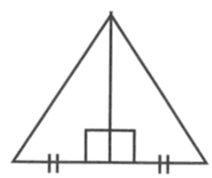
Place an X in <u>all</u> the boxes that can be applied to prove the triangles congruent. The remaining letters will reveal the name of a process used by Archimedes to determine the volume of a sphere using equal weights.

1.



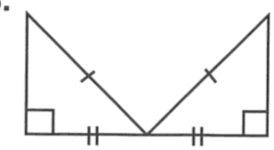
T	H	A	M	E

2.



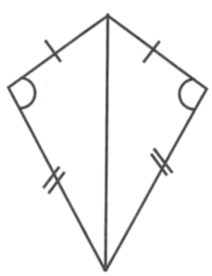
M	P	E	T	H

3.



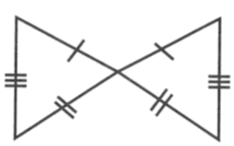
O	D	O	F	R

4.



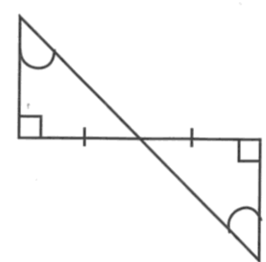
T	H	E	Q	U

5.



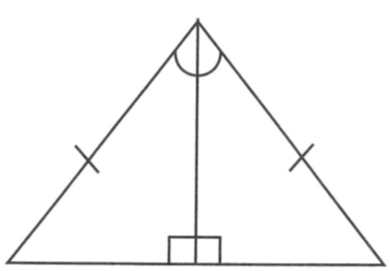
S	T	I	L	I

6.



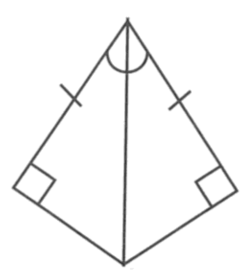
B	R	U	S	I

7.



U	H	E	A	P

8.



M	T	H	O	D

___ ___ ___   ___ ___ ___ ___ ___ ___   ___ ___

___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

Using the tools provided, construct each of the triangles indicated below. If possible, for each prompt, make a second triangle that is not congruent.

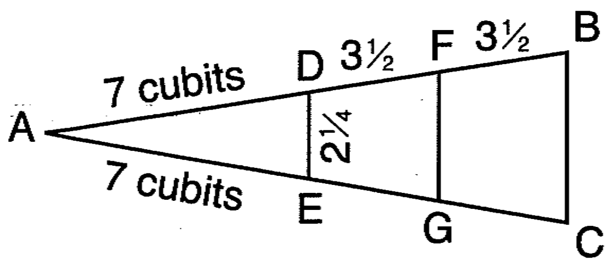
1) Construct triangle ABC such that:
   $AB = 5$ cm $BC = 6$ cm $AC = 7$ cm
2) Construct triangle DEF such that:
   $DE = 5$ cm $EF = 6$ cm angle $E = 40°$
3) Construct triangle GHI such that:
   $GH = 5$ cm $HI = 2$ cm angle $G = 20°$
4) Construct triangle KLM such that:
   $KM = 5$ cm angle $K = 40°$ angle $M = 50°$
5) Construct triangle NPQ such that:
   $NP = 5$ cm angle $P = 50°$ angle $Q = 60°$
6) Construct triangle RST such that:
   $RS = 5$ cm $ST = 7$cm angle $T = 100°$
7) Construct triangle UVW such that:
   angle $U = 40°$ angle $V = 60°$ angle $W = 80°$

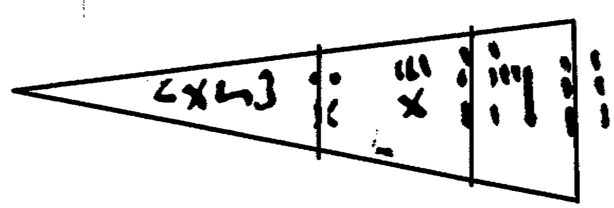
Similar Triangles in Egypt

Ancient Egypt

Students in ancient Egypt studied geometry to solve practical problems. Geometry provided answers for questions that arose in surveying and in construction of the pyramids and temples. The properties of similar triangles were used by mathematicians, artists, and architects in their daily work.

1. In 1650 B.C.E., the scribe Ah'mose copied a mathematical papyrus that was already 200 years old. He drew the diagram shown here. Given that line segments $\overline{DE} \,||\, \overline{FG} \,||\, \overline{BC}$, lengths of $\overline{DF}$ and $\overline{FB}$ are $3\frac{1}{2}$ cubits, and length of $\overline{DE}$ is $2\frac{1}{4}$ cubits, find the lengths of segments $\overline{EG}$, $\overline{GC}$, $\overline{FG}$, and $\overline{BC}$.

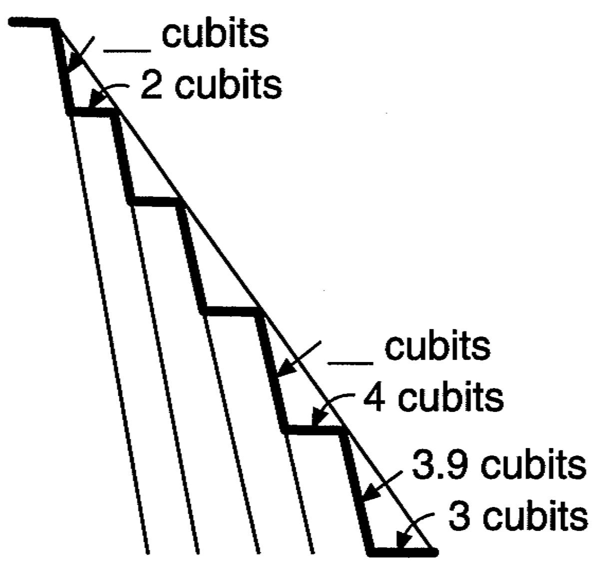




after Chace, 1927–29

Lengths in cubits: $EG =$ ___________ $GC =$ ___________
$FG =$ ___________ $BC =$ ___________

2. The first Egyptian stone pyramid did not have the true pyramid shape but rose in steps, each step set back from the step below. The steps at King Huni's pyramid at Meydum were filled in, creating a true pyramid shape. To assure smooth sides, each step had to have the same slope. Use your knowledge of similar triangles to fill in the blanks:

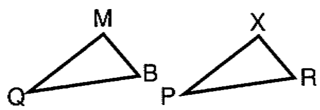


Detail from a cross section of Pyramid at Meydum (Lengths are approximate.)

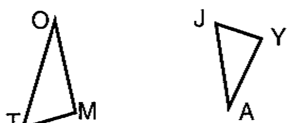
Directions: In each problem there are congruent triangles with three answers. Decide which answer has corresponding parts that are congruent. Notice the letter next to each correct answer. Write that letter on the blank line which corresponds to the problem you have just solved. (There will only be one correct answer per problem.)

1. $\triangle ABC = \triangle DEF$ (D) $\angle D \cong \angle C$ (E) $\overline{BC} \cong \overline{EF}$ (F) $\overline{CA} \cong \overline{DE}$

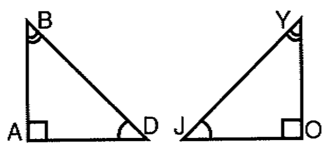
2. $\triangle RST = \triangle UWV$ (A) $\angle T \cong \angle U$ (E) $\overline{RS} \cong \overline{WU}$ (I) $\overline{TS} \cong \overline{UW}$

3. 

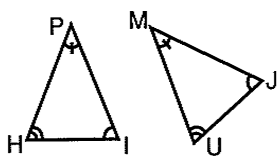
$\triangle MQB = \triangle XPR$ (H) $\angle B \cong \angle R$ (G) $\overline{XR} \cong \overline{QM}$ (I) $\angle Q \cong \angle R$

4. 

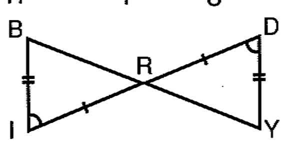
$\triangle TOM = \triangle JAY$ (S) $\angle JYA \cong \angle OTM$ (U) $\angle OMT \cong \angle AJY$ (T) $\overline{JA} \cong \overline{OT}$

5. 

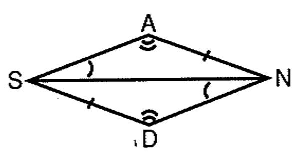
(R) $\triangle BAD \cong \triangle JOY$ (S) $\triangle BAD \cong \triangle YOJ$ (T) $\triangle ABD \cong \triangle OJY$

6. 

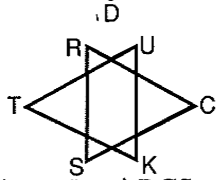
(A) $\angle JUM \cong \angle HIP$ (E) $\overline{UM} \cong \overline{PI}$ (I) $\overline{JU} \cong \overline{HI}$

7. 

(C) $\overline{BR} \cong \overline{YR}$ (D) $\angle BIR \cong \angle DRY$ (G) $\angle BRI \cong \angle RDY$

8. 

(P) $\angle DNS \cong \angle ANS$ (R) $\triangle ASN \cong \triangle DNS$ (T) $\triangle ANS \cong \triangle DNS$

9. 

$\triangle KTU = \triangle RCS$ (U) $\triangle TUK \cong \triangle CRS$ (E) $\triangle UKC \cong \triangle SRT$ (O) $\triangle KTU \cong \triangle RCS$

10. $\triangle DOG \cong \triangle CAT$ (K) $\angle OGD \cong \angle ATC$ (I) $\angle OGD \cong \angle TAC$ (T) $\angle ODG \cong \angle CTA$

Question: Where is Sylvester Stallone's favorite vacation spot?

___ ___ ___   ___ ___ ___ ___ ___ ___ ___

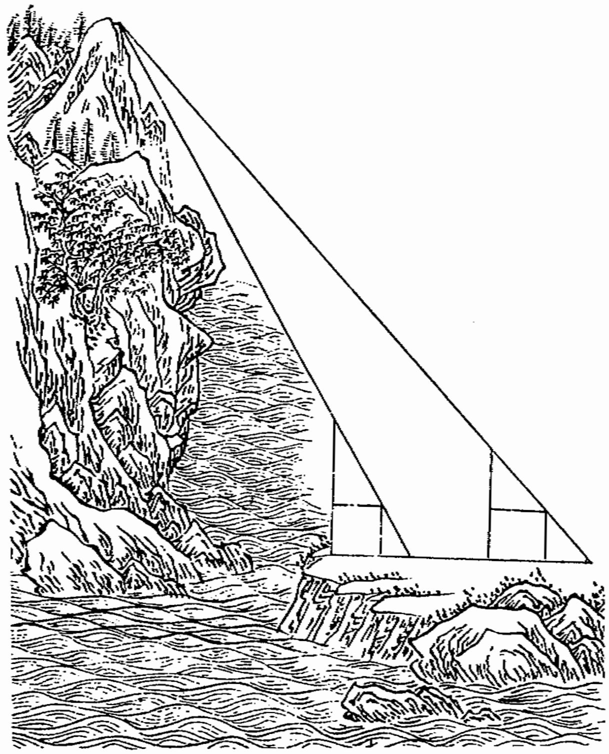
4 3 1   8 9 7 10 6 2 5

An ancient Chinese surveying problem asks for the height and distance from shore of a mountainous sea island. The solution to the problem is given in the *Haidao Suanjing*, translated by Frank Swetz. The idea is to find the height of the mountain without climbing it and the distance to the island without getting wet.

The solution begins "Now for looking at a sea island, erect two poles of the same height." The basic unit of length used here is the Chinese bu, about 1.8 m.

Two vertical poles, each 5 bu high, are placed on the ground 1,000 bu apart, aligned to sight the top of the island. When a surveyor moves back 123 bu from the first pole, the surveyor can sight the island top in line with the pole. To sight the island top in line with the second pole, the surveyor steps back 127 bu behind the second pole. (Ignore the height of the surveyor.)

How high is the top of the mountainous sea island, and how far is it from the first pole? *Method:* Draw a diagram and use proportions derived from similar triangles.

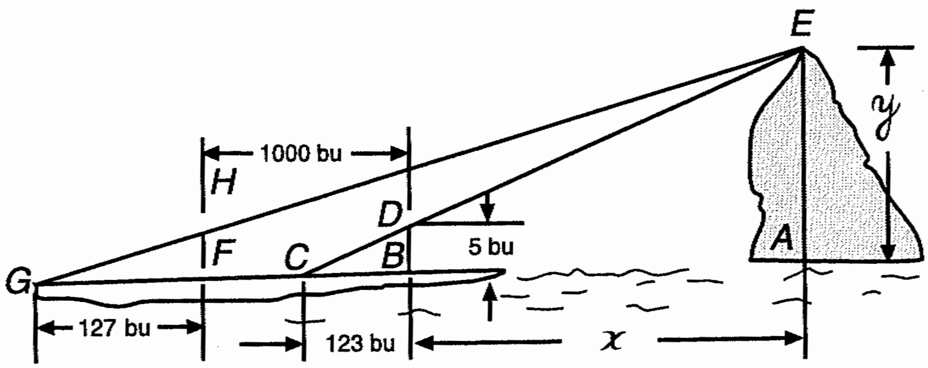


after Needham, 1959

Solve the following proportions.

Here x = distance of island from the first pole, and y = height of mountain.

$$\frac{y}{x+123} = \frac{5}{123} \qquad \frac{y}{x+1,127} = \frac{5}{127}$$



after Swetz, 1977

Eratosthenes was born in Cyrene, a city in the country now called Libya. By that time, Greek mathematics had merged with the ancient mathematics of North Africa and western Asia. Eratosthenes won a high position at the famous university, or museum of Alexandria, and did his best known work in Egypt.

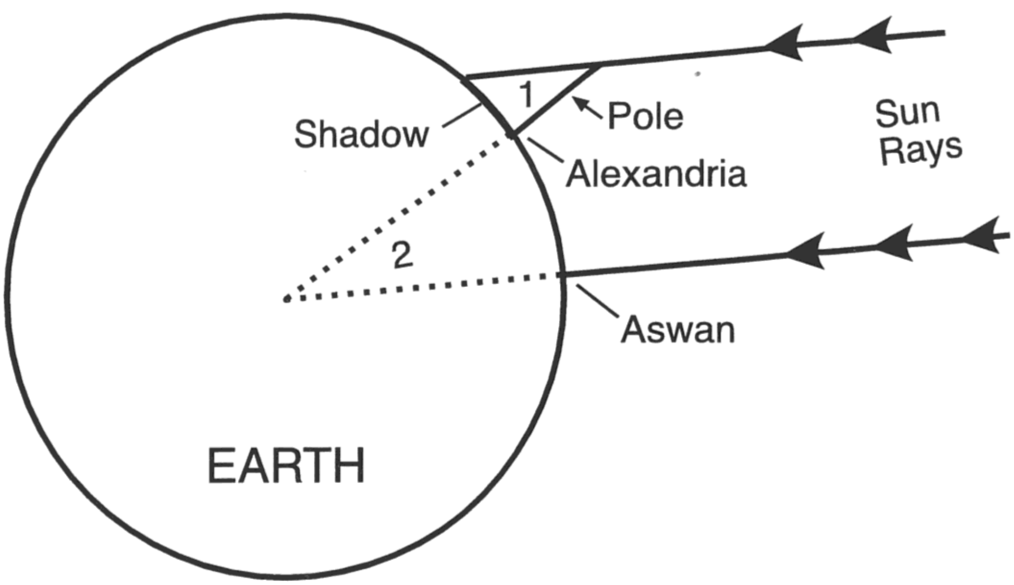
Scientists already knew that the earth was round. As sailboats sailed out to the horizon, people on land lost sight of the bottom of the ship first. The masts were the last part that was visible. Eratosthenes wanted to know how large the earth's circumference was.

Eratosthenes had a brilliant idea. He knew that on June 21 the sun would be directly overhead and cast no shadow at Aswan. At the same moment, the sun would cast shadows in Alexandria, about 600 miles north of Aswan. So at noon on June 21, in the year 230 B.C.E., Eratosthenes measured the height of a pole and the length of its shadow in Alexandria. He found that $\angle 1$ measured about 7.2°. He used the fact that sun's rays are parallel and that alternate interior angles of parallel lines are congruent to show that $\angle 1 \cong \angle 2$. Therefore, the measure of $\angle 2 = 7.2°$, $\frac{1}{50}$ of a circle.

Taking the distance between Alexandria and Aswan as 5000 stades, Eratosthenes found the circumference of the earth from this proportion:

$$\frac{7.2}{360} = \frac{5000}{\text{circumference of Earth}}$$

Eratosthenes found the circumference of the earth to be 250,000 stades. It is believed that he took 1 stade as equal to 300 royal cubits. The royal cubit is 0.525 m long. That gave Eratosthenes a very accurate value for the circumference of the earth.



Proportionally, the pole and shadow are much shorter than shown.

Questions for Critical Thinking

1. Use Eratosthenes's calculation of 250,000 stades as the circumference of the earth and the conversion factors 1 stade = 300 royal cubits and 1 royal cubit = .525 m, to find:

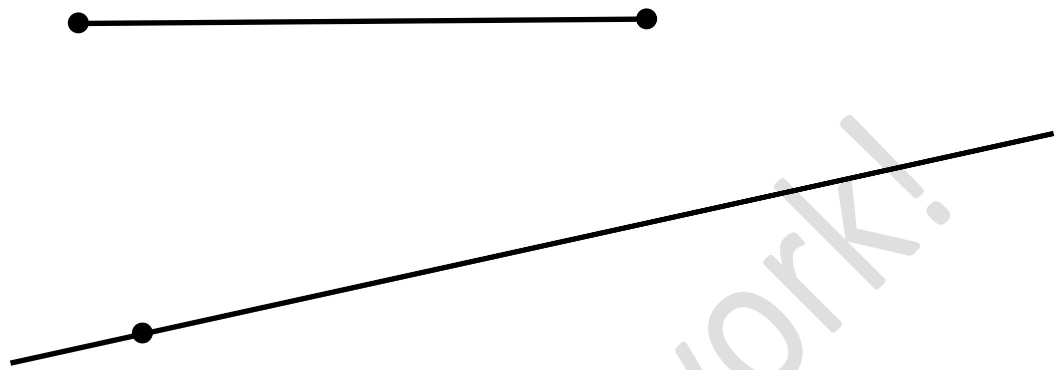
   The equivalent, in kilometers and in miles, for Eratosthenes's figure for the circumference of the earth.

2. The circumference of the earth measured around the poles is given today as 24,859 miles.
   (a) How big was Eratosthenes's error?

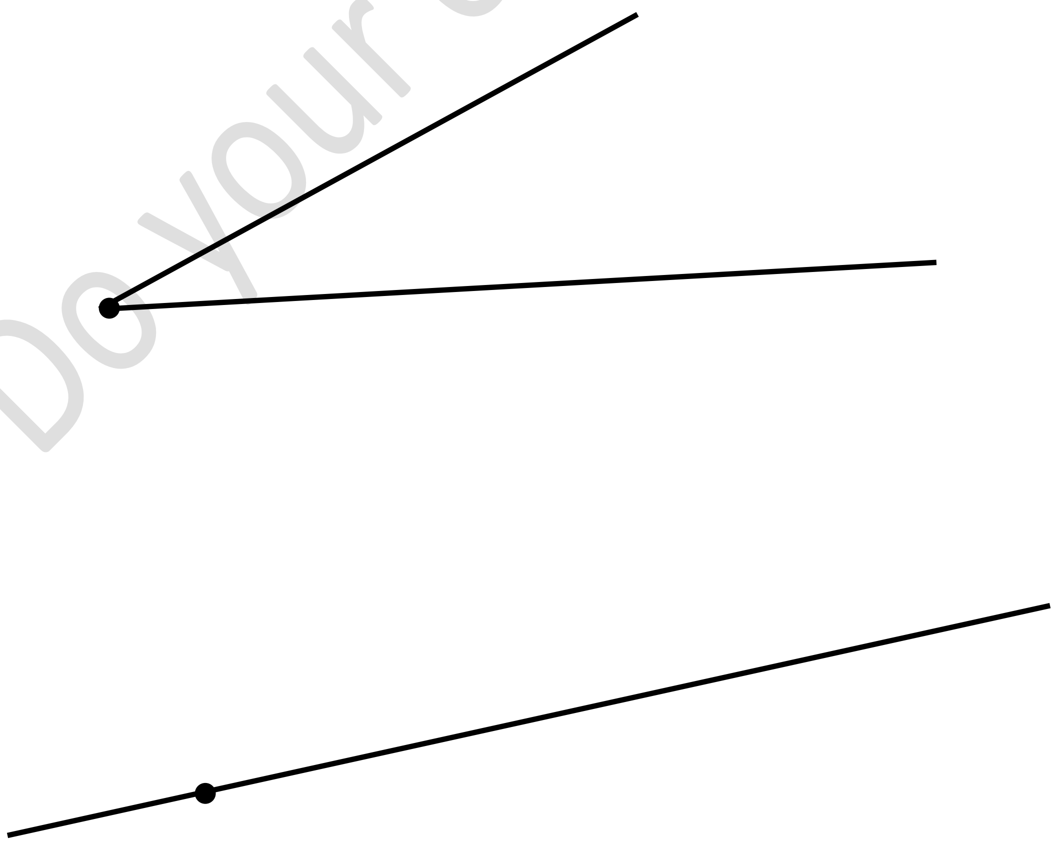
   (b) What was his percent error?

Perform the following Euclidean(Compass and Straightedge) constructions.

1. Construct a copy of the given segment on the indicated segment.

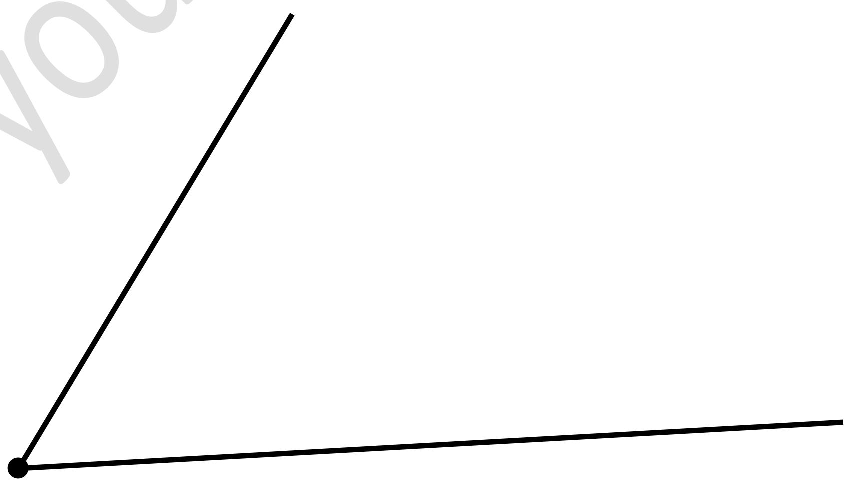


2. Construct a copy of the given angle on the indicated segment.

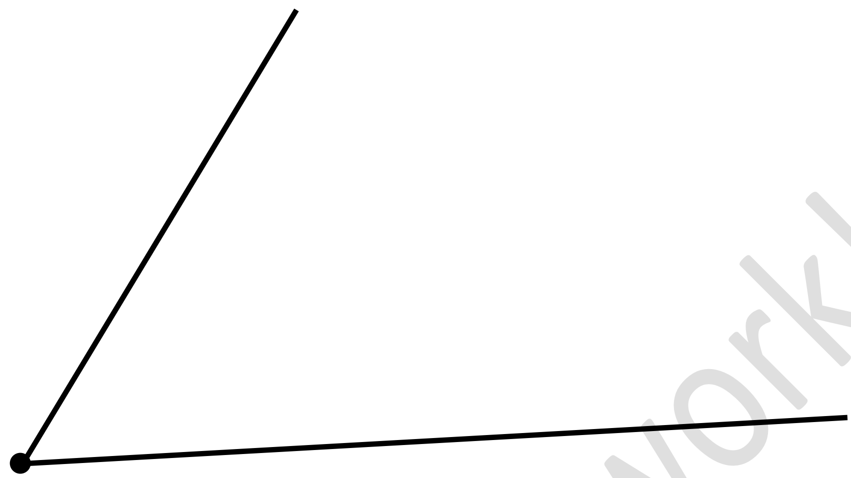


3. Bisect the given segment.

4. Bisect the given angle.

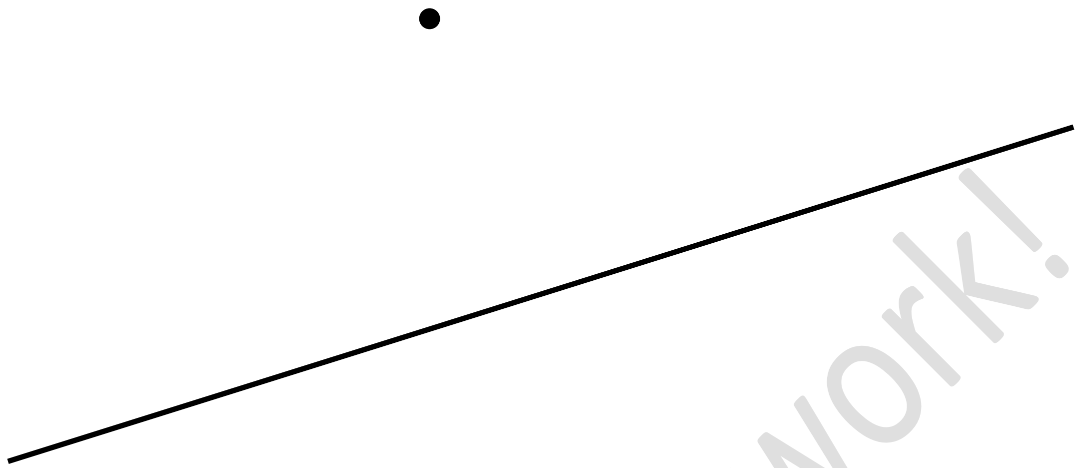


5. Construct an angle whose measure is one-fourth the measure of the given angle.

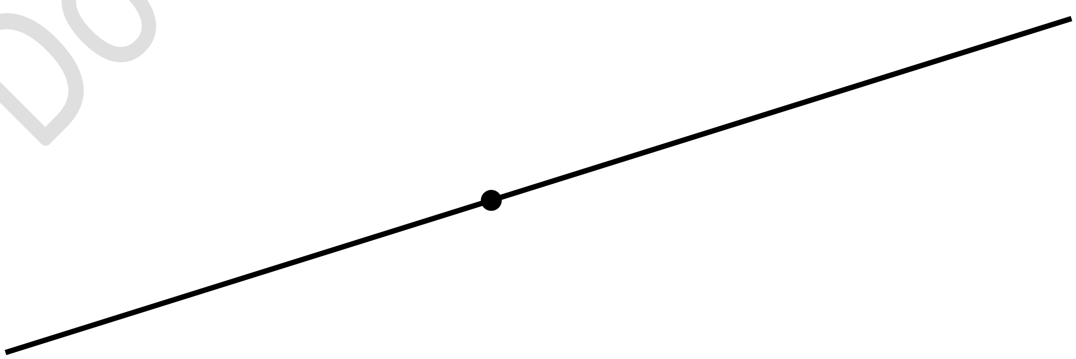


6. Construct a segment whose length is one-fourth the length of the given segment.

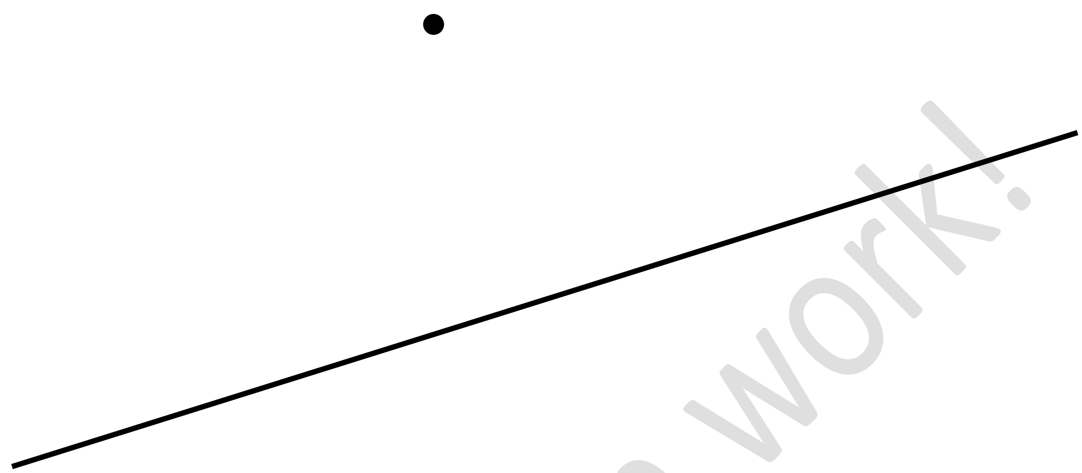
7. Construct a segment through the given point that is perpendicular to the given segment.



8. Construct a segment that is perpendicular to the given segment at the given point on the segment.



9. Construct a segment through the given point that is parallel to the given segment.



10. Construct an equilateral triangle using the given segment as one of its sides.