Review of Relations and Functions:

$R = \{(1,2),(2,4),(3,5)\}$

$S = \{(1,2),(1,3),(2,6)\}$

The domain is the set of first numbers, and the range is the set of second numbers.

Domain of *R*?

$\{1,2,3\}$

Domain of *S*?

$\{1,2\}$

Range of *R*?

$\{2,4,5\}$

Range of *S*?

$\{2,3,6\}$

Is *R* a function? Explain.

Yes. Each domain value is associated with exactly one range value.

Is *S* a function? Explain.

No. The domain value 1 is associated with two different range values.

Relations and Functions from equations: ***The equation defines y as a function of x if when each domain value is substituted in for x, there is a unique value for y that makes the equation true.***

$$x + y = 6$$

$y = 6 - x$, so each domain value *x* is associated with exactly one range value. So the equation defines *y* as a function of *x*.

$$x = y^2$$

The domain value 1 is associated with the range values 1 and -1. So the equation doesn't define *y* as a function of *x*.

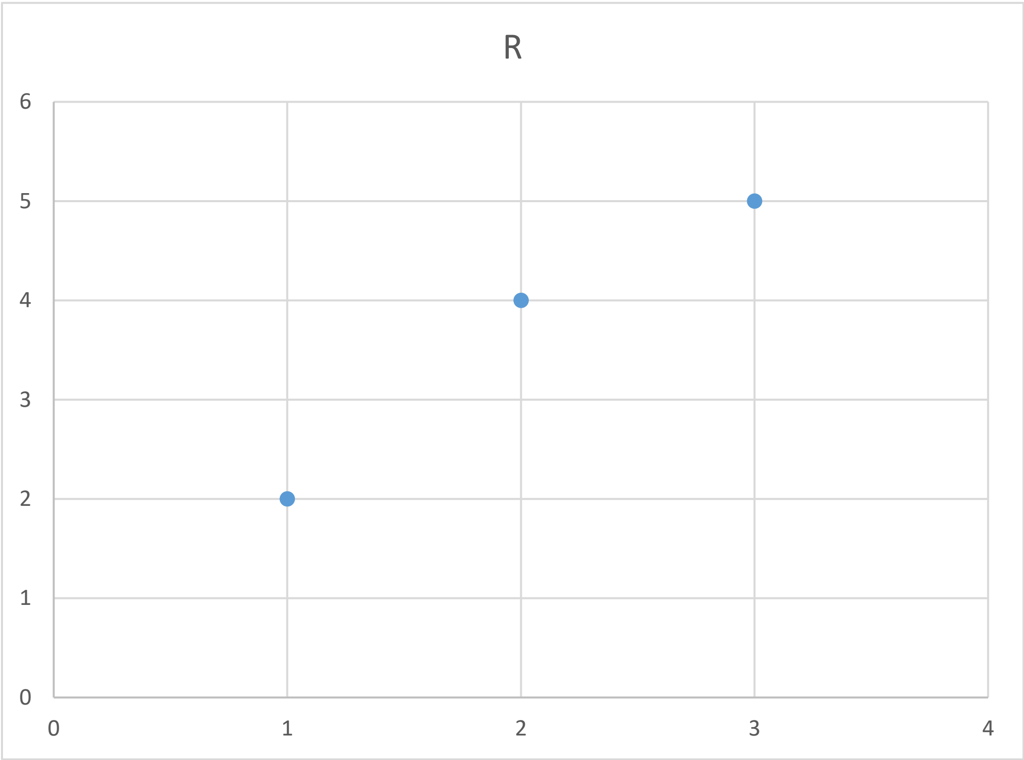
$$2x^2 + 3y^2 = 0$$

The only solution pair of this equation is $(0,0)$, so the equation defines *y* as a function of *x*.

$$|y| = x$$

The domain value 1 is associated with the range values 1 and -1. So the equation doesn't define *y* as a function of *x*.

Relations and Functions from graphs: The domain is the set of *x*-coordinates(left to right), and the range is the set of *y*-coordinates(bottom to top).



Domain?*(x-coordinates)*

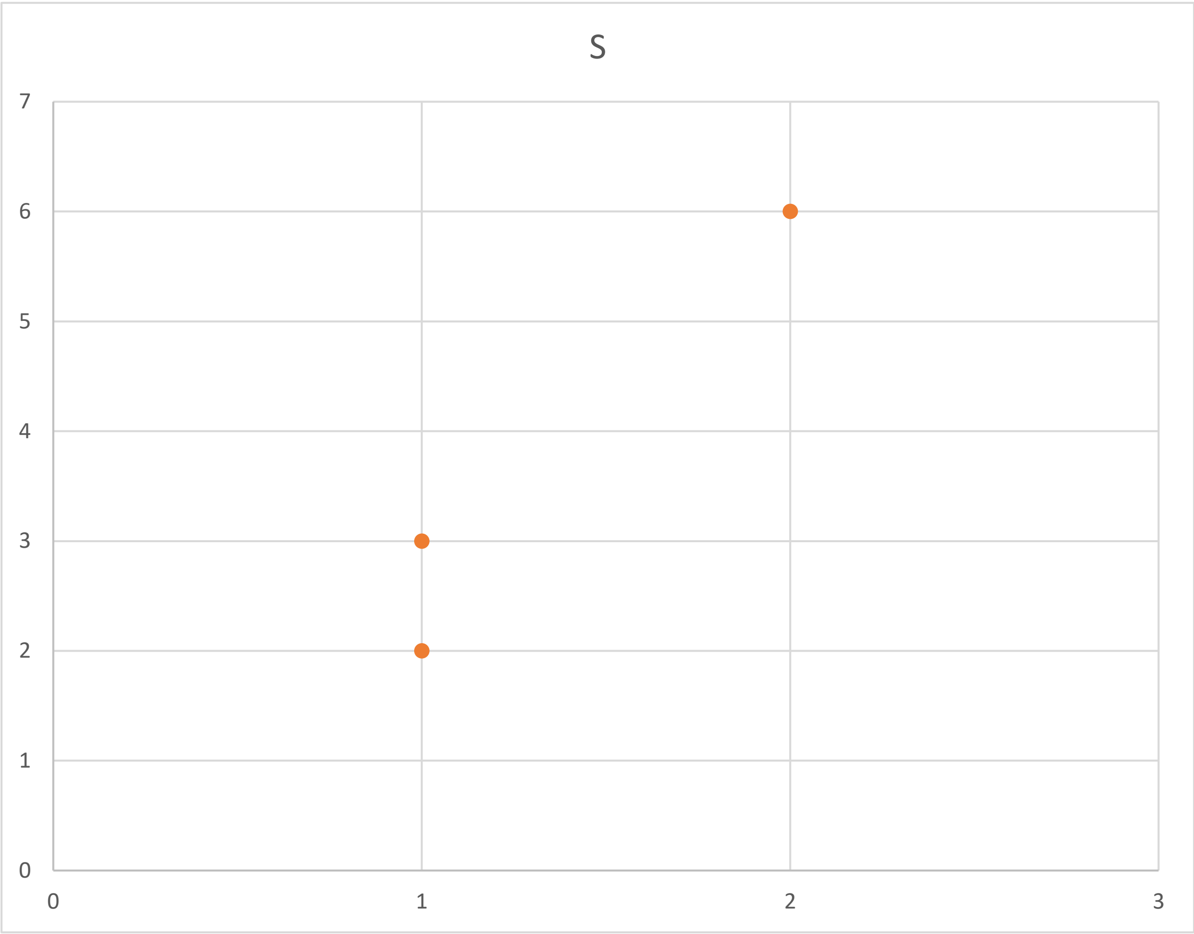
$\{1,2,3\}$

Range? *(y-coordinates)*

$\{2,4,5\}$

Function?*(vertical line test)*

Yes. There is no vertical line that touches or crosses the graph.



Domain?

$\{1,2\}$

Range?

$\{2,3,6\}$

Function?

No. The vertical line $x = 1$ touches the graph in two places.

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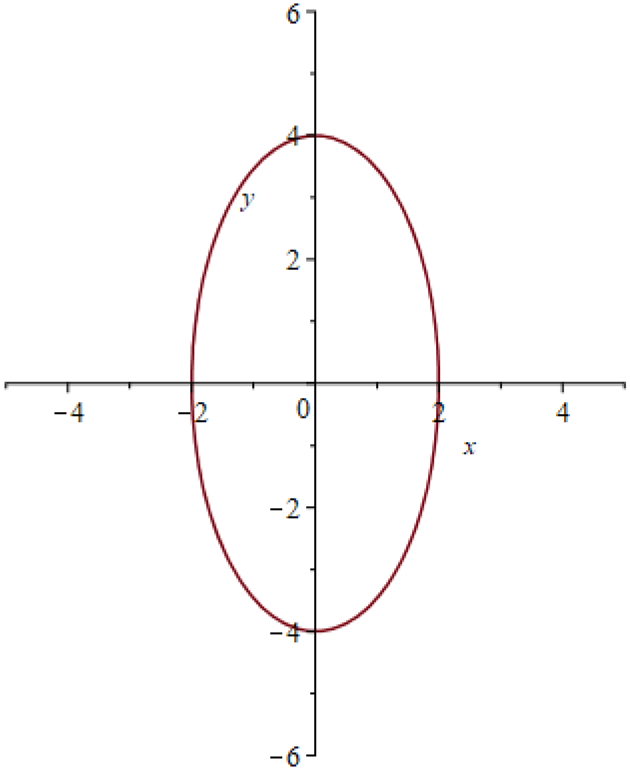


Domain?*(left to right)*

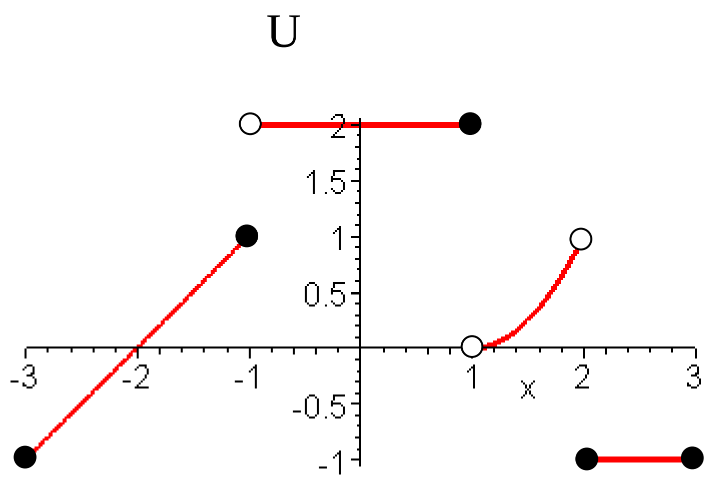
$[-2, 2]$

Range?*(bottom to top)*

$[-4, 4]$

Function?*(vertical line test)*

No. Many vertical lines cross the graph in two places.



Domain?

$[-3,3]$

Range?

$[-1,1]\cup\{2\}$

Function?

Yes. There is no vertical line that touches or crosses the graph in more than one point.

Function Notation and Function Evaluation:

Function evaluation is accomplished by substituting a given domain value into a formula and simplifying.

$$f(x) = x^2 - 1$$

$$f(-1)$$

$$f(-1) = (-1)^2 - 1 = \boxed{0}$$

$$f(0)$$

$$f(0) = (0)^2 - 1 = \boxed{-1}$$

$$f(2)$$

$$f(2) = (2)^2 - 1 = \boxed{3}$$

$$f\left(\tfrac{1}{2}\right)$$

$$f\left(\tfrac{1}{2}\right) = \left(\tfrac{1}{2}\right)^2 - 1 = \boxed{-\tfrac{3}{4}}$$

$$g(x)=\begin{cases} x & ; x \le -1 \\ 2x+1 & ; x > -1 \end{cases}$$

When there is more than one formula, choose the formula associated with the condition that's satisfied by the domain value.

$$g(-2)$$

Which formula?

Is $-2 \le -1$, or is $-2 > -1$?

$$g(-2) = -2$$

$$g(-1)$$

Which formula?

Is $-1 \le -1$, or is $-1 > -1$?

$$g(-1) = \boxed{-1}$$

$$g(1)$$

Which formula?

Is $1 \le -1$, or is $1 > -1$?

$$g(1) = 2(1)+1 = \boxed{3}$$

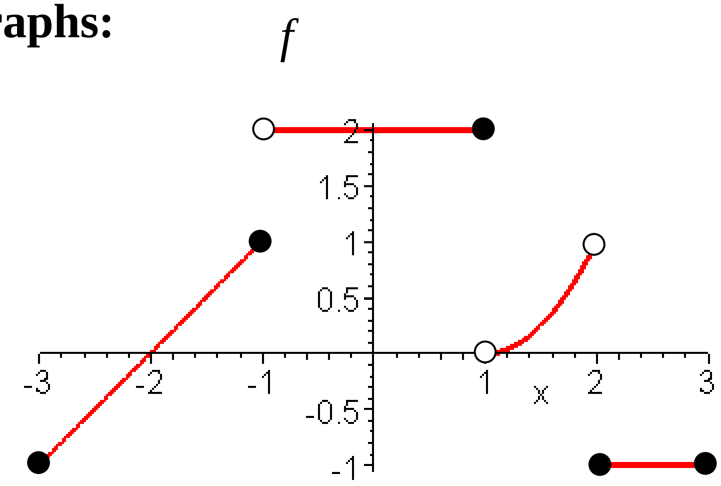
$$g\left(\tfrac{1}{2}\right)$$

Which formula?

Is $\tfrac{1}{2} \le -1$, or is $\tfrac{1}{2} > -1$?

$$g\left(\tfrac{1}{2}\right) = 2\left(\tfrac{1}{2}\right)+1 = \boxed{2}$$

Function values from graphs:



$f(-3)$

What is the y-coordinate of the point on the graph with x-coordinate -3?

$\boxed{-1}$

$f(-2)$

0

$f(-1)$

1

$f(0)$

2

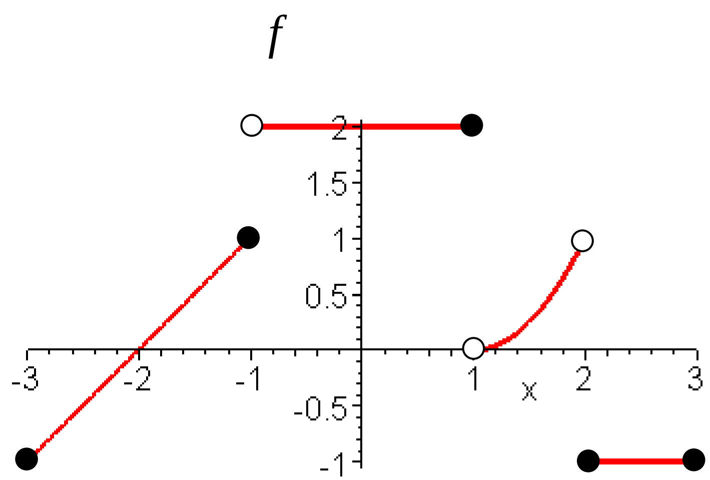
$f(1)$

2

$f\left(\frac{5}{2}\right)$

-1

$f(-4)$ undefined



Solve the equations:

$f(x)=1$

Find the *x*-coordinates of all the points on the graph with *y*-coordinate 1.

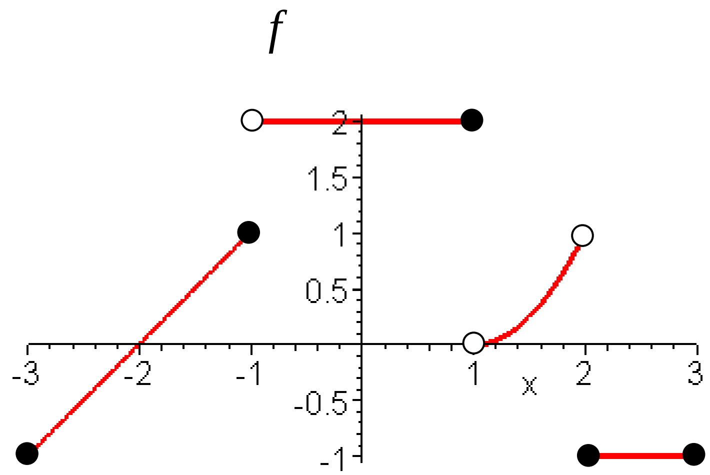
$x = \boxed{-1}$

$f(x)=2$

$(-1,1]$

$f(x)=\frac{3}{2}$

No solution.



Solve the inequalities:

$f(x) > 1$

Find the x-coordinates of all the points on the graph with y-coordinates larger than 1.

$\boxed{(-1, 1]}$

$0 \le f(x) < 1$

$[-2, -1) \cup (1, 2)$

$f(x) \le -1$

$\{-3\} \cup [2, 3]$

Domains from function formulas:***(Avoid division by zero and even roots of negative numbers.)***

$$f(x)=\frac{1}{x^2+3x+2}$$

$x^2+3x+2=0$

$(x+1)(x+2)=0 \Rightarrow x=-1,-2$

Domain is $(-\infty,-2)\cup(-2,-1)\cup(-1,\infty)$

$$g(x)=\sqrt{1-2x}$$

$1-2x\geq 0$

$-2x\geq -1$

$x\leq \frac{1}{2}$

Domain is $\left(-\infty,\frac{1}{2}\right]$

$$h(x)=\frac{\sqrt{x-2}}{x-3}$$

$x-2\geq 0,\ x\neq 3$

$x\geq 2,\ x\neq 3$

Domain is $[2,3)\cup(3,\infty)$

Combinations of functions:

$$f+g,\ f-g,\ fg,\ \frac{f}{g},\ f\circ g$$

$$(f+g)(x)=f(x)+g(x)$$

$$(f-g)(x)=f(x)-g(x)$$

$$(fg)(x)=f(x)g(x)$$

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$$

$$(f\circ g)(x)=f(g(x))$$

$$f(x) = x^2 - 2, \quad g(x) = \sqrt{x+1}$$

$$(f+g)(0)$$
$$f(0)+g(0)$$
$$-2+1=\boxed{-1}$$

$$(f-g)(3)$$
$$f(3)-g(3)$$
$$7-2=\boxed{5}$$

$$(fg)(3)$$
$$f(3)\cdot g(3)$$
$$7\cdot 2=\boxed{14}$$

$$\left(\frac{f}{g}\right)(3)$$
$$\frac{f(3)}{g(3)}=\boxed{\frac{7}{2}}$$

$$(f\circ g)(3)$$
$$f(g(3))$$
$$f(2)=\boxed{2}$$

$$(f+g)(-1)$$
$$f(-1)+g(-1)$$
$$-1+0=\boxed{-1}$$

$$(f-g)(-2)$$
$$f(-2)-g(-2)$$
$$2-\textit{undefined}=\boxed{\textit{undefined}}$$

$$\left(\frac{f}{g}\right)(-1)$$
$$\frac{f(-1)}{g(-1)}=\frac{-1}{0}=\boxed{\textit{undefined}}$$

$$\left(\frac{f}{g}\right)(-2)$$
$$\frac{f(-2)}{g(-2)}=\frac{2}{\textit{undefined}}=\boxed{\textit{undefined}}$$

$$(g\circ f)(3)$$
$$g(f(3))$$
$$g(7)=\boxed{\sqrt{8}}$$

$$(g\circ f)(0)$$
$$g(f(0))$$
$$g(-2)=\boxed{\text{undefined}}$$

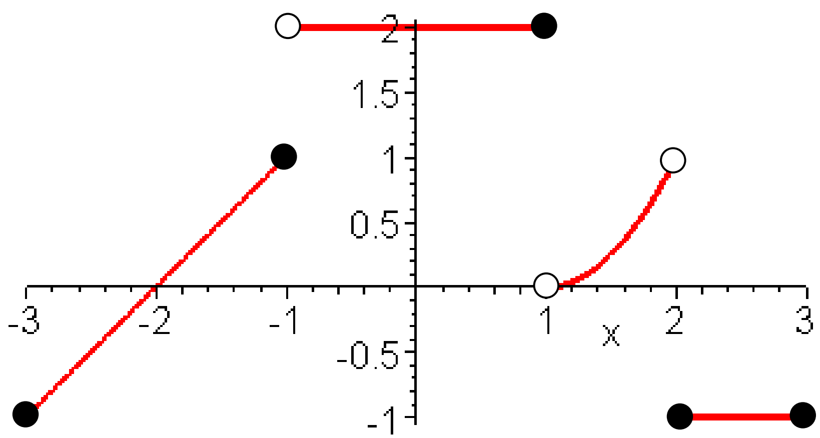
$$(f\circ f)(3)$$
$$f(f(3))$$
$$f(7)=\boxed{47}$$

$$(g\circ g)(3)$$
$$g(g(3))$$
$$g(2)=\boxed{\sqrt{3}}$$

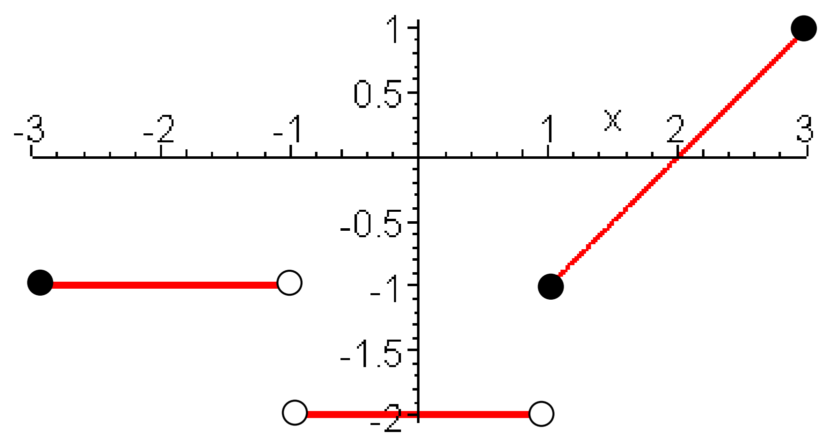
$$(f\circ g\circ f)(\sqrt{5})$$
$$f\left(g\left(f\left(\sqrt{5}\right)\right)\right)$$
$$f(g(3))=f(2)=\boxed{2}$$

Combinations from graphs:

Graph of *f*



Graph of *g*



$(f+g)(0)$

$2+-2=\boxed{0}$

$\left(\dfrac{f}{g}\right)(1)$

$\dfrac{2}{-1}=\boxed{-2}$

$(fg)(-3)$

$(-1)(-1)=\boxed{1}$

$(f\circ g)(2)$

$f(0)=\boxed{2}$

$(g\circ f)(2)$

$g(-1)=\boxed{undefined}$

$(g\circ g)(0)$

$g(-2)$

$\boxed{-1}$

$(f\circ f)(-1)$

$f(1)$

$\boxed{2}$

$(f\circ g\circ f)(1)$

$f(g(2))$

$f(0)=\boxed{2}$