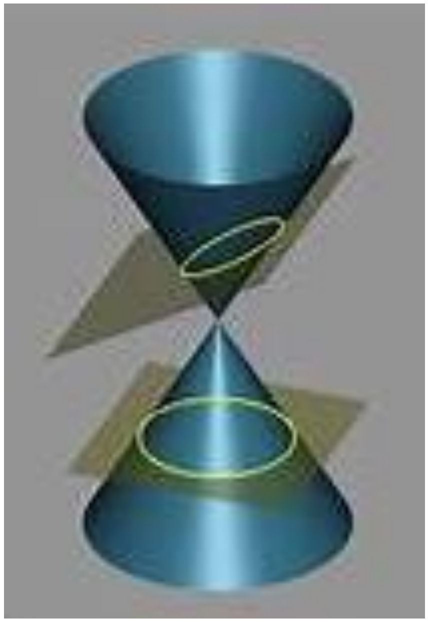
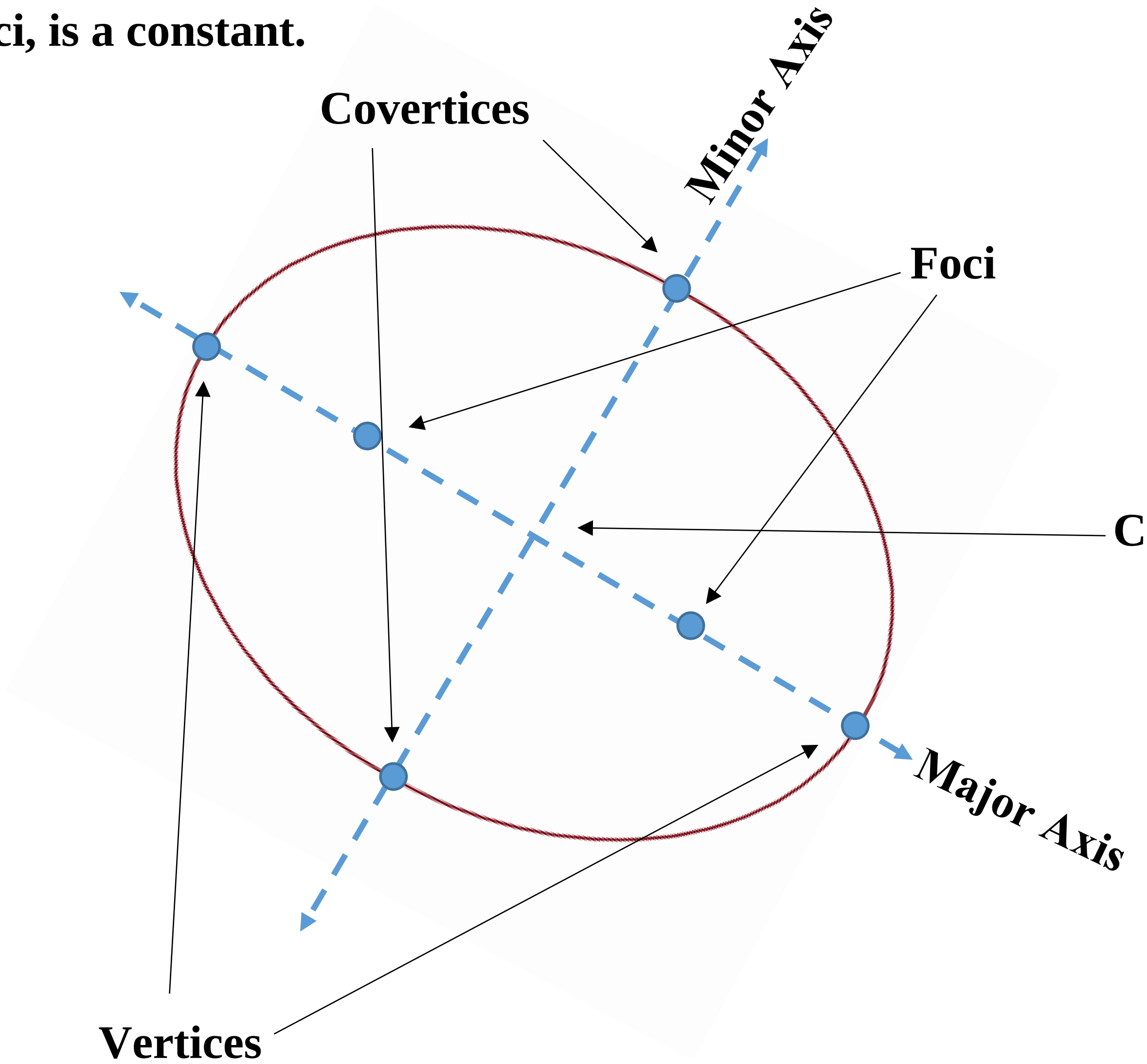
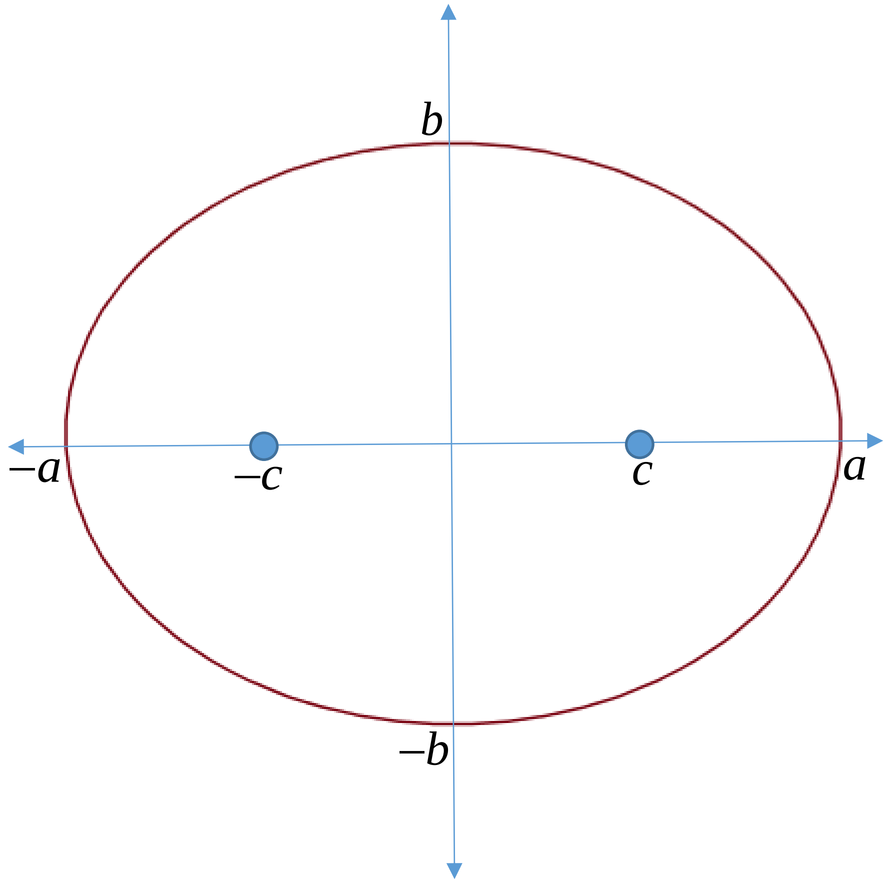
<u>Ellipse:</u> **The set of points in the plane the sum of whose distances to two fixed points, called foci, is a constant.**

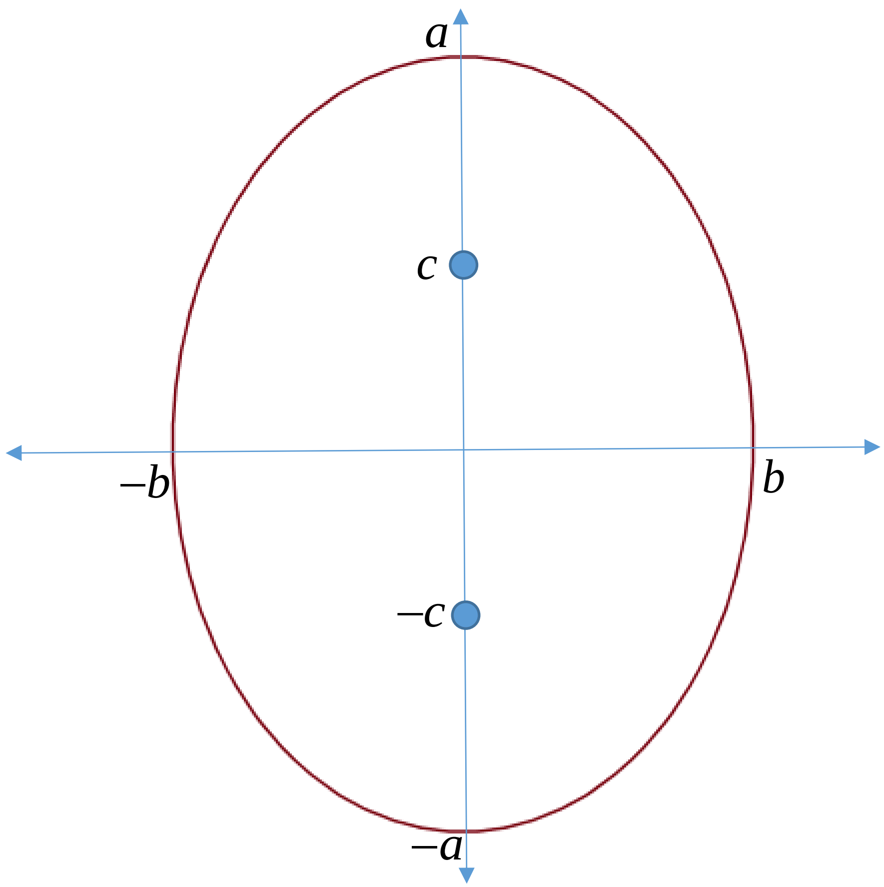




Ellipses centered at the origin with major and minor axes of the *x* and *y* axes.



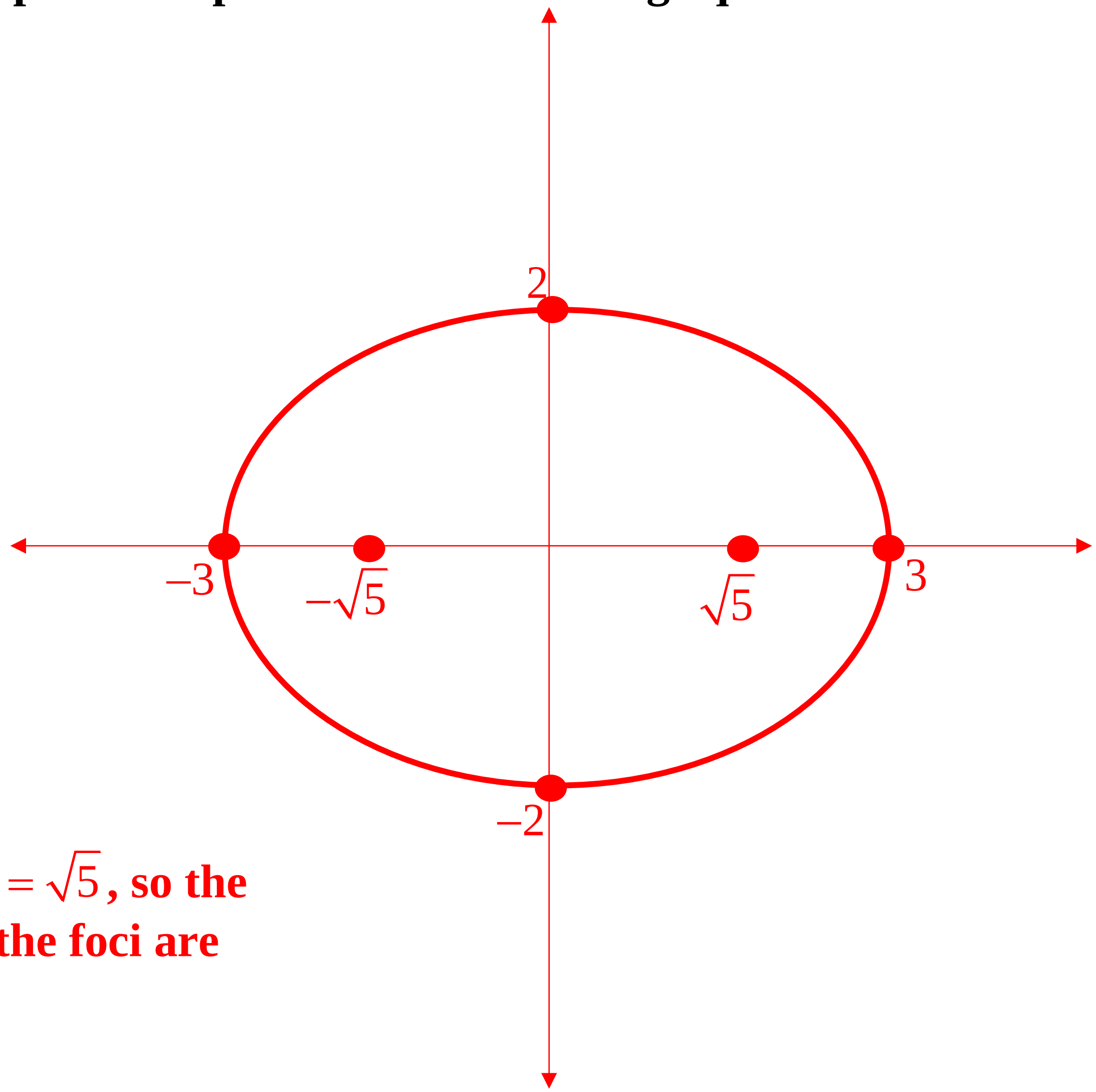
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ c^2=a^2-b^2, a\geq b$$



$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1,\ c^2=a^2-b^2, a\geq b$$

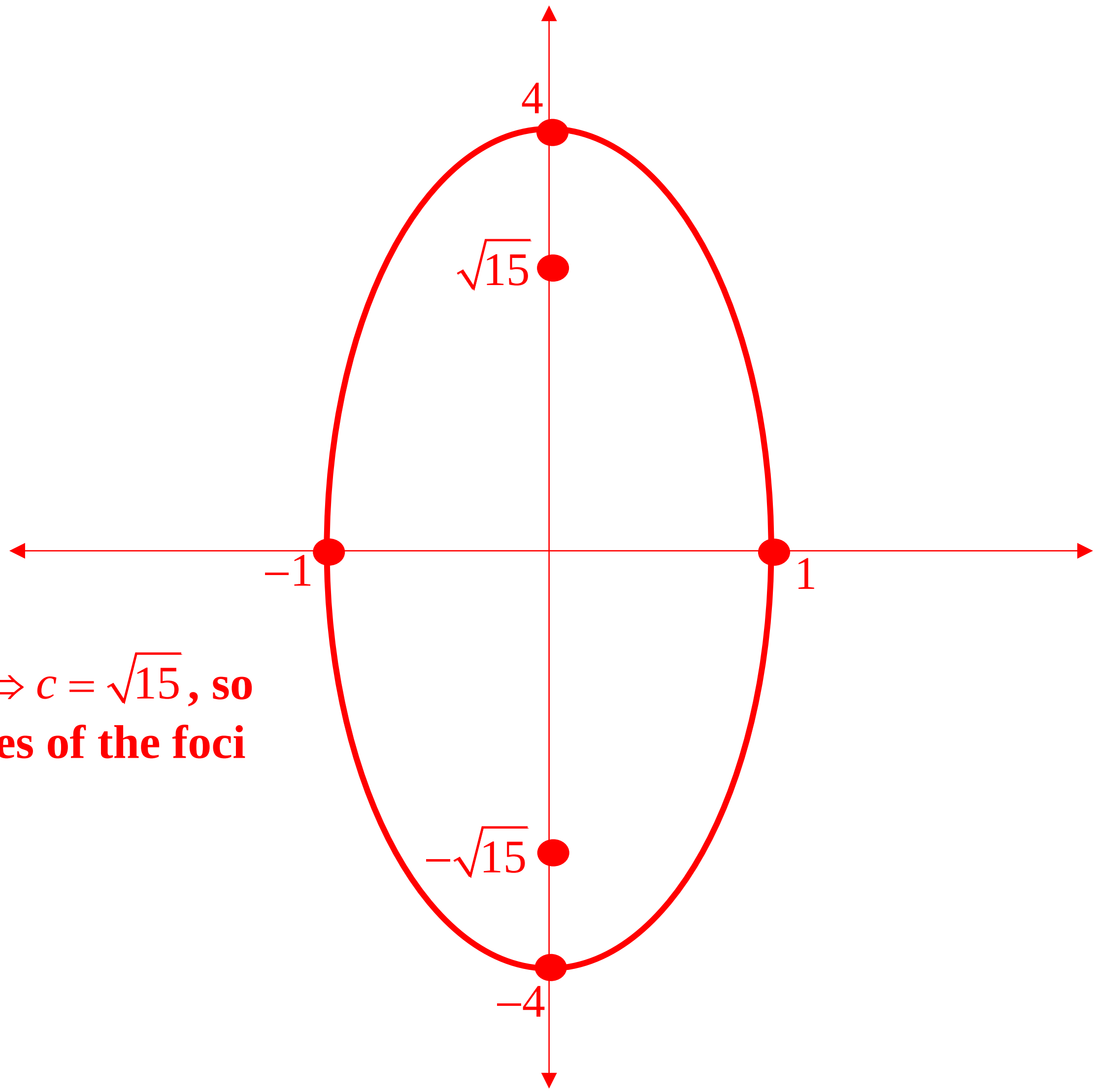
<u>Examples:</u> Graph the ellipses with the following equations.

1. $\frac{x^2}{9}+\frac{y^2}{4}=1$



$c^2=9-4=5 \Rightarrow c=\sqrt{5}$**, so the *x*-coordinates of the foci are** $\pm\sqrt{5}$**.**

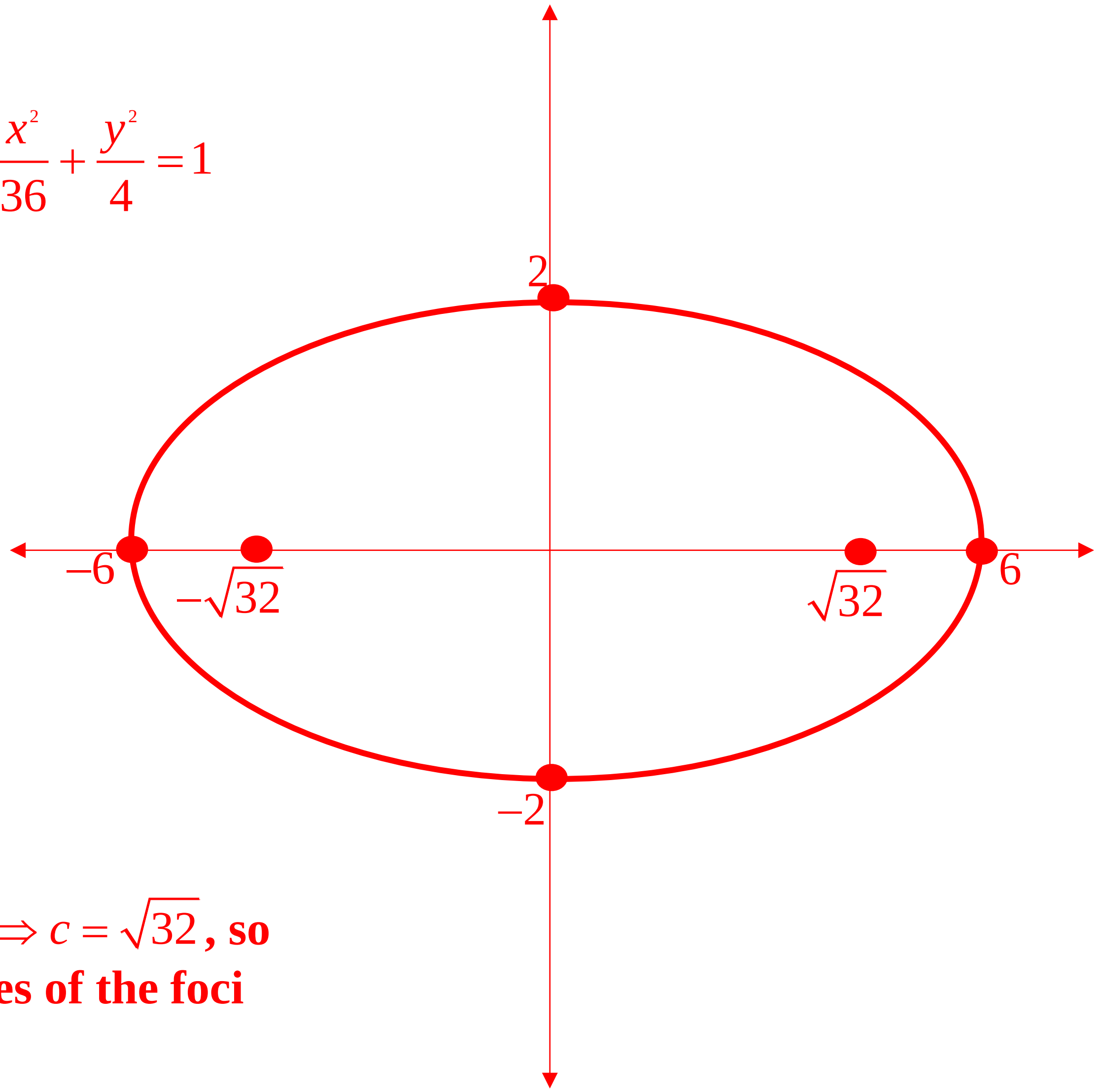
2. $x^2 + \frac{y^2}{16} = 1$



$c^2 = 16 - 1 = 15 \Rightarrow c = \sqrt{15}$**, so the *y*-coordinates of the foci are** $\pm\sqrt{15}$**.**

3. $x^2 + 9y^2 = 36$

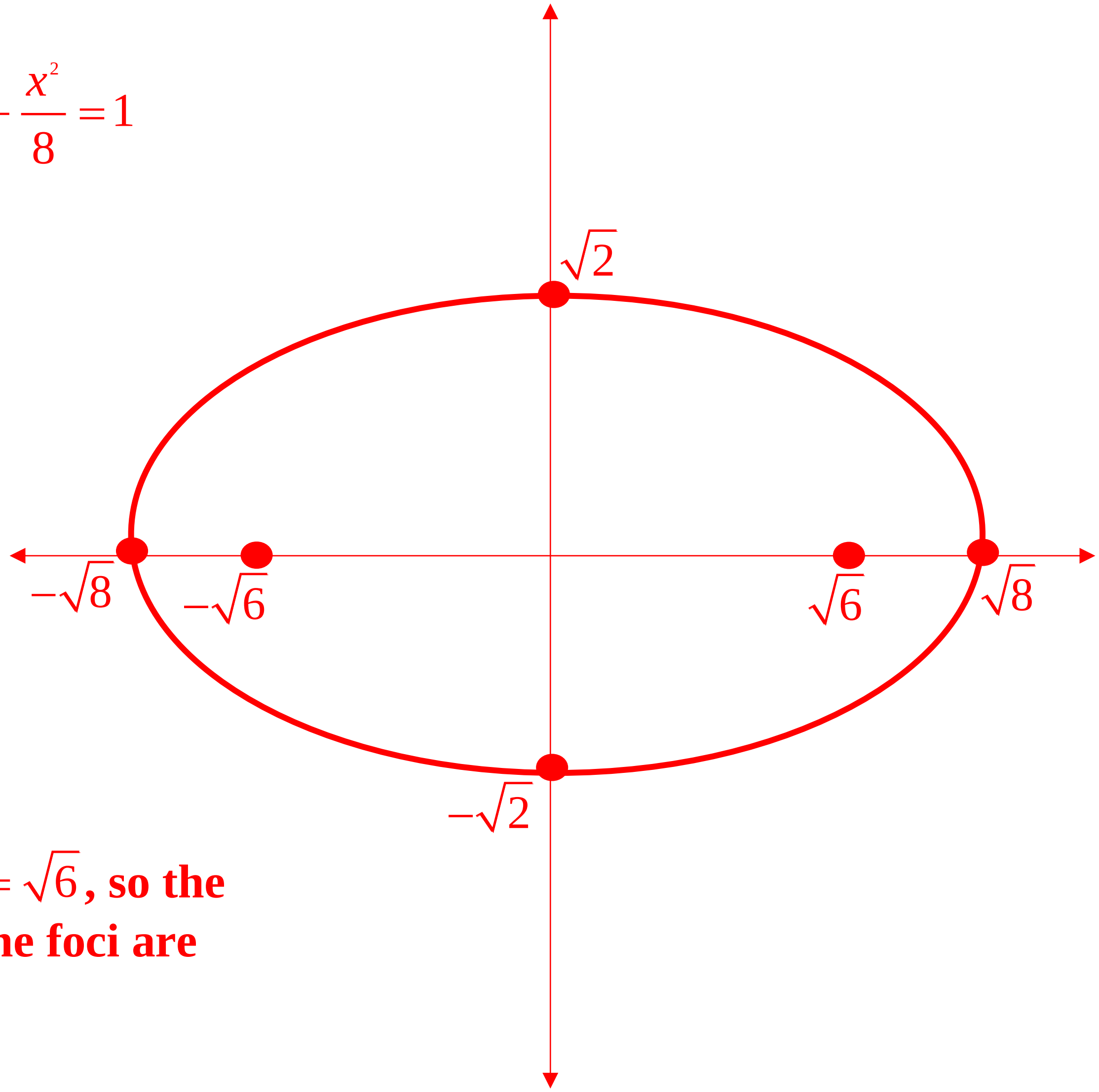
$$\frac{x^2 + 9y^2}{36} = \frac{36}{36} \Rightarrow \frac{x^2}{36} + \frac{y^2}{4} = 1$$



$c^2 = 36 - 4 = 32 \Rightarrow c = \sqrt{32}$**, so the *x*-coordinates of the foci are** $\pm\sqrt{32}$**.**

4. $4y^2 + x^2 = 8$

$$\frac{4y^2 + x^2}{8} = \frac{8}{8} \Rightarrow \frac{y^2}{2} + \frac{x^2}{8} = 1$$



$c^2 = 8 - 2 = 6 \Rightarrow c = \sqrt{6}$**, so the *x*-coordinates of the foci are** $\pm\sqrt{6}$**.**

Not necessarily centered at $(0,0)$:

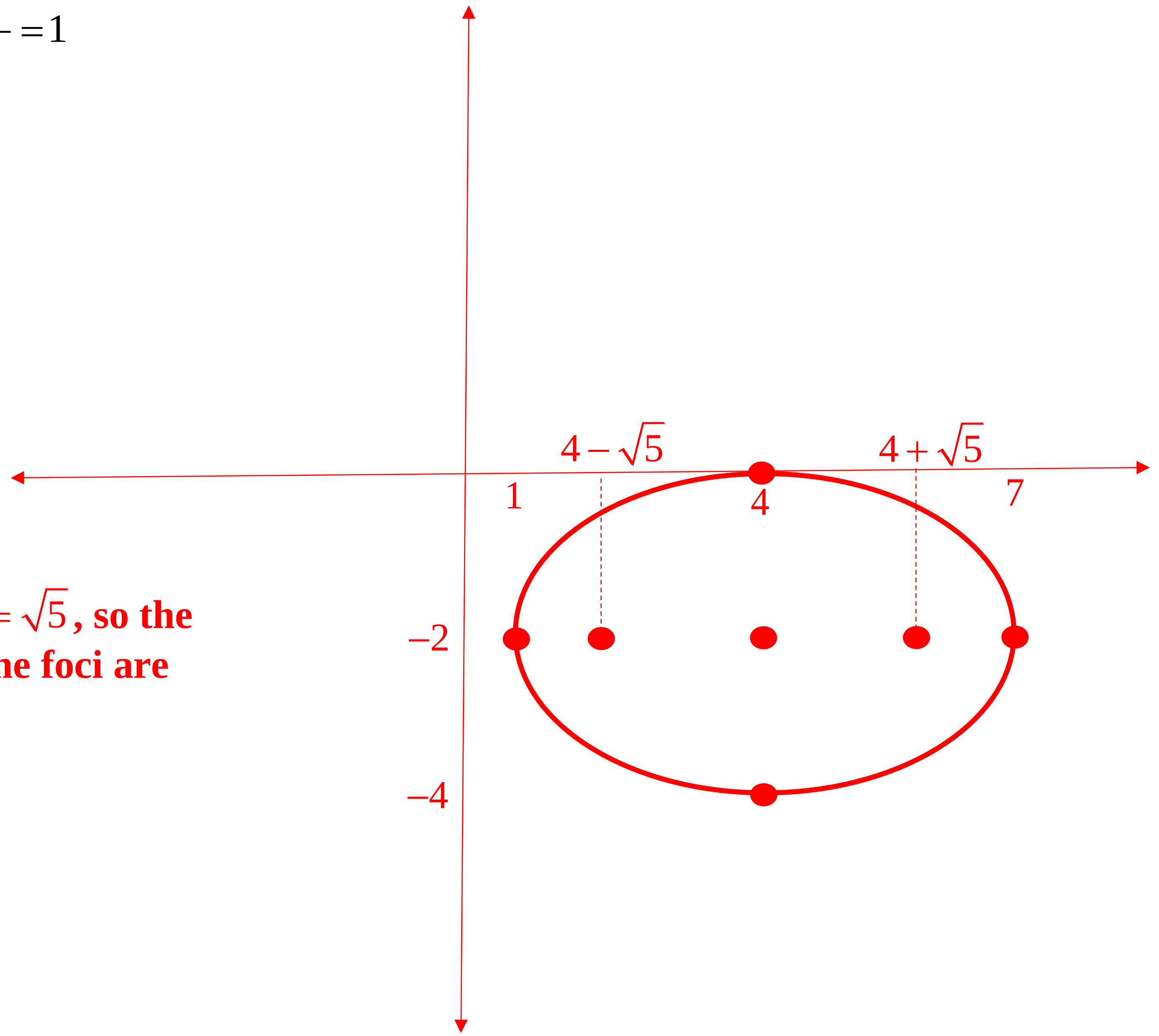
$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Or

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

Graph:

1. $\dfrac{(x-4)^2}{9}+\dfrac{(y+2)^2}{4}=1$

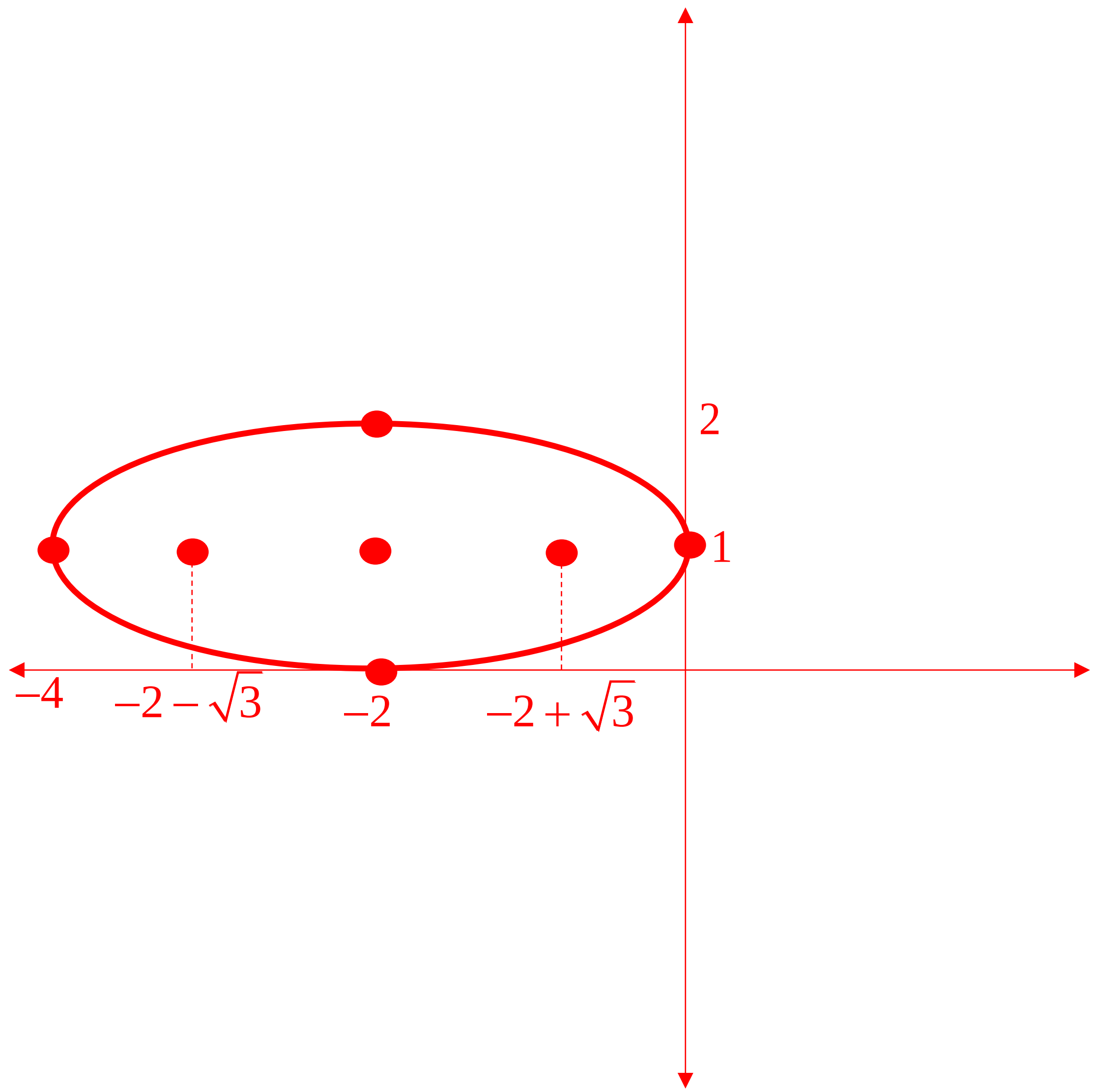


$c^2=9-4=5 \Rightarrow c=\sqrt{5}$**, so the** ***x*****-coordinates of the foci are** $4\pm\sqrt{5}$**.**

2. $x^2 + 4x + 4y^2 - 8y + 4 = 0$

$(x^2 + 4x) + 4(y^2 - 2y) = -4 \Rightarrow (x^2 + 4x + 4) + 4(y^2 - 2y + 1) = -4 + 4 + 4 \Rightarrow (x+2)^2 + 4(y-1)^2 = 4$

$\Rightarrow \frac{(x+2)^2}{4} + \frac{(y-1)^2}{1} = 1$



$c^2 = 4 - 1 = 3 \Rightarrow c = \sqrt{3}$**, so the** ***x*****-coordinates of the foci are** $-2 \pm \sqrt{3}$**.**

Find an equation for the ellipse that is described.

1. Center at $(0,0)$, focus at $(-1,0)$, and vertex at $(3,0)$.

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, **so from the vertex,** $a=3$, **from the focus,** $c=1$, **so** $b^2=9-1=8$, **and therefore an equation of the ellipse is** $\boxed{\frac{x^2}{9}+\frac{y^2}{8}=1}$.

2. Foci at $(0,\pm 2)$, and the length of the major axis is 8.

$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, **so from the foci,** $c=2$, **from the length of the major axis,** $a=4$, **so** $b^2=16-4=12$, **and therefore an equation of the ellipse is** $\boxed{\frac{x^2}{12}+\frac{y^2}{16}=1}$.

3. Center at $(2,-2)$, focus at $(4,-2)$, and vertex at $(7,-2)$.

$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$, **so from the center,** $\frac{(x-2)^2}{a^2}+\frac{(y+2)^2}{b^2}=1$, **from the vertex,** $a=5$, **from the focus,** $c=2$, **so** $b^2=25-4=21$, **and therefore an equation of the ellipse is** $\boxed{\frac{(x-2)^2}{25}+\frac{(y+2)^2}{21}=1}$.

4. Center at $(1,2)$, focus at $(1,4)$, and passes through $(2,2)$.

$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$, **so from the center,** $\frac{(x-1)^2}{b^2}+\frac{(y-2)^2}{a^2}=1$, **from the focus,** $c=2$**, from the point** $\frac{1}{b^2}=1\Rightarrow b^2=1$, **so** $a^2=1+4=5$, **and therefore an equation of the ellipse is** $\boxed{\frac{(x-1)^2}{1}+\frac{(y-2)^2}{5}=1}$.