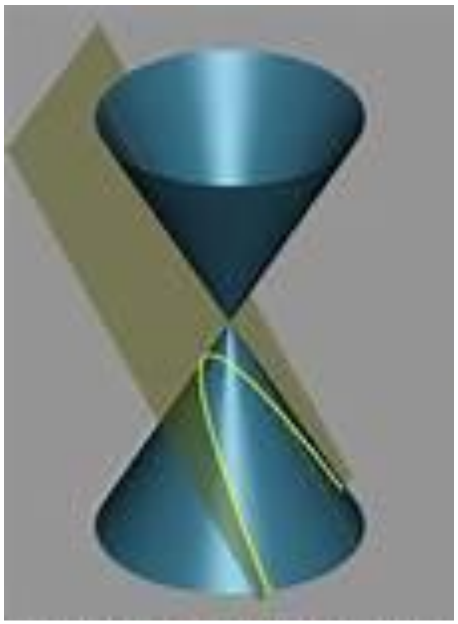
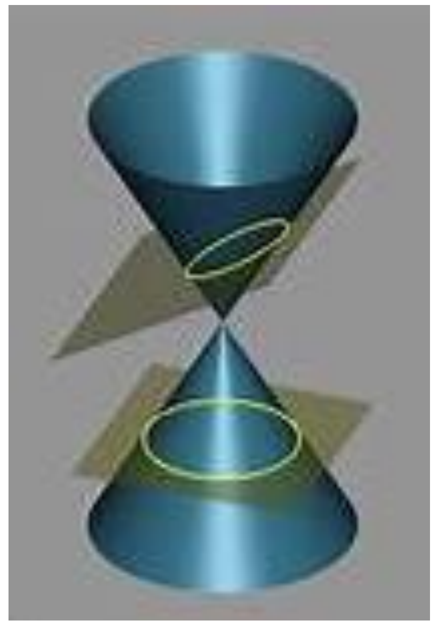
Conic Sections:

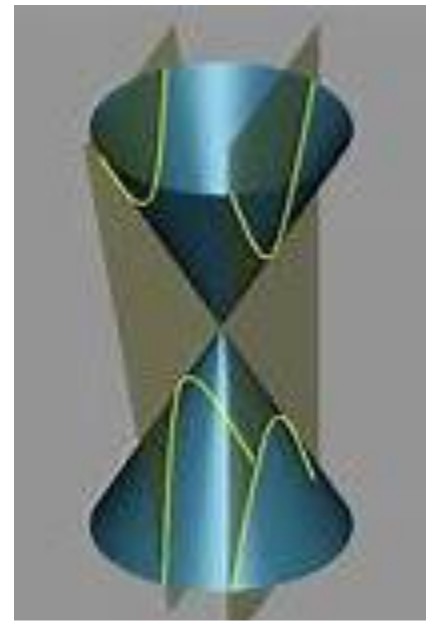
Curves formed by the intersection of a plane with a double cone.



Parabola



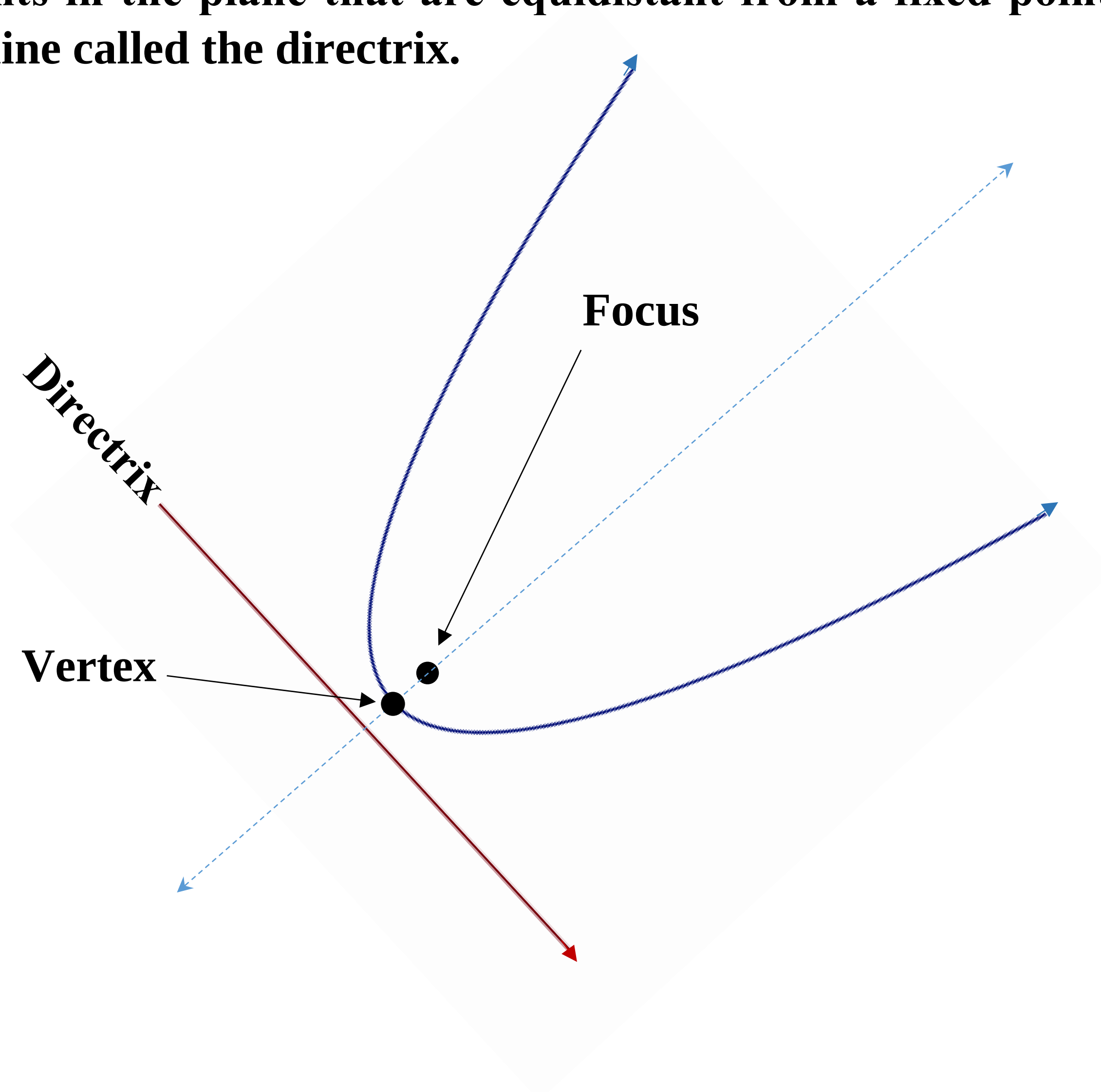
Ellipse or Hypobola



Hyperbola

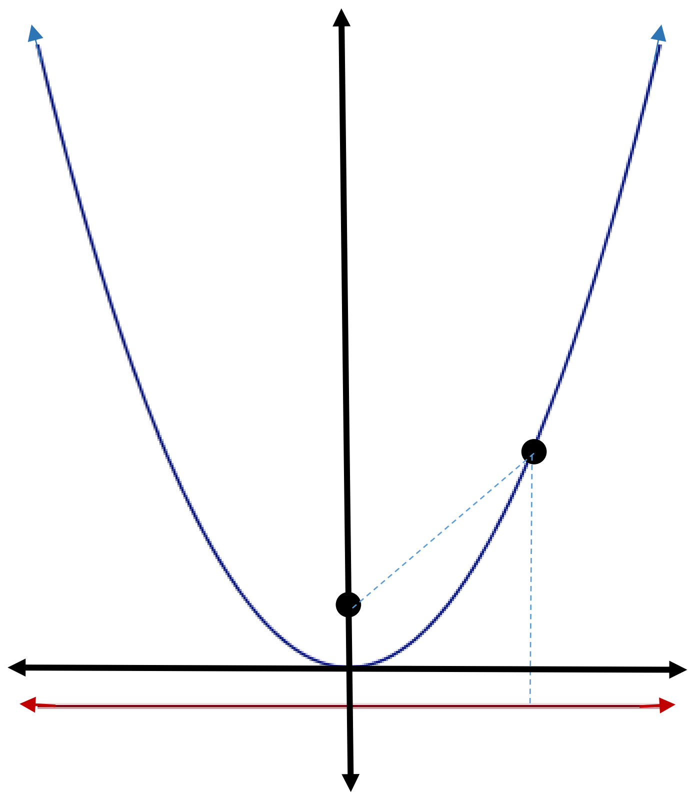
Parabola:

It's the set of points in the plane that are equidistant from a fixed point called the focus and a fixed line called the directrix.



Vertical Parabolas with Vertex at the Origin:

The focus will be at $(0,a)$ **with** $a>0$**, and the directrix will be** $y=-a$**.**



$$\sqrt{(x-0)^2+(y-a)^2}=y+a$$

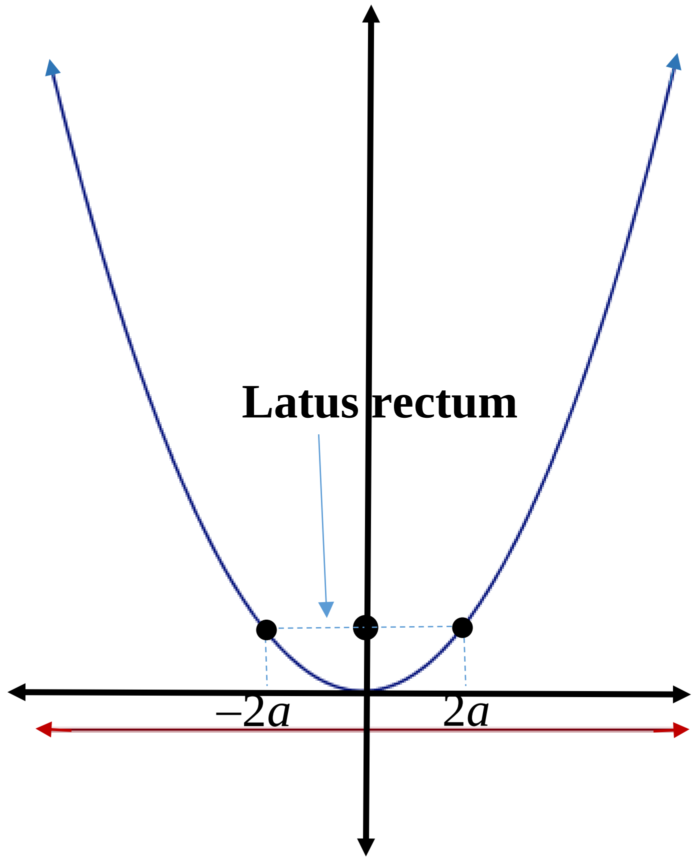
$$x^2+(y-a)^2=(y+a)^2$$

$$x^2+y^2-2ay+a^2=y^2+2ay+a^2$$

$$x^2-2ay=2ay$$

$$x^2=4ay$$

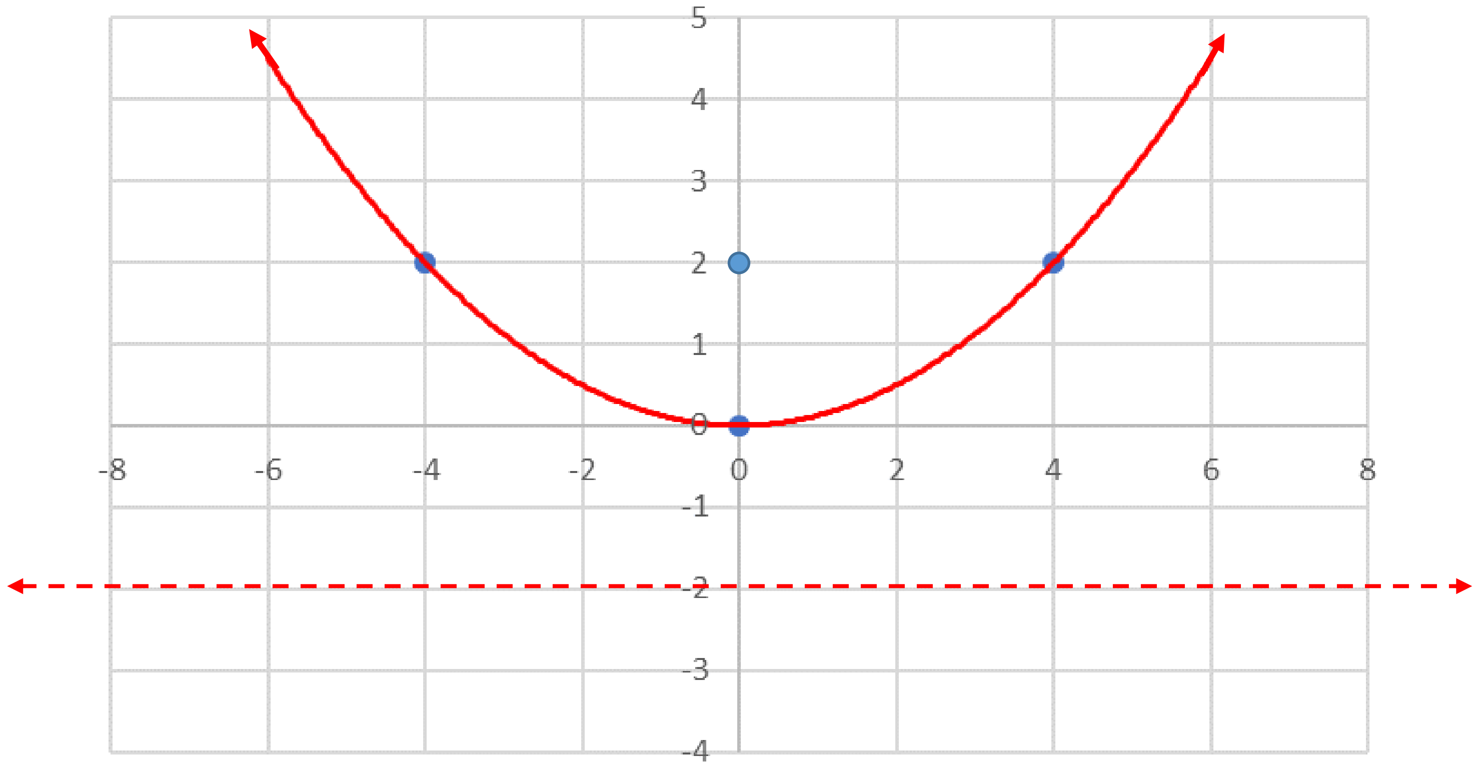
The Latus Rectum is the segment through the focus, parallel to the directrix, with endpoints on the parabola. The endpoints are always 2*a* units from the focus.



When graphing parabolas, you should indicate the vertex, focus, endpoints of the latus rectum, and the directrix.

Sketch the graph of the parabola with the equation $x^2 = 8y$**.** **up parabola**

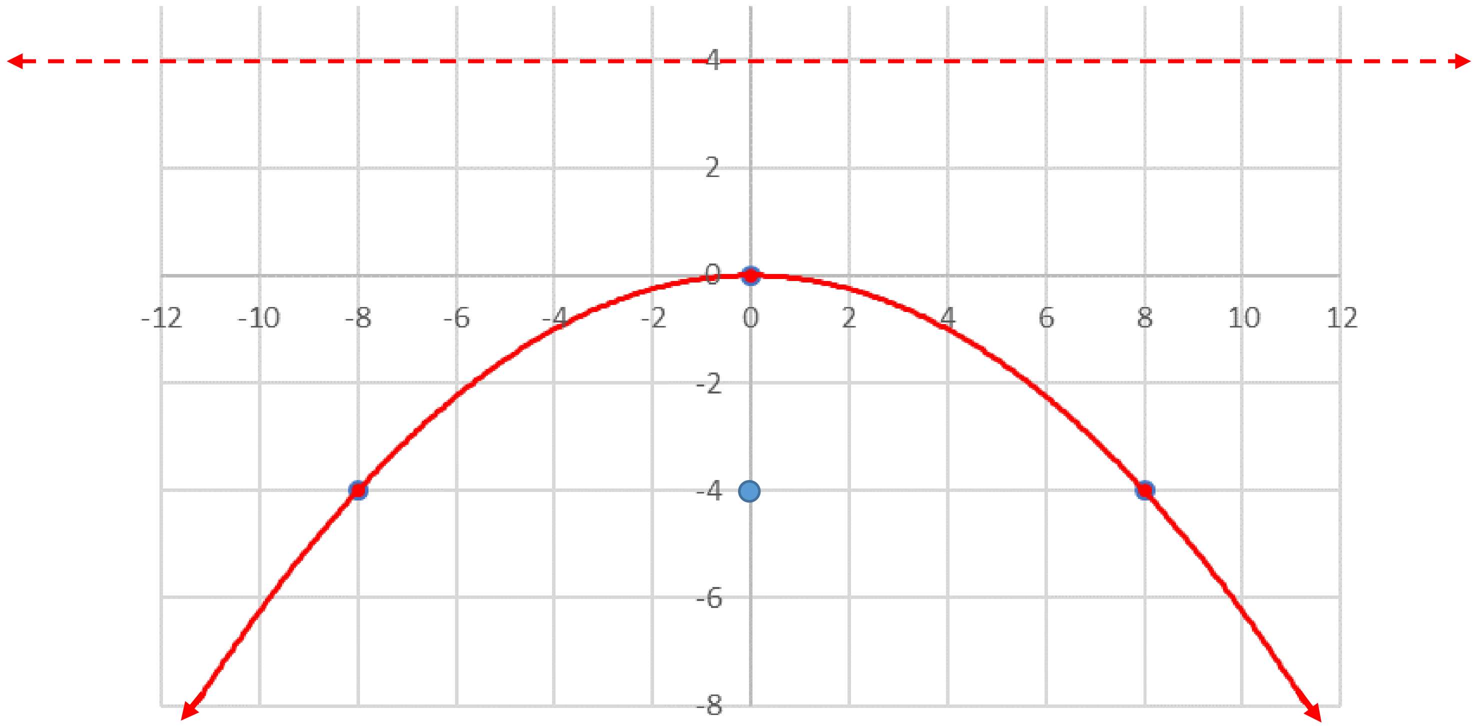
$x^2 = 4(2)y \Rightarrow a = 2$, **the vertex is at the origin, the focus is at** $(0,2)$**, the directrix is** $y = -2$**, and the endpoints of the latus rectum are** $(4,2)$ **and** $(-4,2)$**.**



For a vertical parabola with vertex at the origin that opens down, the equation can be written as $x^2 = -4ay$ **with** $a > 0$**, and the focus at** $(0,-a)$ **and the directrix** $y = a$**.**

Sketch the parabola with the equation $x^2 = -16y$. **down parabola**

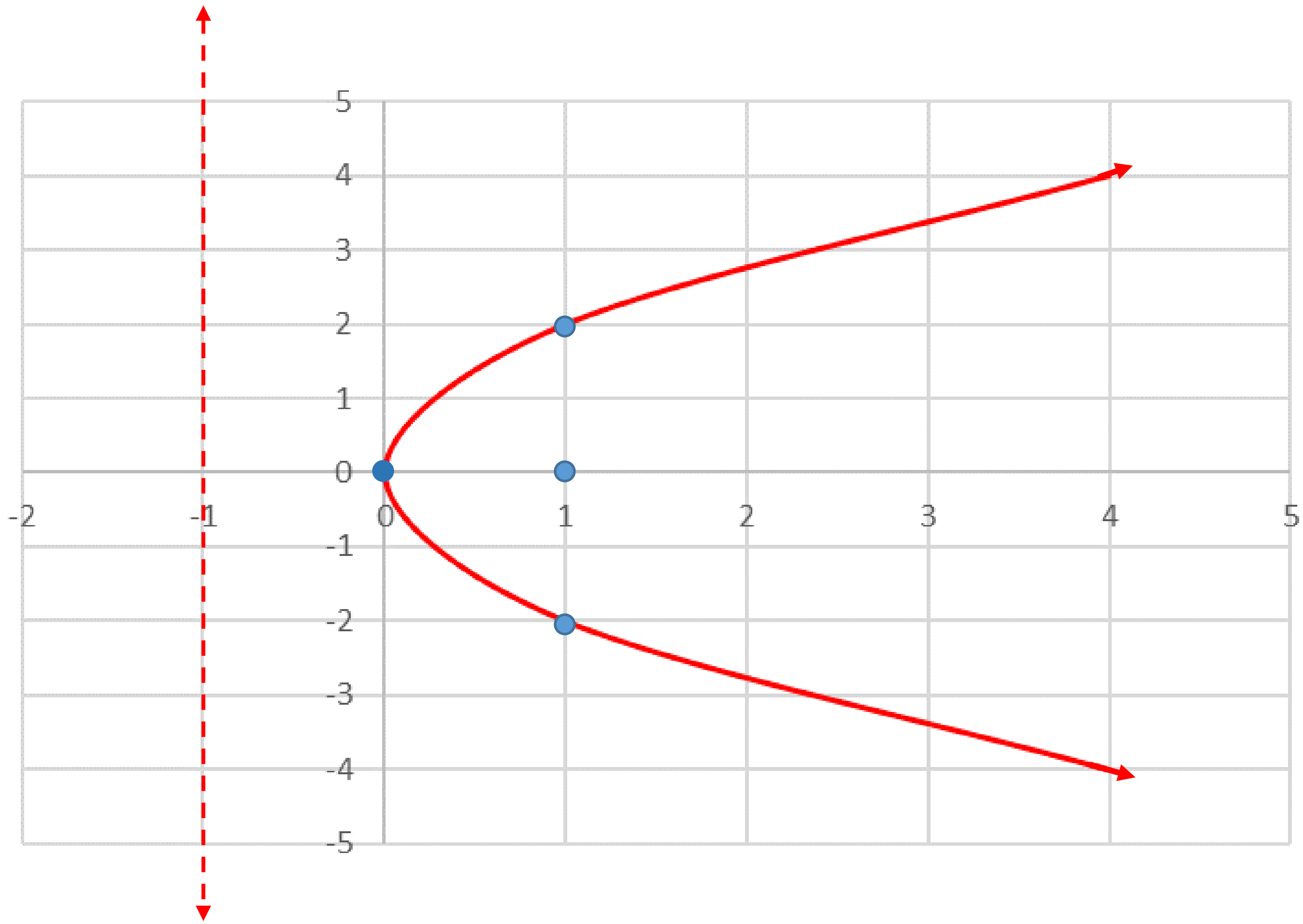
$x^2 = -4(4)y \Rightarrow a = 4$, **the vertex is at the origin, the focus is at** $(0,-4)$, **the directrix is** $y = 4$, **and the endpoints of the latus rectum are** $(8,-4)$ **and** $(-8,-4)$.



Similarly, for horizontal parabolas, the equations are(right parabola) $y^2 = 4ax$**, with focus at** $(a,0)$ **and directrix** $x = -a$**, and (left parabola)** $y^2 = -4ax$**, with focus at** $(-a,0)$ **and directrix** $x = a$.

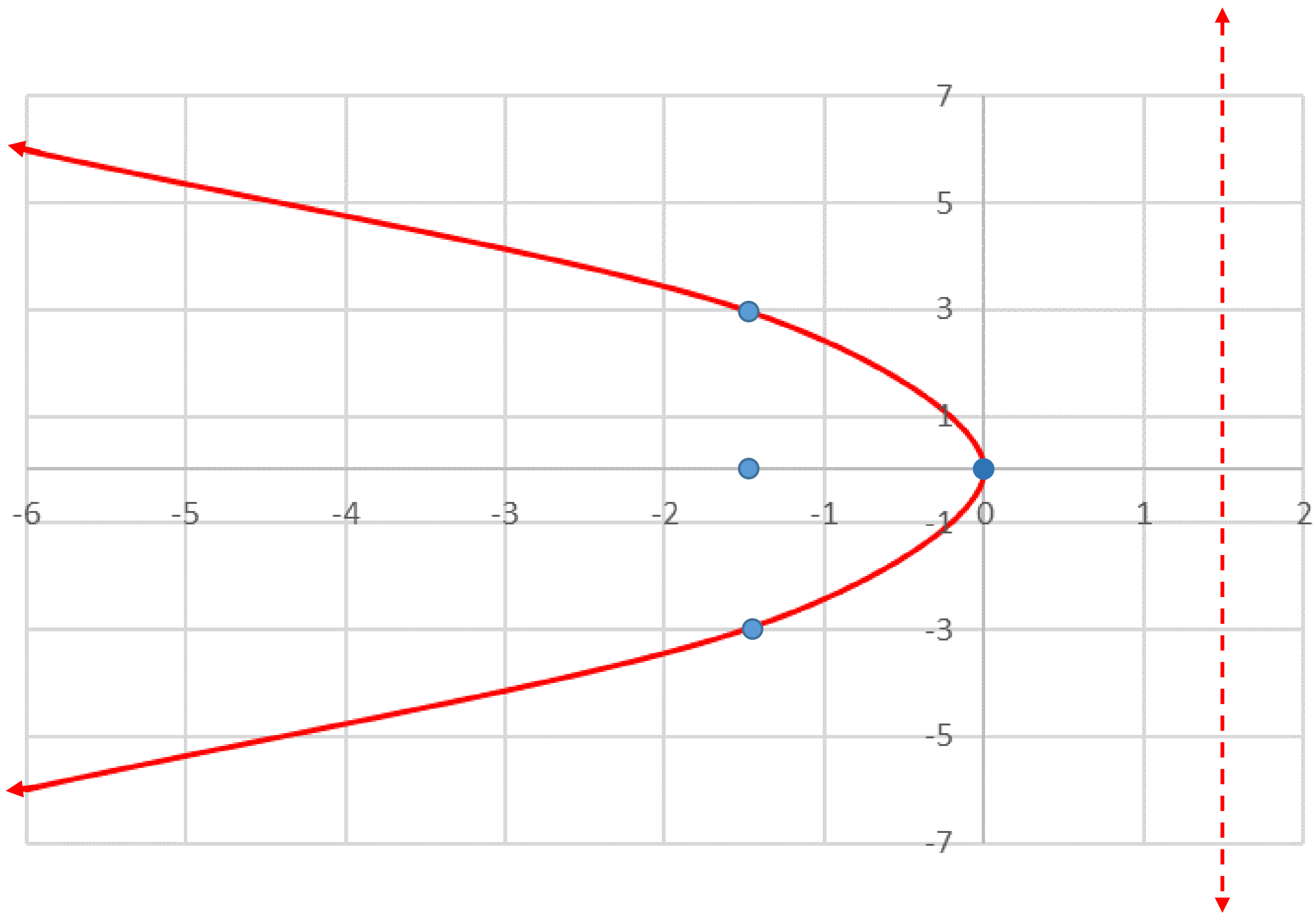
Sketch the graph of the parabola with the equation $y^2 = 4x$. right parabola

$y^2 = 4(1)x \Rightarrow a = 1$, the vertex is at the origin, the focus is at $(1,0)$, the directrix is $x = -1$, and the endpoints of the latus rectum are $(1,2)$ and $(1,-2)$.



Sketch the graph of the parabola with the equation $y^2 = -6x$**.** **left parabola**

$y^2 = -4\left(\frac{3}{2}\right)x \Rightarrow a = \frac{3}{2}$**, the vertex is at the origin, the focus is at** $\left(-\frac{3}{2}, 0\right)$**, the directrix is** $x = \frac{3}{2}$**, and the endpoints of the latus rectum are** $\left(-\frac{3}{2}, 3\right)$ **and** $\left(-\frac{3}{2}, -3\right)$**.**



For parabolas with vertices at (h,k), the equations are

$$(x-h)^2 = 4a(y-k)$$

$$(x-h)^2 = -4a(y-k)$$

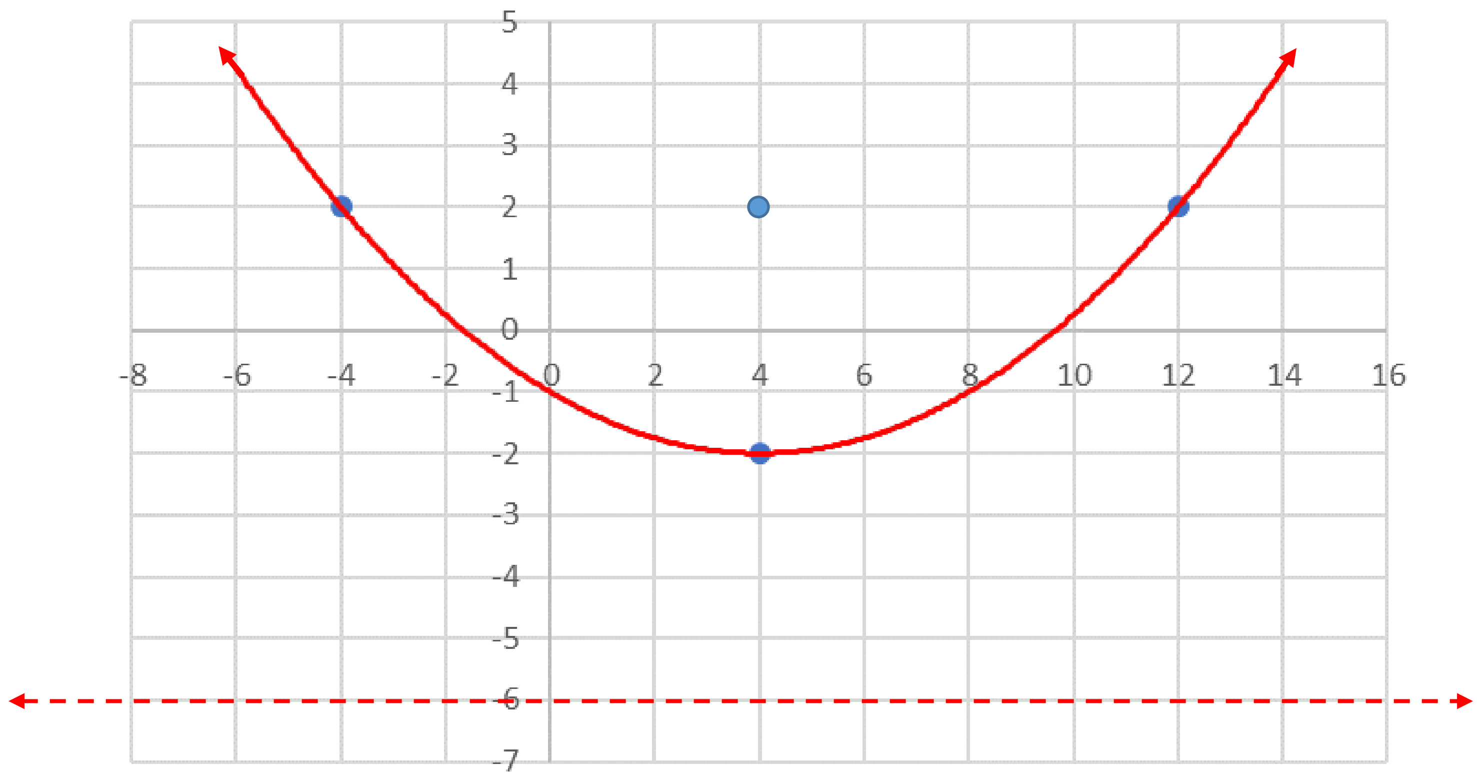
$$(y-k)^2 = 4a(x-h)$$

$$(y-k)^2 = -4a(x-h)$$

Sketch the following parabolas:

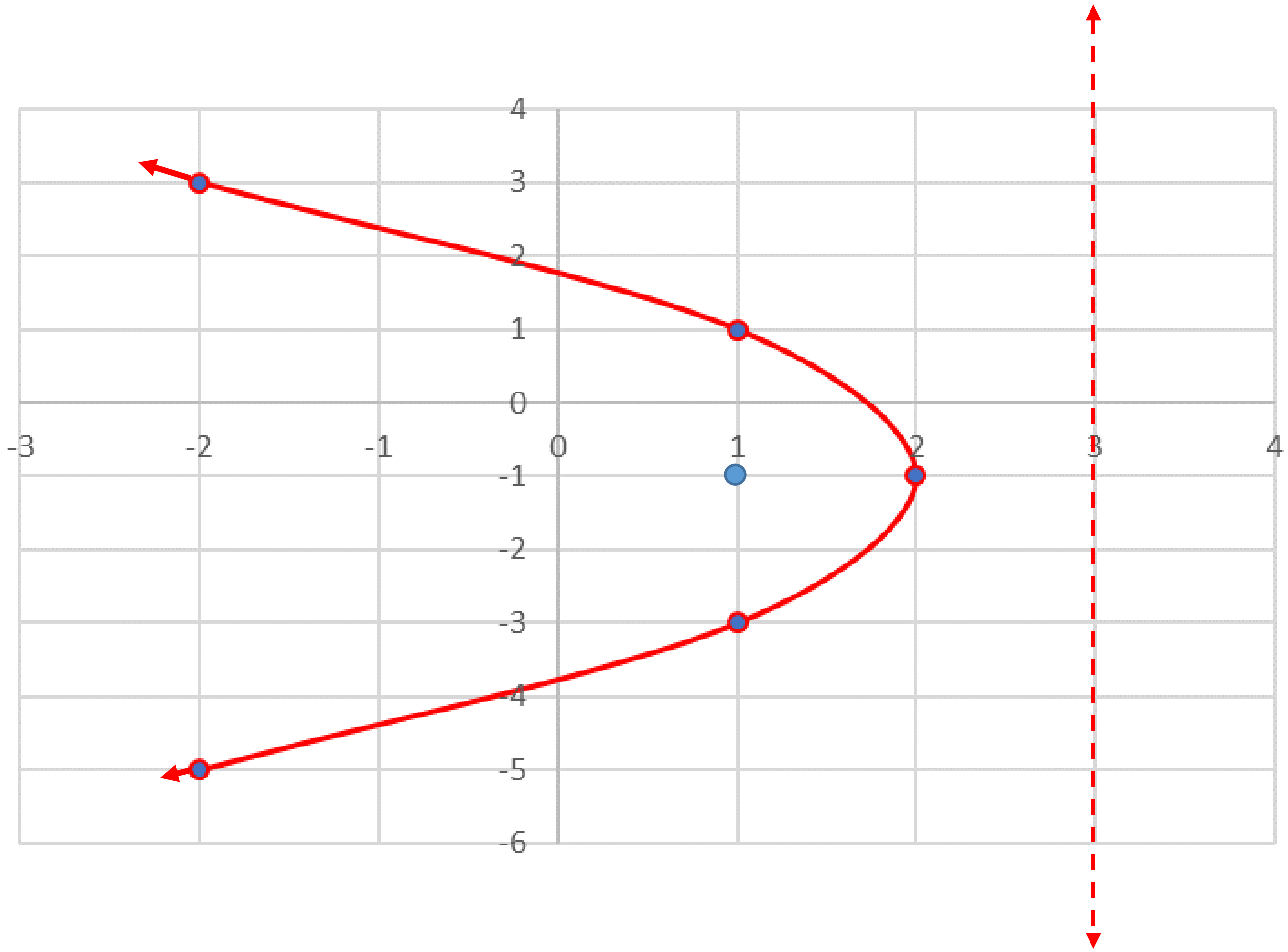
1. $(x-4)^2 = 16(y+2)$ **up parabola**

$(x-4)^2 = 4(4)(y+2) \Rightarrow a = 4$, **the vertex is at** $(4,-2)$, **the focus is at** $(4,2)$, **the directrix is** $y = -6$, **and the endpoints of the latus rectum are** $(-4,2)$ **and** $(12,2)$.



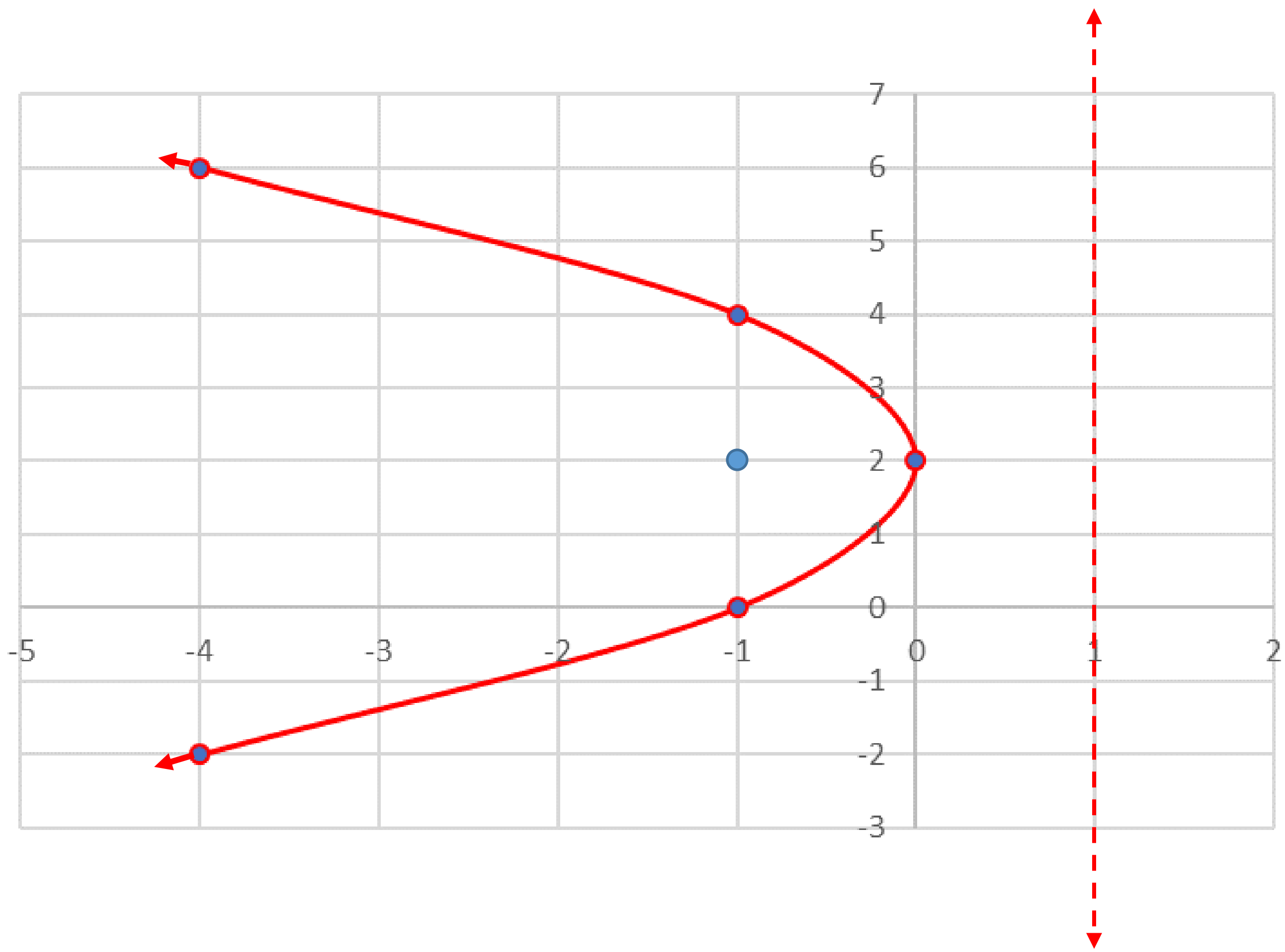
2. $(y+1)^2 = -4(x-2)$ **left parabola**

$(y+1)^2 = -4(1)(x-2) \Rightarrow a = 1$**, the vertex is at** $(2,-1)$**, the focus is at** $(1,-1)$**, the directrix is** $x = 3$**, and the endpoints of the latus rectum are** $(1,1)$ **and** $(1,-3)$**.**



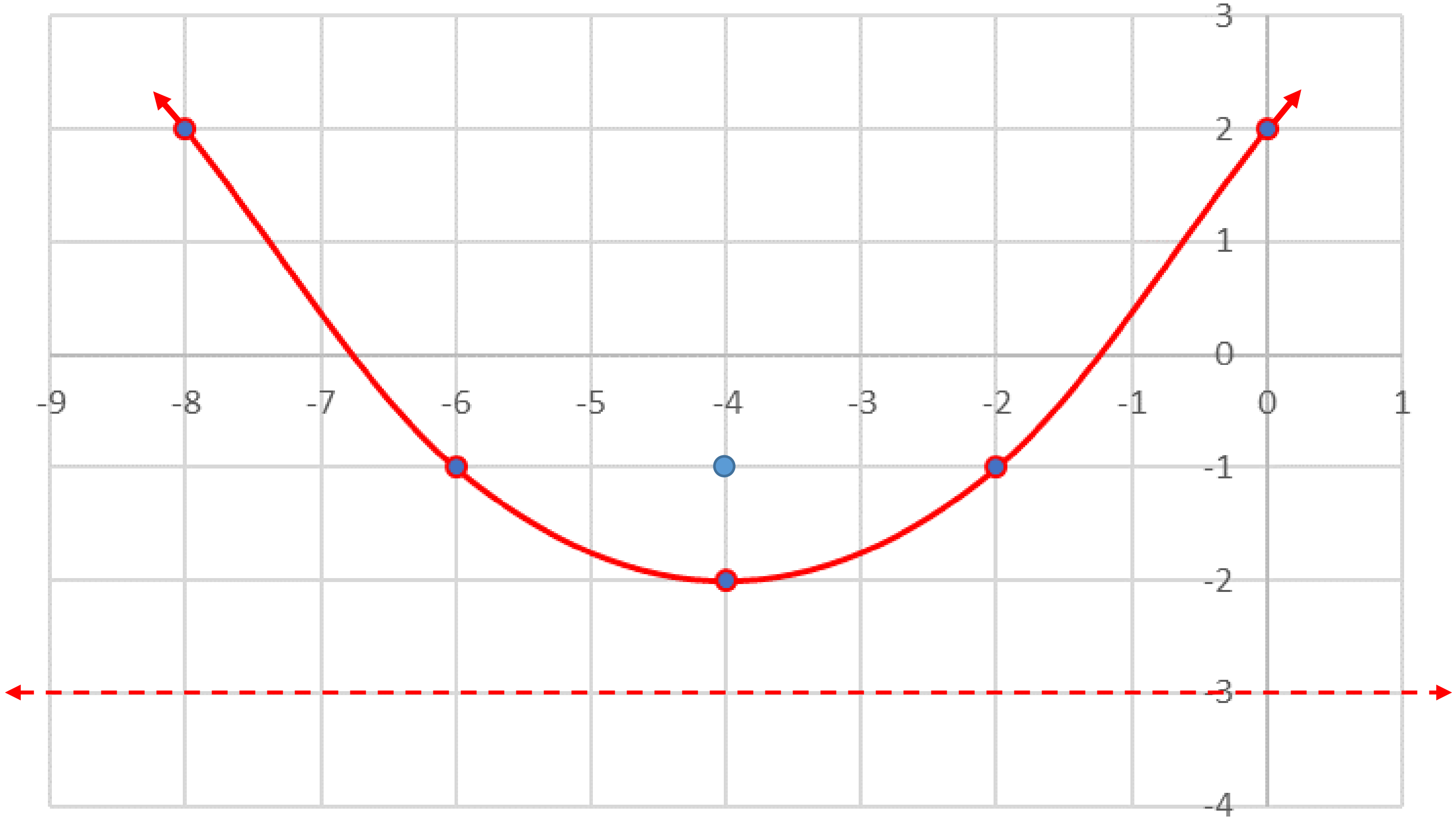
3. $y^2 - 4y + 4x + 4 = 0$

Complete the square in *y*. $y^2 - 4y + 4 = -4x \Rightarrow (y-2)^2 = -4(1)x \Rightarrow a = 1$**, it's a left parabola, the vertex is at** $(0,2)$**, the focus is at** $(-1,2)$**, the directrix is** $x = 1$**, and the endpoints of the latus rectum are** $(-1,0)$ **and** $(-1,4)$**.**



4. $x^2 + 8x = 4y - 8$

Complete the square in *x*. $x^2 + 8x + 16 = 4y + 8 \Rightarrow (x+4)^2 = 4(1)(y+2) \Rightarrow a = 1$**, it's an up parabola, the vertex is at** $(-4,-2)$**, the focus is at** $(-4,-1)$**, the directrix is** $y = -3$**, and the endpoints of the latus rectum are** $(-2,-1)$ **and** $(-6,-1)$**.**



Find equations for the following parabolas:

1. Focus: $(0,2)$ **and Vertex:** $(0,0)$

It must be an up parabola with equation $x^2 = 4ay$**. From the focus information, we know that** $a = 2$**, so the equation is** $\boxed{x^2 = 8y}$**.**

2. Focus: $(-4,0)$ **and Vertex:** $(0,0)$

It must be a left parabola with equation $y^2 = -4ax$**. From the focus information, we know that** $a = 4$**, so the equation is** $\boxed{y^2 = -16x}$**.**

3. Focus: $(0,-1)$ **and Directrix:** $y=1$

From the focus and directrix, it must be a down parabola with vertex at the origin with equation $x^2=-4ay$**. From either the focus or directrix we know that** $a=1$**, so the equation is** $\boxed{x^2=-4y}$**.**

4. Focus: $(6,-2)$ **and Vertex:** $(4,-2)$

It must be a right parabola with equation $(y+2)^2=4a(x-4)$**. From the focus and vertex, we know that** $a=2$**, so the equation is** $\boxed{(y+2)^2=8(x-4)}$**.**

5. Focus: $(-4,4)$ **and Directrix:** $y=-2$

It must be an up parabola with equation $(x-h)^2=4a(y-k)$**. From the focus and directrix, we know that the vertex is at** $(-4,2)$ **and** $a=2$**, so the equation is** $\boxed{(x+4)^2=8(y-2)}$**.**